2022

A Case Study of Classroom Practices for Additional Strategies in First Grade

Lori StClair Rhodes
University Of Alabama At Birmingham

A CASE STUDY OF CLASSROOM PRACTICES FOR ADDITION STRATEGIES IN FIRST GRADE

by

LORI STCLAIR RHODES

KELLY HILL, COMMITTEE CHAIR
HOLLAND BANSE
ANN DOMINICK
LYNN KIRKLAND
JENNA LACHENAYE

A DISSERTATION

Submitted to the graduate faculty of The University of Alabama at Birmingham,
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

BIRMINGHAM, ALABAMA

2022

A CASE STUDY OF CLASSROOM PRACTICES FOR
ADDITION STRATEGIES IN FIRST GRADE

LORI STCLAIR RHODES

EARLY CHILDHOOD EDUCATION

ABSTRACT

This study addresses first graders' conceptual knowledge exhibited in addition strategies in relation to instructional practices and teachers' beliefs in a suburban Alabama school. To address the purpose of the study, the central question was "What is the nature of the relationship between curricular practices and beliefs and students' advanced strategy use for addition in first grade?" The study employs a case study design to obtain a deeper and more comprehensive view of students' conceptual understanding exhibited in advanced addition strategies in the context of the first-grade classroom. Student addition strategies were compared to instructional practices and teachers' beliefs to examine how practices and beliefs re related to student strategy use.

Keywords: addition, instructional practices, conceptual understanding, procedural fluency, advanced addition strategies

DEDICATION

To my children and grandchildren

My favorite form of addition

ACKNOWLEDGEMENTS

I would like to acknowledge all those who have supported me in my journey. I would first like to thank my committee for guiding and supporting me. To Dr. Holland Banse, many thanks to you for sharing your expertise and investing in a stranger. To Dr. Lynn Kirkland, my friend Lynn, so many thanks for being an example to us all for how to do it all and keep on doing it. To Dr. Jenna LaChenaye, thank you for adding me to your long list of candidates eager to take advantage of your expertise. A heartfelt thank you Dr. Ann Dominick for going above and beyond, no less than you always do. Many thanks to Dr. Kelly Hill for keeping me on track. I might still be trying to lasso my big ideas.

I would also like to acknowledge the many people in my life who cheer me on—seeing more in me than I see in myself.

To the principals who have believed in me enough to let me be, especially our dear Dr. Charlotte Brown, I thank you. Charlotte, not only are you able to imagine us as better than we are, but you have the ability, a gift, to inspire us to be what you see.

Many thanks to all the teachers, colleagues, and work friends and family, especially that amazing team of teachers that started Quest and that amazing group of kindergarten teachers.

Thanks to my family, especially my mom—always proud, but never surprised. Will we ever be able to impress you?

To my children and grandchildren who have cheered me on. To Julian, Gracie, and Maddie, your Lovie is back.

To my sister, colleague, friend, Tami Puchta, thank you for always pushing me to be more. I am so glad you convinced me to fill out the first college application.

To my precious husband, thank you for always supporting me. I wish I could see myself through your eyes. Now, let’s finish this house and go on an adventure!

TABLE OF CONTENTS

LIST OF TABLES

Table *Page*

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Mathematical competence has the potential to affect many areas, including later school success, sense of self-worth, career choices, and even satisfaction in life (Gilmore, 2018; McCoy et al., 2017; OECD, 2013; Parsons & Bynner, 2005; Ritchie & Bates, 2013; Watts et al., 2014). Foundational skills in early mathematics can pave the way for later mathematical success, and counting is one of the first and most essential skills children learn in mathematics. Children build on their counting skills and typically begin progressing to more complex skills including addition and subtraction (between 1 and 20) around six or seven years old. Proficiency in addition includes the ability to flexibly use known combinations to solve unknown combinations exhibiting key understandings of the way numbers are related in our number system (National Research Council, 2001). Students show their understanding in the strategies they use to solve problems (Geary, 2004). However, many children continue to use simpler counting strategies- either lacking the key understandings or the ability to flexibly use them (Carr & Alexeev, 2011; Hopkins, 2020; Rhodes, 2019). The instructional practices used at this crucial time in development can potentially have long-lasting effects.

This case study explores classroom instructional practices for addition strategies in first grade in a well-supported elementary school. The use of case study allows for an in-depth look at how practices are implemented and how they relate to students' addition strategies. The researcher utilizes case study to provide a more complete picture of the classroom. Data gathered through observation, interviews, and documents provide context for students' addition strategies. Student data about addition strategies are gathered from individual interviews, classroom observations, and student work. Classroom practice data include observations of math lessons and teacher interviews.

Statement of Problem

Research supports that as children develop, they will "use multiple strategies and choose among these strategies adaptively" (Siegler, 2006), but this development does not occur in all children. It is well documented that children with difficulties in mathematics do not consistently adopt or adapt multiple strategies (Geary, 2013). However, research shows that it is not just students with mathematical difficulties who still use immature counting strategies into upper elementary grades (Hopkins, 2020) and even into adolescence (Rhodes, 2019). Many students continue using (immature) counting strategies past the point that "it is advantageous to do so" (Hopkins, 2020, p. 1). Instructional practices that do not encourage advanced strategy usage in early grades affect achievement for students as mathematical complexity increases in later grades.

Significance of the Study

Instructional practices affect learning, and researchers are calling for more observational studies of actual teacher practices (Ball & Forzani, 2011). Instructional practices can include curriculum, lesson content, grouping, interactions, methods, or how time is spent, among many other decisions. This study examines instructional practices influencing how addition strategies are being taught in first grade. Information about classrooms is included, specifically teachers' philosophical beliefs about how children develop and learn early math. The study has the potential to provide needed clarification about what is happening in the classroom to connect research to practice. Additionally, findings from this study may be beneficial in identifying variables for future studies adding to the body of knowledge concerning specific practices that encourage or inhibit conceptual understanding.

Benefits

In spring 2022, the Alabama state legislature passed the Alabama Numeracy Act (SB171, 2022) which outlines a plan for evaluation and intervention for schools whose students show low performance in mathematics. The proposed purpose of the act is "relating to public education:...to implement steps to improve mathematics proficiency of public school K-5 grade students and ensure that those students are proficient in mathematics at or above grade level by the end of fifth grade by monitoring the progression of each student from one grade to another, in part, by his or her proficiency in mathematics" (p. 1). A great deal of the written legislation concerns the framework for establishing leadership as well as identifying and mandating policy for schools and students in most need of support. However, the Alabama Numeracy Act does offer specific recommendations for the State Department of Education, for administrators, for instructional coaches, and for teachers. Among other mandates, the bill specifically states:

> Each K-5 teacher who is providing instruction in mathematics, with the full support of his or her principal, shall: Build fluency with procedures on a foundation of conceptual understanding, strategic reasoning, and problem solving over time; provide a learning environment that promotes student reasoning, student discourse, and student questioning and critiquing the reasoning of their peers" (p. 14); [and] an elementary school teacher should not engage in any practice that minimizes sense making and understanding of mathematics concepts. (p. 15)

The findings and their implications from this study can potentially benefit all stakeholders by illuminating a path to these aspirations. Examining practices that inhibit or encourage conceptual understanding through qualitative case study allows consumers of the research to vicariously experience (Stake, 1995) this particular case and relate to their personal situation. With a firmer grasp of what teaching for conceptual understanding looks like, stakeholders can have a clearer vision for how to translate policy mandates into classroom practice that leads to mathematical proficiency for more Alabama students.

Purpose of the Study

A better understanding of classroom practices during mathematics instruction is needed. As the research indicates the need for students to achieve mathematical proficiency by moving beyond immature counting strategies and using advanced addition strategies, it is necessary to identify which whole-group instructional practices teachers employ to encourage mature addition strategies. Numerous studies have looked at how to identify and address the needs of students with difficulties (Geary et al., 1991, 2012; Jordan et al., 2003, 2010a; Ostad & Sorensen, 2007), but few studies look at the reasons a typical student might hold on to immature counting strategies instead of moving on to more advanced strategies (Hopkins et al., 2020).

Currently, a great deal of attention, funding, and research is directed toward preschool, which reflects the line of reasoning that children are not adopting more sophisticated strategies because of a deficit in preschool skills. With this reasoning, finding the gaps and going back to preschool to fill them would be the goal. Whether or not this line of reasoning is correct, it has resulted in a great deal of research concerning preschool math. In contrast, little research is available concerning the instructional practices at the crucial time that students transition to advanced strategies.

Current research is predominately looking for causal relationships between a narrowly defined intervention and a predetermined performance outcome. In contrast,

> A case study is expected to catch the complexity of a single case…. We study a case when it itself is of very special interest. We look for the detail for interaction with its contexts. Case study is the study of the particularity and complexity of a single case, coming to understand its activity within important circumstances. (Stake, 1995, p. xi)

The exploratory nature of this study allows for unforeseeable possibilities or factors that encourage or inhibit advanced addition strategies at the crucial time that students should be making connections between counting strategies and advanced addition strategies.

Examining all the influences of the learning environment (e.g., teacher, curricula, students) allows the researcher to look beyond the practice to the evidence of the relationship between the strategy use and practice. The more holistic approach illuminates the interaction and how the practices relate to one another and the students' strategy use.

Research Questions

Central Research Question

How do teachers' beliefs and practices influence student strategy use for addition in first grade?

Sub Questions

- What addition strategies do students use to solve early addition?
- Is addition treated as a procedure or as an understanding?

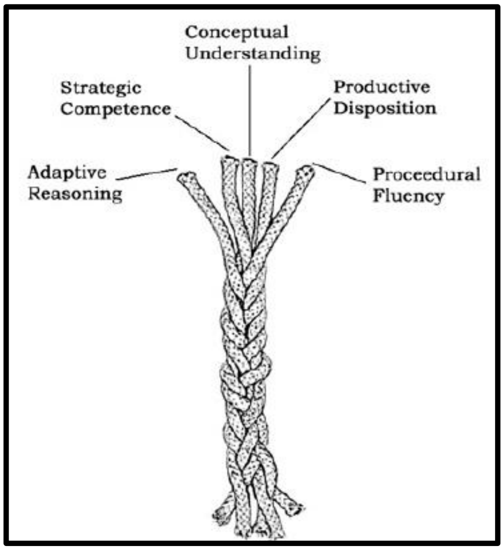
- What conceptual knowledge about addition do students exhibit?
- What are teachers' beliefs about the role and impact of conceptual knowledge in addition strategies?
- What curricular practices are teachers enacting to encourage or inhibit students' conceptual understanding needed for advanced addition strategies?
- How do teachers' practices align or diverge from their stated beliefs?

Theoretical Framework

The researcher in this study has learned and worked as a constructivist educator for more than 30 years. The essence of constructivism is the notion that students construct their own meaning, which is in opposition to the long-held belief that knowledge is a matter of transfer from the giver of knowledge to the child who is the receiver. In Piagetian constructivist theory, learning is a cognitive process of integrating new knowledge with prior knowledge (Kamii & Ewing, 1996). The student constructs and reconstructs relationships internally. Each child invents knowledge instead of discovering something that externally exists. In Vygotsky's (1978) social constructivism, meaning and understanding grow from social interactions. With adult guidance and collaboration with peers, students can learn at a level is above their developmental level in a zone of proximal development. The research was also guided by The National Research Council publication, *Adding It Up: Helping Children Learn Mathematics* (NAP, 2001). Mathematical proficiency, which the authors believe is necessary for anyone to learn mathematics, is illustrated as five interwoven components sometimes represented as the strands of rope (Figure 1).

Figure 1

Intertwined Strands of Proficiency

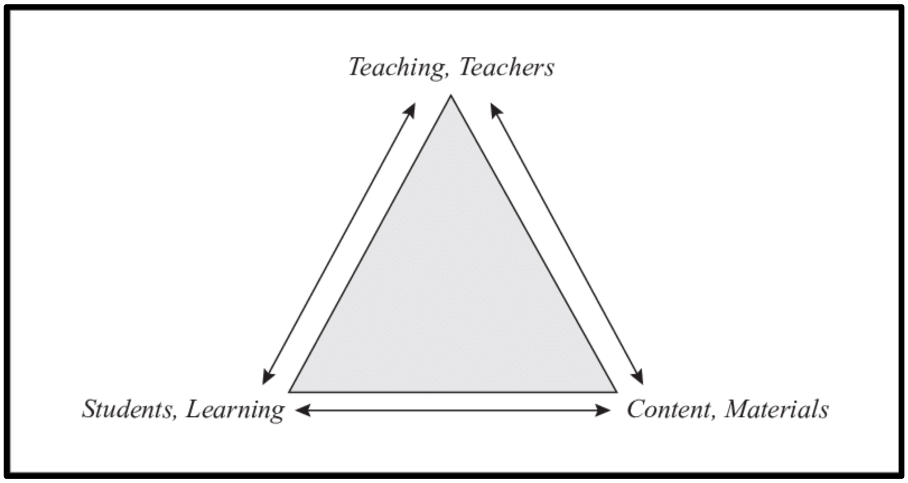


Note: From Adding It Up: Helping Children Learn Mathematics by National Research Council, 2001, p. 5. Copyright 2001 by The National Academies Press. Reprinted with permission.

The authors of *Adding It Up* (NAP, 2001) also offer a model describing the interaction between teacher, students, and content. Murata et al. (2012) offer a model to descrive the interaction (Figure 2). The researcher based this study on the belief that all three sides of the triangle must be considered in examining how students learn addition strategies.

Figure 2

Interactions Among Teacher, Students, and Content



Note: From Murata et al., 2012, p. 620.

Definition of Terms

The purpose of this section is to define key terms and their definitions in the context of this study. The definition explains the meaning of each term as it pertains to this study.

algorithm: a process or set of rules to be followed in calculations or other problem-solving operations.

algorithmic thinking: a way of getting to a solution through the clear definition of the steps needed

cardinality: the knowledge of how many things are in a set and the number name for that quantity

conceptual knowledge (CK):

> Conceptual knowledge is characterized most clearly as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of

> information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network. (Hieber, 1986, pp. 3–4)

conceptual understanding (CU): defined by the National Research Council (2001) as "an integrated and functional grasp of mathematical ideas" (p. 118)

early childhood: children between infancy and eight years old.

early addition: problems with both addends less than 20

hierarchical inclusion: Understanding that all numbers preceding a number can be or are systematically included in the value of another selected number

MD: Mathematical Difficulties

MLD: Mathematical Learning Disability

mathematical proficiency: the ability to competently blend and apply the five interwoven components, which are conceptual understanding, productive disposition, strategic competence, procedural fluency, and adaptive reasoning.

NCTM: National Council of Teachers of Mathematics.

part-part-whole: the idea that numbers can be split into parts.

procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately (National Research Council, 2001).

procedural knowledge: knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

procedures: a sequence of steps or actions that lead to the correct answer or solution.

Limitations

As the purpose of the case study is to evaluate a small population to thoroughly examine classroom practices, the findings will not be generalizable to a larger sample.

Additionally, the exploratory nature of the study may provide questions for later empirical research but does not allow for causal relationships to be drawn between student understanding and classroom practice.

Delimitations

The researcher chose a small sample to study various aspects of the classes in depth. The case for the study is an elementary school where mathematical proficiency is valued and emphasized and teachers and students are supported to learn and enact best practices. The school was chosen because of its standards-based curriculum and supportive environment for best math practice. The administration is knowledgeable, encouraging, and supportive of mathematical practices based on conceptual understanding. The school, and its district, provide professional development regularly to all teachers in standards-based math practice. The district employs a math coach at each school to support teachers with implementation and teachers are provided with sufficient resources. Findings from this study might provide an ideal for mathematics education, but it is not typical for most Alabama schools. The school's percentage of free and reduced lunch (32%) is also below the typical Alabama school. While the school does not represent a typical Alabama school, the researcher chose the school because it offers the optimal setting for the researcher to isolate the relationship between teacher instructional practices and the impact on students with minimum negative inferences to obscure the influence of the teacher.

Organization of the Study

The researcher hopes to present a holistic account of teaching and learning involved in the development of addition strategies in first grade. In Chapter One, the researcher provides an introduction, overview, background information, and research questions guiding the study. Within Chapter Two, the researcher outlines the significant research leading to justification for the study. Within Chapter Three, the researcher presents the methodology for the study. The researcher will detail the analysis and present findings in Chapter Four. In Chapter Five, the researcher will offer discussion and research implications.

CHAPTER II

LITERATURE REVIEW

What happens in the early math classroom matters. Math matters and classroom practices matter. This chapter will look at the research detailing the importance of early math success, how early math success is defined, and a specific area of math content needed for math success. The researcher explains the need for proficient mathematicians in the world and the need for all to have the opportunity to be mathematicians. The researcher explains mathematical proficiency as the goal for mathematics education. Specifically, research which points to early addition strategies as one area of particular interest in leading to later math success or difficulty is reported. A thorough explanation of advanced addition strategies along with the relation to conceptual understanding and procedural fluency follows.

For the past three decades, as research has shown more about how math is learned, math experts and reformists have refined thinking about instructional practices that foster mathematical content knowledge, dispositions, and student practices. Learning Trajectories have been offered as a bridge from what is known about how students learn to practical applications of that knowledge in the classroom. The study will consider what instructional practices are exhibited in two general education first-grade classrooms. Because looking at the classroom holistically as opposed to only one aspect will give a more comprehensive view of how student thinking is supported, encouraged, or inhibited, the study will look at many facets of the instructional environment. The chapter will conclude with the justification of the study.

Math Matters

There is a growing awareness of the importance of mathematics for the individual as well as for society collectively. Those who do not have mathematical knowledge can be denied access to opportunities professionally and to a better life. Health, wealth, and quality of life are affected by mathematical abilities (Gilmore, 2018; OECD, 2013; Parsons & Bynner, 2005). For the individual, mathematical proficiency is often the gateway to opportunities, and success in mathematics can have long-lasting effects including the cost of college, the range of career opportunities, and even the level of salary. Collectively, competent, and inventive problem-solvers are needed to address and overcome the problems facing our world. To address these complex global problems, the world needs people who are proficient in areas requiring mathematical knowledge. Lack of access to quality math instruction presents the problem of equity from the classroom to the global level.

Equity

Given the importance of mathematical success, lack of mathematics skills becomes a matter of access and achievement at multiple levels. At the personal and individual level, the belief that some people are just not good at math encourages some people, especially young girls, to equate struggle with lack of ability, causing many to abandon math (Boaler, 2002).

At the community and national levels, students are not offered the same opportunities, and society reaps the social and economic consequences (Dunphy, 2014). The National Math Advisory Panel stated, “Unfortunately, most children from low-income backgrounds enter school with far less knowledge than peers from middle-income backgrounds, and the achievement gap in mathematical knowledge progressively widens throughout their pre-K-12 years" (2008, p. xvii). Even more unfortunate is the reality that as students move through school this gap in mathematical knowledge continues to grow alongside the growing gap between higher and lower resource communities (Bachman et al., 2015; Reardon, 2013). At the global level, lack of mathematics skills could limit access for individuals as well as countries' access to a global economy where "all people in each country" are able to participate (Clements, 2020, p. 1).

Early Math Matters

"Math is a language best learned early" (Clements and Sarama, 2020).

Early math has been shown to be particularly important. Over the past two decades, research has shown that early math matters for future success in school. Early math knowledge has been shown to predict high school graduation (McCoy et al., 2017; Watts et al., 2014) and arithmetic and number knowledge at age seven has been shown to predict socioeconomic status at age 42 (Ritchie & Bates, 2013).

Research has shown early math skills as a predictor for later math achievement as well as educational achievement in general. A study by Duncan et al. (2007) showed school entry math skills to be the strongest predictor for later achievement in reading as well as math. More specifically, certain math skills and abilities have shown greater relevance. Teaching more advanced content is more important than teaching basic content (Engel et al., 2013; Le et al., 2019). In particular, advanced numeracy skills have been found to be the most predictive of later mathematics ability (Geary, 2013; Jordan et al., 2010; Nguyen, 2016). Number sense in kindergarten and first grade is especially important in predicting applied problem solving in later grades (Jordan, 2010).

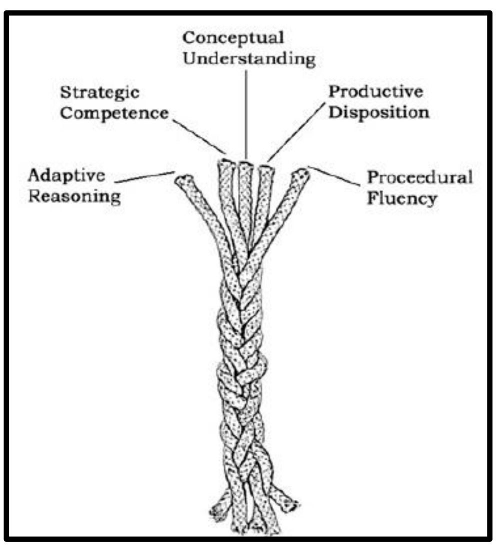
Mathematical Proficiency

What is Mathematical Proficiency?

Given the significance of mathematical success, especially in the early years, it is important to understand the meaning of mathematical proficiency. Though there are numerous interpretations of mathematical proficiency, for this study, the research is guided by The National Research Council publication, *Adding It Up: Helping Children Learn Mathematics* (NAP, 2001). Mathematical proficiency, which the authors believe is necessary for anyone to learn mathematics, is illustrated as five interwoven components sometimes represented as the strands of rope (Figure 1).

Figure 1

Intertwined Strands of Proficiency



From Adding It Up: Helping Children Learn Mathematics by National Research Council, 2001, p. 5. Copyright 2001 by The National Academies Press. Reprinted with permission.

The strands of the rope include the following:

- conceptual understanding—comprehension of mathematical concepts, operations, and relations;
- procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- strategic competence—ability to formulate, represent, and solve mathematical problems;
- adaptive reasoning—capacity for logical thought, reflection, explanation, and justification; and
- productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (National Research Council, 2001, p. 116)

The strands of the rope represent the interconnectedness of the strands of competence. To have a deep understanding, students must connect and apply to productively solve problems. While all the strands are vital, the researcher in this study will look closely at the special relationship between conceptual understanding and procedural fluency.

Conceptual Knowledge

The National Research Council states that "conceptual understanding refers to an integrated and functional grasp of mathematical ideas" (p. 118). The distinction between conceptual understanding and conceptual knowledge is not definitive, and literature sometimes treats conceptual knowledge as actually encompassing both. Star (2005) noted, "The term conceptual knowledge has come to encompass not only what is known (knowledge of concepts), but also one way that concepts can be known (e.g., deeply and with rich connections)" (p. 408). This definition is based on Hiebert and LeFevre's definition in the seminal book edited by Hiebert (1986), as follows:

> Conceptual knowledge is characterized most clearly as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network. (pp. 3–4)

The importance of conceptual understanding to success in mathematics is widely recognized in the literature (Rittle-Johnson & Schneider, 2015). Benefits of conceptual knowledge include flexible problem-solving, facility in procedure selection and novel situations, reasonableness of solutions as well as long-term benefits such as deeper and longer-lasting mathematical understanding. A deeper understanding of conceptual knowledge will be of value for researchers "espousing a range of theoretical perspectives who are interested in mathematical thinking, learning, and instruction" (Crooks & Alibali, 2014, p. 346).

Procedural Fluency

The National Research Council refers to procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" and procedural knowledge as the "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (p. 115). In Rittle-Johnson and Schneider's (2001) review of literature, the authors claim that there is "a consensus that procedural knowledge is the ability to execute action sequences (i.e., procedures) to solve problems." Clearly, there is a distinction between procedural fluency that is described as simply following steps accurately and the ability to flexibly solve problems. However, as the content of many studies reveal that the term procedural knowledge is used to mean flexibly using knowledge procedures, the researcher has tried to represent the literature accurately in making claims clarifying whether the claims address fluency or simply the ability to carry out the procedures. For this study, procedures are a sequence of steps or actions that lead to the correct answer or solution (Hiebert, 1986).

Conceptual Knowledge WITH (not vs.) Procedural Knowledge

As there is debate concerning definitions of conceptual knowledge and procedural knowledge (Rittle-Johnson & Schneider, 2015), it is important to note that, for this study, the researcher is referring to not only the type of knowledge, but also the quality of the knowledge. Procedural knowledge and conceptual knowledge have sometimes been treated as opposite ends of the spectrum (Saxon & Cakir, 2006), and the significance of the order, amount, and influence of each has been debated (Canobi, 2009; McNeil et al., 2012; Rittle-Johnson & Alibali, 1999; Schneider et al., 2011). However, recent research points to a more reciprocal/symbiotic relationship much more in line with the intertwined model offered by NAP (2001). Rittle-Johnson et al. (2001) proposed a model of iterative development, where gains in one type of knowledge positively affect gains in the other type of knowledge which in turn lead to more gains in the first.

The researcher anticipated that the relationship between procedural knowledge and conceptual knowledge could emerge as a theme in this study. It is unlikely that the young children in this study would be using the procedural knowledge usually associated with the steps in a formal algorithm. Yet, young children use a strategic process, knowing how to do a strategy to solve a problem, as a form of procedural knowledge (Clements, 2021). These solution strategies can be important in learning mathematics and could play a role in developing mathematical proficiency.

Addition Strategies

Strategy Theory

Rhodes et al. (2019) described the Strategy Choice Model as outlining "the processes underlying people's use of one problem-solving approach or another to solve any particular problem as well as the mechanisms that govern developmental change in the mixture of strategies used during problem solving." In this model, the student must balance many factors including speed and accuracy, demands of the problem, and cognitive abilities in choosing a solution strategy. Strategy use has been shown to be a specific predictor for later mathematics performance (Carr & Alexeev, 2011; Jordan, 2007; Jordan et al., 2010; Rhodes et al., 2019). Even the willingness to attempt strategy use showed promise (Carr and Alexeev, 2011). Differences in strategy choice have been shown to indicate ability with typically developing adolescents (Rhodes et al, 2019).

Simple/Advanced or Mature/Immature Addition Strategies

Most children use their knowledge of counting in their early addition strategies. Students typically progress in a typical way (whether in waves, stages, or iteratively is debated) from counting all objects (* * * + * * * *) to counting on from one set (3 + * * * *) to using number sense/knowledge to solve problems (3 + 3 = 6, so 3 + 4 =7). Addition strategies have been named and labeled simple and advanced with counting strategies from counting all to counting on from the larger number referred to as less advanced, simple, or immature. Advanced strategies include decomposition and retrieval, and these advanced addition strategies have been shown to predict later math achievement (Geary, 2004). The advanced mathematical concepts required to decompose justify its inclusion as an advanced or mature addition strategy. Prior knowledge of part-part-whole relations of numbers, cardinality, and associativity have been proposed as skills requisite for decomposition (Clements, 2020). While the progression from simple to advanced is well documented, many students seem to get “stuck” counting on from one set and do not ever utilize advanced addition strategies.

Retrieval

To discuss how retrieval will be treated in this study, it is important to specify the definition of fluency used. Computational fluency is described as the ability to compute with accuracy, flexibility, and efficiency (Adding It Up, 2001). Fluency is not just speed and accuracy, but also the flexibility to use relationships in novel situations with speed and accuracy. For this study, retrieval without further evidence of understanding will not be counted as a strategy as it is impossible to know from an answer alone if the child has the conceptual knowledge to derive the fact and is now fluent or has simply memorized the combination as a fact. As memory is a key factor in mathematical fluency, this study does not refute the role of memory, but uses a definition of strategy more in line with Bisanz and LeFevre (1990), who discriminated strategies from other cognitive procedures.

Persistence/Overuse of Immature Counting Strategies

One of the most fundamental and essential skills that young children learn in early mathematics is counting. As such, counting is heavily emphasized in early education. In fact, research on counting strategies in preschool suggests that an overemphasis on counting strategies as the main or only strategy can have negative consequences, such as a delay in development of more advanced mathematical skills (Cheng, 2012; Contreras, 2002). The children prefer the comfort of the counting strategy and are reluctant to try new and more advanced strategies (Cheng, 2012). This reluctance can delay important conceptual understanding such as part-part-whole relationships needed to compose and decompose (Baroody & Cannon, 1984; Cheng & Chan, 2005).

Immature Counting Strategies

Geary et al. (2004) and others have done extensive research on children with mathematical difficulties (MD) or mathematical learning disabilities (MLD) who hold on to counting strategies and how that affects later math achievement, finding that children with MLD and low-achieving children use counting strategies for more years and show evidence of poor executive function processing (Geary, 2004; Geary & Brown, 1991; Wu 2008). However, studies show that the protracted use of counting-based strategies for simple addition is more prevalent than generally viewed and is not only a concern for children with a mathematics learning disability or persistent low achievement (Carr & Alexeev, 2011; Hopkins, 2020; Rhodes, 2019). Rhodes (2019) showed that even some adolescents continue to rely on immature counting strategies, and these same students were also struggling with broad mathematical achievement. Contreras (2002) found that 13% of pre-service secondary mathematics teachers continued to use counting strategies in their computations. There are many students who are not MD or MLD who continue to use immature counting strategies past the time “when it is advantageous to do so” (Hopkins, 2020). These students who continue to use counting strategies past the advantageous point exhibit a lack of mathematical proficiency. Although the students might exhibit a limited procedural fluency, the procedure they use is immature and ceases to be efficient. Carr and Alexeev (2011) contended that "to have improved chances of success in mathematics, children need to begin second grade using cognitive strategies at a higher rate than most children currently do" and that developmental trajectories and differences in later competency were significantly influenced by fluency and accuracy as measured in the second grade, suggesting that early skills have long-term consequences for students. The current study hopes to examine what is happening as these students should be transitioning to more advanced strategies before the second grade.

Use of Manipulatives (Fingers)

One aspect of immature counting strategies that is often noted is the use of manipulatives, including fingers. Use of manipulatives, especially fingers, does not generalize well as the numbers in the problem get larger because of the limitation of the number of fingers or cumbersomeness of large sets of objects. Fingers can also encourage unitary, or one-by-one counting, which might delay advancing to more part-part-whole understanding (Murata, 2004; Murata & Fuson, 2006). While some might be tempted to discourage manipulative use at all, the use of manipulatives can be effective if they are used to encourage students to make connections (Lee & Ginsburg, 2009), and the manner and timing of which manipulatives are used is important (Bodrova & Leong, 1996).

Counting On as Algorithm

Webster broadly defines an algorithm as "a step-by-step procedure for solving a problem or accomplishing some end." Algorithms are necessary for efficient mathematical computation. However, an argument has been made that the introduction (and overuse) of algorithms in multi-digit addition/subtraction without regard for conceptual understanding can be harmful and have long-lasting negative effects (Kamii

& Dominick, 1998). It is not that algorithms should not be utilized; it is when algorithms should be introduced and encouraged, who oversees the thinking, and if understanding is encouraged. Students might stop trying to make sense if they have an answer. Student invention should also be encouraged before algorithms are taught (Clements, 2021). This study aims to look at the student strategy considering invention versus procedure.

Instructional Practice

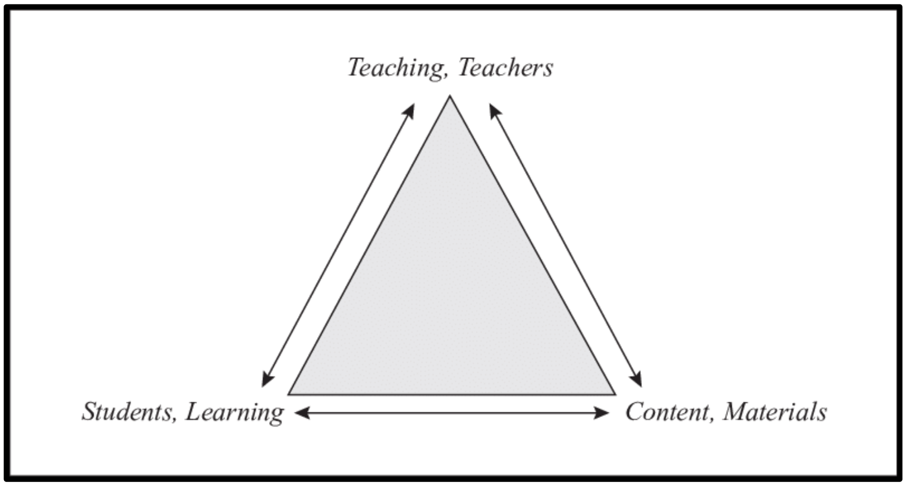
To fully understand student strategy, it is important to note how students are being encouraged to use strategies, or if accuracy only is emphasized. Looking at where, when, and how conceptual understanding and procedural fluency are emphasized in the classroom is important in learning where, when, and how students move to advanced strategies. To fully examine classroom practices, the researcher seeks to look at more than the program or the teacher or the specific practice, but all of it taken together as the environment for learning.

What Happens in the Classroom Matters

"An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (*Principles to Action*, 2014, p. 7). As important as preschool math instruction has shown to be, we are cautioned that the predictive power is decreased if "subsequent environments do not adequately build upon the competences students have already developed" (Nguyen, 2016). In *Adding it Up* (2001), the authors broadened the scope of teaching to include the interactions among teachers and students around content in a model with each as a side of a triangle, as depicted by Muraski et al. (2012) (Figure 2).

Figure 2

Interactions Among Teacher, Students, and Content



Note: From Murata et al., 2012, p. 620.

Not only are the teachers' beliefs, knowledge, decisions, and actions thought to affect what is taught and learned, but also the students' expectations, knowledge, interest, and responses are considered crucial. Math content, represented in different ways such as curricular materials or tasks, make up the third side of the triangle in which all sides interact and affect the other sides. For example, the teacher must decide how to use the task, the students must engage with the teacher and the task, and with each other. The context in which the learning occurs is also relevant. The context includes all parts of the learning environment that influence the instruction, such as educational practices, policies, assessments of teachers and students, leadership, etc. This section of the Literature Review will look at specific influences and recommendations concerning what happens in the learning environment affecting the way addition is taught.

Research on How Math is Taught for How Children Learn

A great deal of research affecting mathematics education has involved intervention studies from a medical or psychological paradigm rather than as a social construct (Gervosoni, 2018). Such research includes brain research, which leans toward an information-processing view of learning. Current brain research is beyond the scope of this study. However, while the social context of the learning environment is not considered, a review of the literature shows that the information processing model offers suggestions for instruction which include comparing solution methods, self-explanation, and opportunities for exploration before instruction (Rittle-Johnson & Schneider, 2011).

Learning Trajectories Approach

Clements and Samara (2020) synthesized research about elementary mathematics learning into learning trajectories. Of particular interest to this study are the trajectories concerning counting and addition. The Clements and Sarama developmental model moves beyond simply offering a path or development progression where students move through levels of increasingly sophisticated thinking. It also includes a goal and practical classroom suggestions, instructional tasks, and strategies to help students move toward meeting the goal.

In Clements and Sarama's Learning Trajectories, the Addition Trajectory progresses from counting strategies to part-part-whole understanding to numbers in numbers to deriver to problem solver. Ages and progressions that pertain to multi-digit addition and subtraction are included but are not considered in this study. At the "Numbers in Numbers" level, the description is as follows:

> Evidence of the next level is when a child recognizes that a number is part of a whole and can solve problems when the start is unknown with counting strategies.

> For example, when asked, "You have some balls, then you get 4 more balls, now you have 9. How many did you have to start with?" this child counts, putting up fingers, "5, 6, 7, 8, 9." Looks at fingers, and says, "5!"

At the Deriver level, a child can use flexible strategies and derived combinations (for example, "7 + 7 is 14, so 7 + 8 is 15") to solve all types of problems. For example, when asked, "What's 7 plus 8?" this child thinks, 7 + 8 = 7 + (7 + 1) = (7 + 7) + 1 = 14 + 1 = 15. A child at this level can also solve multi-digit problems by incrementing or combining 10s and ones. For example, when asked "What's 28 + 35?" this child thinks, 20 + 30 = 50; 50 + 8 = 58, 2 more is 60, and 3 more is 63. Combining 10s and ones, 20 + 30 = 50, 8 + 5 is like 8, plus 2 and 3 more, so it's 13, and 50 + 13 is 63.

The problem solver would indicate procedural fluency given the flexibility of strategy use. "As children develop their addition and subtraction abilities, they can solve all types of problems by using flexible strategies and many known combinations. For example, when asked, "If I have 13 and you have 9, how could we have the same number?" this child says, "9 and 1 is 10, then 3 more to make 13. 1 and 3 is 4. I need 4 more!"

Because the age for the necessary understanding for part-part-whole in progressing from counting strategies to deriving and problem-solving is six to seven years, first grade is the most logical grade for this study.

Clements (2021) offered recommendations in moving from counting to problem-solving, specifically working with different problem-types. The same recommendation is echoed in Common Core Math Standards.

Possible Aspects of Learning Environment and Practices that Inhibit Use of Advanced Addition

Timing

Students must have some conceptual understanding to decompose. On Clements et. al (2021) trajectory, one major requisite understanding is part-part-whole understanding. To understand that 5 can be decomposed into 2 and 3, a child must understand that 2 and 3 are inside of 5, so to speak. If this conceptual knowledge is not secure before the child learns a procedure to get an answer, then it is not likely there can exist the supportive relationship between conceptual understanding and procedural fluency. Crafting procedural lessons to encourage students to notice underlying concepts can promote a stronger link from improved procedural knowledge to gains in conceptual knowledge (Rittle-Johnson & Schneider, 2011). Yet sometimes the answer becomes paramount, and students are not encouraged.

Over Teaching of Counting

Encouraging the use of counting beyond a certain point has shown to have long-lasting negative effects in preschool studies. “Continued encouragement of counting as children’s primary and unitary strategy” may delay children’s development of more advanced mathematical skills (Cheng & Chan, 2005) as they are reluctant to try new and more advanced strategies. This study considers if these low-level counting strategies are still being encouraged in first grade.

Role of Invention

For decades, there has been debate about the significance of the role of student strategy invention in “accuracy, problem-solving ability, base-ten number concepts, and flexibility of transferring knowledge to novel situations. (Carpenter et al., 1998; Clements & Samara, 2020). Rittle-Johnson (2006) suggested that it is the importance of engaging in cognitive processes and not the invention itself. This study considers the role of student strategy invention in the two classrooms.

Count-On Strategy Taught as an Algorithm

In the researcher's experience as a primary mathematics coach, students often explain their addition strategy as “putting the big number in my head, and then counting on” with or without fingers or other manipulatives. The same thinking behind this procedure can be shown on a number line by putting your finger on the larger number and counting on by ones or writing the first number and drawing and counting tally marks for the second number, and so on. Even though the representation changes, the strategy is the same counting-on strategy. Although the debate about the teaching of algorithms, specifically if conceptual understanding is not secure, is not new, it is still pertinent today (Clements, 2021). This study considers if the counting-on strategy is taught and/or implemented algorithmically.

Supportive Classroom Environment and Practices

Although "the critical interaction in education is between the teacher and the student, with student learning reliant upon teacher instructional practices” (Ball & Forzani, 2011), the teacher does not work in a vacuum and has many influences (e.g., curriculum, mandates, assessment pressures). Also, the classroom learning environment can intentionally and unintentionally contribute to instruction. So, while the teacher may be the most critical, there is value in examining the classroom as a whole and not just the teaching practices. Since this study is exploratory in nature, the aspects of the classroom environment that affect the teaching and learning will be allowed to emerge. The learning environment might include interactions between students and teacher and content, use of time, choice and decisions, materials, or dialogue.

Research supports the notion that what teachers do in the classroom matters. A knowledgeable and responsive adult is critical to a high-quality educational environment (National Research Council, 2009; Sarama & Dibiase, 2004). Research shows differences in mathematical achievement are accounted for by differences in mathematical instruction (Gordon et al., 2006); yet, there is little research about specific skills and practices of what this looks like in high-quality instruction (Walkowiak, 2014). More observational studies are needed including research of actual teacher practices in the classroom to complement commonly used self-reports (Fishman et al., 2003) and in-depth studies of specific instructional practices (Ball & Forzani, 2011). Reports from the current study will contribute to observational studies, and also provide guidance for more in-depth observational and experimental studies in the future.

Math Curriculum

Research has shown that the math curricula vary in many ways and suggests that some curricula are more effective than others in improving math achievement and interaction between teacher and curriculum may mediate the effects (Agodini & Harris, 2016). For this reason, the prescribed curriculum, at least as much as the lessons observed, will be part of the classroom evidence. The math curriculum used at SouthBridge School is EnVision Mathematics Alabama (Savvos Learning Company, 2022). The U.S. Department of Education’s Institution of Education Sciences (2016) described enVisions Mathematics as follows:

> EnVisionMATH, published by Pearson Education, Inc., is a core mathematics curriculum for students in grades K–6. The curriculum aims to help students develop an understanding of mathematics concepts through problem-based instruction, small-group interaction, and visual learning, with a focus on reasoning and modeling. Differentiated instruction and ongoing assessment are used to meet the needs of students at all ability levels. Within each grade, the curriculum is organized around clusters of Common Core standards and consists of 120–130 teacher-led lessons, with the intention that one lesson is completed per day. Each lesson includes daily review and a small-group, problem-based activity, followed by guided and independent, paired, or small-group practice activities. Instructors use daily assessments to track student progress and enable targeting of additional practice and homework activities for students that need more support. Lessons are organized into a customizable sequence of topics and use texts, workbooks, manipulatives, online web-based materials, and technology within group and individual activities.

One study (Agodini et al., 2013), cited by enVision, compared the program to three other prominent programs, showing that enVision considers the teacher's role as explaining, teaching, and guiding; however, the text offers "minimal description of teacher actions"

I-Ready. "I-Ready is an online program for reading and/or mathematics that will help your student's teacher(s) determine your student's needs, personalize their learning, and monitor progress throughout the school year. i-Ready allows your teacher(s) to meet your student exactly where they are and provides data to increase your student's learning gains. i-Ready consists of two parts: Diagnostic and Personalized Instruction.
i-Ready Personalized Instruction provides students with lessons based on their individual skill level and needs, so your student can learn at a pace that is just right for them. These lessons are fun and interactive to keep your student engaged as they learn" (Curriculum Associates, 2022).

XtraMath. "XtraMath is an online math fact fluency program that helps students develop quick recall and automaticity of basic math facts. Students with a strong foundation have greater confidence and success learning more advanced math like fractions and algebra" (XtraMath, 2022).

Summary

Research shows that math success is important and that early math performance predicts later math success. Recent research suggests that many students are not moving past immature addition strategies, and the preponderance of the research concerns students with mathematical difficulties. Research and theory suggest what students need to be proficient with addition strategies, and the time that students generally start using advanced strategies. Yet, little research has been done at the exact place where students would be most likely to move to advanced addition strategies.

Research tells us that what happens in the classroom matters. Therefore, it would benefit the body of research to have a better understanding of what is happening in the general education first grade classroom at the time when more advanced addition strategies should emerge.

CHAPTER III

PLAN OF INQUIRY

Within Chapter Three, the researcher describes and explains the approach and methods employed in this qualitative case study, followed by the measures and methods of analysis that were used (Stake, 1995; Yin, 2012). The researcher outlines the methods of the study and explains how case study is most appropriate for answering the following research questions:

Central Research Question

How do teachers' beliefs and practices influence student strategy use for addition in first grade?

Sub-Questions

What addition strategies do students use to solve early addition? Is addition treated as a procedure or as an understanding? What conceptual knowledge about addition do students exhibit? What are teachers' beliefs about the role and impact of conceptual knowledge in addition strategies? What curricular practices are teachers enacting to encourage or inhibit students' conceptual understanding needed for advanced addition strategies? How do teachers' practices align or diverge from their stated beliefs?

Philosophical Assumptions

According to Creswell and Creswell (2018), qualitative researchers "support a way of looking at research that honors an inductive style, a focus on individual meaning, and the importance of reporting the complexity of the situation." Merriam (2002) explained that learning how individuals experience and interact with their social world, the meaning it has for them, is considered an interpretive approach. The researcher in this study adopts a constructivist framework for the study in that the focus is on specific contexts in which people live and work (teach and learn) in order to understand the historical and cultural (educational) settings of the participants. Schwandt (2000) suggested a basic assumption of the constructivist paradigm is that knowledge is socially constructed by people involved in the research process and that researchers should make the effort "to understand the complex world of lived experiences from the point of view of those who live it." The interpretivist/constructivist case study researcher holds to the view that knowledge is not discovered, but is constructed (Stake, 1995). The researcher in this study solicits the views and observes the actions and interactions of participants with the intent to interpret and understand. Creswell and Creswell (2018) further explained that researchers "recognize that their own background shapes their interpretation, and they position themselves in the research to acknowledge how their interpretation flows from their personal, cultural, and historical experiences." The researcher in this study explains the role of the researcher and specific lens through which the experiences will be interpreted. Another assumption in qualitative research is that, instead of starting with a theory to be tested, the process is inductive, and meaning is generated by the inquirer based on the data collected in the field.

The researcher uses open-ended questions and observations to allow students and teachers to present their own perspectives. Data analysis is allowed to reveal classroom practices that potentially support or inhibit student development of conceptual understanding pertaining to addition strategies. The qualitative approach allows the researcher to better capture the cognition involved in addition strategies as well as the complexity of the classroom practices. In this study, the researcher purposefully lets themes emerge in order to allow for or see beyond what is expected which allows the researcher to examine particular practices that encourage, inhibit, or otherwise relate to the development of more advanced strategies. Additionally, qualitative data can capture the unexpected such as spontaneous strategy use and hidden curriculum.

Research Design

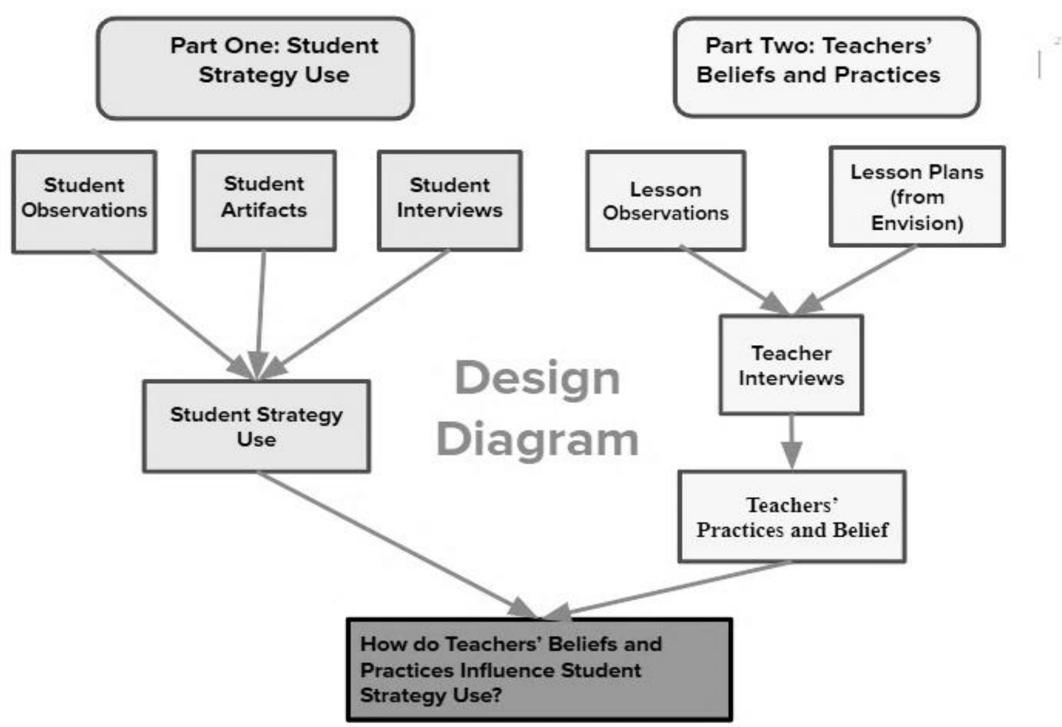
The research design is case study. Case studies are a design of inquiry found in many fields in which the researcher develops an intensive description and analysis of a case which can be a program, institution, community, event, activity, process, or one or more individuals, usually selected because it is unique, successful, typical, or experimental (Merriam, 2002; Stake, 1995). Cases are bounded by time and activity. According to Stake (1995), in case study, the researcher can study the "particularity and complexity" of a single case, allowing for an understanding within those important circumstances (p. xi). The researcher chose to use case study in this study to examine the complexity of how addition is taught at a particular well-supported school. Stake uses the term an instrumental case study for this type of case study when the objective of the researcher is to understand a general phenomenon in a given context, the case chosen here is "instrumental" in understanding how teachers can support addition strategies. In this particular school, the case, teachers are given extensive professional development, supported in their planning and practice, and offered quality resources and assistance in planning and implementation. According to Stake (1994), "My choice would be to take that case from which we feel we can learn the most…. Often it is better to learn a lot from an atypical case than a little from a magnificently typical one" (p. 243). While this case is not typical, the optimal setting is purposefully chosen so the researcher is better able to isolate the relationship between teacher instructional practices and the impact on students with minimum negative inferences to obscure the influence of the teacher. Stake explains: "This use of case study is to understand something else…and we may call it our inquiry instrumental case study" (1995, p. 3). Classroom instruction is complex with multiple influences, interactions, opinions, and decisions. To capture the complexity, Knapp (1997) noted, "Case studies, which involve direct observation of teaching practices, allow for an in-depth examination, and provide rich descriptions of contextual factors pertinent to classrooms." Because of the intense labor of case studies, a limited number of students and teachers were analyzed in this study.

In this study, multiple sources of qualitative data were collected over a sustained amount of time. Qualitative data were analyzed on 28 students' strategy use and conceptual knowledge and on instructional practices of their two teachers (Figure 3). In Part One: Student Strategy Use, the collection and analysis of student strategy data, the researcher collected individual student data using a strategy interview (described below) along with individual student data using explanations, student artifacts, and student explanations during classroom observations. For Teachers' Beliefs and Practices data, lesson observations and curriculum materials were analyzed together and compared to teacher interview analysis to determine findings for Part Two: Teacher Practices and Beliefs Data. Part One: Student Strategy Data and Part Two: Practices and Beliefs Data were analyzed to answer the research question concerning the nature of the relationship between curricular practices and beliefs and students' advanced strategy use for addition in first grade.

Figure 3

Design Diagram

CASE DATA AND ANALYSIS



Participants

Before research began, the researcher contacted the principal of Southbridge Elementary [pseudonym], whose administration, math coach, and first grade teachers were likely to agree to participating in the study. As gatekeepers, the principal and superintendent approval were sought (see Appendix A). The principal and math coach invited individual teachers using information provided by the researcher. Two of the four first-grade teachers agreed to the study.

First grade teachers who agreed to participate were given detailed information about the study including benefits and risks to them, and consent was sought by the researcher (see Appendix B). All students in participating classes were invited and permission was sought from parents/guardians (see Appendix C). For an overview of the class including demographic information, information about the teacher, curriculum, and students was collected from the teacher and school and recorded on the Class Description form (Appendix D).

The sample for the study included two teachers and 28 students from the same school. The school, Southbridge Elementary School, was chosen because of its supportive environment for best math practice. The administration was knowledgeable, encouraging, and supportive of mathematical practices based on conceptual understanding. The school, and its district, provided professional development regularly to all teachers in standards-based math practice. The district employed a math coach at each school to support teachers with implementation. This level of support is not typical in Alabama yet made the school an excellent choice as a study case for what math practice could look like in more optimal circumstances. Additionally, the school offered some ethnic, SES, and gender diversity. The school district is situated in a suburb of a large metropolitan area. There are 11 elementary schools within the district. The school's population is approximately 565 students with approximately 32% receiving free or reduced lunch which is lower than the state average. Sixty-one percent of the student body is proficient in math on the state mathematics achievement test, which is higher than the state average. There are four first-grade classrooms with fewer than 20 students each.

Table 1

Demographics

		White	Black	Asian	Hispanic	Two or more ethnicities	Other
School	565	53.6%	32.4 %	6.8 %	6.6%	0.4%	0.2%
Student Participants	28	50 %	17. 8%	29 %	10.7%	0	0

Demographic data was collected on students that agreed to participate in the study. There were 15 girls and 13 boys. Both participating teachers had multiple professional development opportunities. Both teachers participated in district-level multiple-day professional development from trained presenters on additive reasoning. Ms. Oakley had a master's degree and taught for 13 years, all in first grade, for a few years at a private school in another state, with the majority of her teaching experience at Southbridge Elementary. Ms. Fairfield had a master's degree and had taught for nine years, the past five years in first grade at Southbridge. Both teachers regularly met with the school's math coach, participating in school-level professional development or planning for instruction.

Setting

A major characteristic of case study is "up-close information gathered by actually talking directly to people and seeing them behave and act within their context" (Creswell & Creswell, 2018, p. 181). The researcher collected data in the natural setting where the participants were teaching and learning about addition. All data was collected at the school and most data was collected within the everyday interactions in the classrooms. Normal routines and schedules were disrupted as little as possible.

Part One: Student Strategy Use Data Collection

Prior to beginning any research, Institutional Review Board approval was granted on March 17, 2022 (See appendix E**).** The data for this part of the study included student interviews, observations of students, and student artifacts.

Student Interviews

To establish the student strategy use within a relatable context for researchers in the field, accuracy and strategy use were evaluated using an addition strategy protocol (Appendix F) developed by Geary (2004). However, to better understand the extent to which students exhibited conceptual understanding, the researcher elaborated on Geary's protocol using original and open-ended questions. The interviewer used knowledge of young children and strategy development to encourage students to extend or elaborate answers. When students offered an answer, the interviewer asked the child to explain how they knew or to prove the answer. Possible questions:

- How do you know?
- What if I said the answer was (a close, but inaccurate answer)?
- Can you show me?
- How did you use your fingers?
- What did you see in your head?
- Do you remember how you learned it?

The interviewer then paraphrased what the student said and asked for agreement. All interviews were video and/or audio recorded.

The observer also recorded response times as a measure of automatic retrieval (Geary, 2014). However, retrieval was not coded as a strategy in this study unless there was evidence of conceptual understanding from follow-up questions to exclude memorization as a conceptual strategy.

Student Artifacts

By observing and recording student strategy during independent work and collecting artifacts such as student solutions to word problems given during observed lessons, the researcher gained a better understanding of the students' strategy use. In Rittle-Johnson & Schneider (2001), the researcher is cautioned that it is a "critical feature of conceptual tasks...that they be relatively unfamiliar to participants, so that participants have to derive an answer from their conceptual knowledge, rather than implement a known procedure for solving the task." Because the students were accustomed to solving simple addition problems and even to justify the answer, it was also beneficial to collect work samples and observe the students in varied situations to reveal conceptual understanding. Further, the same problem could reveal different understandings for different students (Dowker, 2019). Analyzing work samples and strategy explanations offered a wider variety of ways for students to show conceptual understanding and allowed the researcher more opportunities to find evidence of conceptual understanding through strategy use.

Student Observations

Within classroom discussions/lessons recorded as classroom observations, students often explained their addition strategies. The researcher took notes at the time of the observation on the classroom observation form and added to and clarified the notes during the transcription of the lesson observations.

Part One: Student Strategy Use Data Analysis Plan

Data analysis began at the same time as data gathering. Stake (1995) offers two strategies for analysis which were both employed in this study. In the first, the researcher finds specific incidents which provide direct interpretation about the case. In the second, the researcher finds similarities in data which allow for a categorical aggregation. As this case study is instrumental, categorical aggregation was used to find "correspondence" and build patterns within the case (Stake, 1995). "Often, the patterns will be known in advance, drawn from research questions, serving as a template for the analysis. Sometimes, the patterns will emerge unexpectedly from the analysis" (Stake, 1995, p. 78). In this study, student responses were initially coded during the interview using the predetermined codes offered by Geary (2004). Merriam (2009) referred to this process as "category construction" and advocated coding, starting with open coding and memoing codes that are repeated under one theme, called "sorting categories" and finally reducing to a manageable level. To analyze the strategy explanations, student observations, and student artifacts, the researcher began initial or open coding with strategy explanations recorded on the addition strategy assessment form (Appendix F). Merriam stated, "The fewer the categories, the greater the level of abstraction, and the greater ease with which you can communicate your findings to others" (Merriam, 2009, p. 187). The codes derived from open coding were used in constant comparative analysis of each viewing of interview videos, notes from student observations, and student artifacts until the point of theoretical saturation, where additional data was no longer providing new or refined concepts. In deciding which categories to keep, the researcher considered the frequency with which something appeared in the data, the credibility to the research audience, uniqueness, or “areas of inquiry not otherwise recognized” or which “provided a unique leverage on an otherwise common problem” (Guba & Lincoln, 1981; Merriam, 2009, p. 187).

Part Two: Teachers’ Practices and Beliefs Data Collection

The data for this part of the study consisted of lesson observations, review of lesson plans and curricular materials, and teacher interviews. The intent was to gather rich descriptive narrative data to complement the data on student understanding. Understanding the instructional practices and teacher’s beliefs allowed the researcher to examine the relationship between student understanding and instructional practices and beliefs.

Classroom Observations

Researchers and practitioners need a better understanding of the prevalence of current practices in classrooms (Hamre & Pianta, 2007), particularly during mathematics instruction. For each classroom, data was collected from several sources. Stake stated,

> During observation, the qualitative case study researcher keeps a good record of events to provide a relatively incontestable description for further analysis and ultimate reporting. He or she lets the occasion tell its story, the situation, the problem, resolution or irresolution of the problem. (Stake, 1995, p. 62)

The researcher conducted classroom lesson observations during what the teacher considered to be the math time over the course of a week to two weeks for each classroom. Because students and teachers were not in the natural environment during an observation because they were being observed and recorded, the researcher visited the school and the classrooms multiple times to lessen the unnatural circumstance of being observed. Getting to know the students, teacher, and classroom routine positioned the researcher in the middle of the participant-observer continuum, which was fitting in the class, but not participating in the activities. Lesson observations were conducted by the researcher and recorded on the Classroom Lesson Protocol (Appendix G) which is an open-ended observation instrument which is informed by Horizon Research's *Inside the Classroom: Observation and Analytic Protocol* (2000). The researcher found the format of the observation and some of the questions to be helpful in developing a more open-ended classroom observation instrument which was consistent with qualitative data collection. In the original protocol, every question was scripted, and most answers were quantified on scales. Additionally, the original protocol collected data on topics unrelated to this study. The format used for this study includes several parts: overview (the facts about the classroom and lesson), field notes during the live observation, topics to look for in the observed lesson, and teacher debrief. In the recorded lesson, the researcher looked for any of the following topics in relation to addition instruction: resources, student-teacher interaction, student-student interaction, mathematical content, teacher confidence/accuracy, classroom culture, degree of sense-making, activities, and use of manipulatives. During this time, the researcher saw 10 whole-group lessons in Mrs. Oakley's class and eight lessons in Mrs. Fairfield's class along with the accompanying small-group or individual instruction and work. However, only the presentation of the content (by teacher, video, etc.) was recorded. The math coach agreed to help cover classes to allow for a short debrief with the teacher after observed lessons and during teacher interviews which allowed the teacher sufficient time to offer clarification or insights into the lessons. Lesson plans and curriculum materials were collected at this time.

Teacher Interviews

Because it was important to understand the teacher's belief system about addition to give context to the decisions made, after the lesson observations ended, each teacher was interviewed using the teacher interview protocol (Appendix H). According to Stake (1995), "Qualitative case study seldom proceeds as a survey with the same questions asked to each respondent; rather each interviewee is expected to have had unique experiences, special stories to tell" (p. 65). In each interview, the teacher was encouraged to talk freely and openly about mathematics in her classroom, and the protocol was utilized as needed to prompt the teacher. To get at the essence of teachers' beliefs about conceptual understanding, every effort was made to answer the following questions in each interview: "What do you want your students to be able to do when they go to the next grade?" and "What understandings do you want your students to have concerning addition/subtraction?" The interviews were recorded and transcribed with editing.

Curricular Materials

The math curriculum used at SouthBridge School is EnVision Mathematics Alabama (Savvos Learning Company, 2022). The U.S. Department of Education's Institution of Education Sciences (2016) described enVisions Mathematics:

> enVisionMATH, published by Pearson Education, Inc., is a core mathematics curriculum for students in grades K–6. The curriculum aims to help students develop an understanding of mathematics concepts through problem-based instruction, small-group interaction, and visual learning, with a focus on reasoning and modeling. Differentiated instruction and ongoing assessment are used to meet the needs of students at all ability levels. Within each grade, the curriculum is organized around clusters of Common Core standards and consists of 120–130 teacher-led lessons, with the intention that one lesson is completed per day. Each lesson includes daily review and a small-group, problem-based activity, followed by guided and independent, paired, or small-group practice

> activities. Instructors use daily assessments to track student progress and enable targeting of additional practice and homework activities for students that need more support. Lessons are organized into a customizable sequence of topics and use texts, workbooks, manipulatives, online web-based materials, and technology within group and individual activities.

The enVision Mathematics curriculum lessons were collected for the corresponding lesson. By examining corresponding lessons, the researcher hoped to better understand content and practices prescribed by curriculum. The school was in the first year of adoption of this text, and teachers had few decisions about the way the lesson was implemented.

Part Two: Teachers' Beliefs and Practices Data Analysis Plan

After the first lesson observation in each classroom, the researcher prepared a transcript of the lesson using notes and recordings. The observation along with the corresponding lesson from teachers' plans and curriculum materials were analyzed using open coding. The codes derived from open coding were used in constant comparative analysis for each subsequent lesson until the point of theoretical saturation, where additional data was no longer providing new or refined concepts (Tashakkori, et al., 2021). As described above, initial coding was ongoing and progressed iteratively as data informed coding and coding informed observations. The researcher organized patterns into categories.

Classroom observation data was collected, recorded, and analyzed for each individual classroom in this first phase of the study. Along with plans, curriculum materials and field notes recorded during the live lesson, the recorded observation notes were analyzed with the following questions as possible guiding questions:

- What teaching strategies are used?

- What teaching strategies are encouraged in curricular materials?
- How are manipulatives used?
- How are students using advanced addition strategies or relying on counting?
- Where and how is Conceptual Knowledge found and/or encouraged?
- Are students encouraged to solve problems algorithmically or inventively?

Limiting the scope of the information allowed the researcher to isolate the relevant data from the immensity of the data collected. As described above, initial coding was ongoing and progressed iteratively as data informed coding and coding informed observations. The researcher organized patterns into categories.

Case Analyses

Analysis began during data collection. Creswell and Plano-Clark (2018) advocated data from one source to inform the need for collection of other data or using data from one source to explain the data from another source. Student responses in assessments led the researcher to look for the source of the response, or classroom observations help explain responses on assessments.

As can be seen in the Design Diagram (Figure 3, p. 37), from lesson observations, lesson plans, and curriculum materials, the researcher codes the data using constant comparative method to the point of theoretical saturation. The interviews were compared to the observations to look for confirming or contradicting themes. The themes and descriptions derived from the two sets of data made up the findings for Part Two: Teacher Practices and Beliefs. To answer the question of the relationship between practices and beliefs and advanced addition strategies, the researcher compared the findings from Student Data with the findings from Teacher Beliefs and Practice to find "correspondence" and build patterns within the case (Stake, 1995).

Legitimation

The researcher sought to achieve transferability of the study by providing rich and descriptive language in participants' own words to allow the reader to rely on the findings and transfer to their own setting or study. The researcher was concerned about researcher and participant effects and therefore spent time in classrooms helping participants become more comfortable with being observed to minimize those effects. The investigator also utilized multiple data sources for methodological triangulation (Stake, 1995), such as interviews, observations, and artifacts; analyzing student strategy use; and using classroom observations, lesson plans, and teacher debriefs for instructional practices and beliefs. The researcher disclosed corroborations as well as contradictions in the data collection, coding, and analysis. In student data, the researcher used multiple data sources to accurately capture the students' meaning and viewpoint. Video and audio taping were utilized to ensure that the researcher presented a factual account.

Participant views were used and presented in this study by using participant's exact words and cross-checking with participants. Stake described this member checking when the participant

> is requested to examine rough drafts of writing where the actions or words of the actor are featured, sometimes when first written up but usually when no further data will be collected from him or her. The actor is asked to review the material for accuracy and palatability. The actor may be encouraged to provide alternative language or interpretation but is not promised that their version will appear in the final report. Regularly, some of that feedback is "worthy of inclusion." (Stake, 1995, p. 115)

The interests, values, and viewpoints of the stakeholders were considered throughout the study.

Ethical Considerations

At every part of the study, the researcher sought to protect the ethical integrity of the participants and the study as a whole. Before research began, the researcher sought and obtained approval for the study from the Institutional Review Board (Appendix E). To apply for approval, the researcher obtained permission from the gatekeepers (Appendix A). For this study, the gatekeepers included the Chief Learning Officer of the school system and the principal of the selected school. All participants were invited to participate and informed of research intentions along with benefits and risks associated with participation in the study. The risks were minimal, and the benefits were not direct but to the body of educational research as a whole. To verify that participants understood the study, benefits, risks, and voluntary participation, the participants were asked to sign an informed consent (Appendix B). For the children in the study, parents and guardians were informed and asked to sign a permission form (Appendix C). When needed, consent forms were translated in the student's family's home language. As the children in the study were not proficient readers, the researcher verbally explained that they could stop at any time in the study (Appendix I).

Participants were informed of their right to confidentiality. All identifying information was replaced with a pseudonym, including names of participants, school, and school district. The researcher alone accessed video and audio recordings and all collected data were stored in a locked cabinet or on a password-protected computer.

Role of the Researcher

According to Stake (1995), the researcher plays an important role in interpreting what is happening, noting that "standard qualitative design call for the persons most responsible for interpretations to be in the field, making observations, exercising subjective judgment, analyzing and synthesizing, all the while realizing their own consciousness." The researcher has 30 years of experience as a primary mathematics educator. The researcher has a background in constructivist mathematics education, working at schools that are supportive of constructivist mathematics, studying with a prominent constructivist mathematics scholar, and leading professional development in constructivist mathematics practices. The researcher intentionally used the lens of a constructivist mathematics educator in the qualitative data collection in the study.

As an early childhood educator with over 25 years of experience working directly with kindergarten and first graders, the researcher is able to work with and talk to children in ways that make them comfortable and willing to share and extend their thinking. The researcher also has extensive experience in analyzing student work to reveal student understanding.

For eight years, the researcher worked as a mathematics instructional coach for teachers and as a university supervisor for pre-service teachers. In both roles, the researcher spent time conducting classroom observations that were designed to be non-threatening and constructive. The researcher used this experience to make the participants in this study comfortable, authentic, natural, and willing to share their thoughts. However, the researcher recognized the different lens required as a researcher in looking for what is happening instead of looking for ways to improve. The researcher consciously combatted the tendency to look for areas for improvement in order to capture the current experience. While acknowledging the lens of a constructivist educator, the researcher was aware of the bias in favor of instructional practices supporting a constructivist philosophy.

Procedural Issues

Teacher participation in the study was an issue especially given the extreme stress endured by teachers through the previous two years of the COVID pandemic, new math curriculum, and time of year. While the principal and math coach were agreeable, only two of the four first-grade teachers were comfortable being observed. Additionally, teachers were concerned that students could be behind academically because of instructional time lost or compromised during school closings and absences and online instruction. The circumstances put additional pressure on teachers, but also could have affected the age range of development of addition strategies. Further, because IRB approval took longer than expected (several months), this constraint limited the time and scope for data collection before the end of the academic year. The researcher had hoped to begin data collection in February 2022. However, final IRB approval was not granted until late March 2022, just before the school's spring break. Consequently, collection of data did not begin until early April, which affected the content taught and the amount of time to observe in the classrooms.

Feasibility of the Study

The study was feasible as the researcher had the potential availability of students, access to necessary resources, as well as supportive and knowledgeable colleagues and mentors. The researcher had classroom knowledge and experience to work with students and teachers. Having worked for many years as a math coach in the geographic area, the researcher knew which schools would be a good fit for the study. Additionally, the researcher was familiar and trusted with administration at the district and school level which allowed access to teachers and students to invite to participate. Permission was sought from the gatekeepers, the Chief Learning Officer for the school district and the principal of the school. After securing district, school, and IRB approval, the researcher sought consent/permission from all participants. As a graduate student and instructor at the University of Alabama at Birmingham, the researcher had access to online databases, peer-reviewed journals, and other resources concerning both content and methodology for the study. The researcher was guided and mentored by a committee knowledgeable about the topic and inquiry plan.

Reporting of the Study

Although the data was collected concurrently, the student strategy data is reported first followed by the teachers' beliefs and practices. The results are presented in charts showing predetermined codes for addition strategies used by student participants. The classroom observation data results are explained with a descriptive narrative including examples from the qualitative data that was representative of the observations. Illustrations, diagrams, drawings, and mathematical notations are used to illustrate the results.

CHAPTER IV

FINDINGS

In this chapter, results from Part One: Student Strategy Use and Part Two: Teachers' Beliefs and Practices are discussed and compared. The results are reported in three separate sections. In the first section, Part One: Student Strategy Use, the researcher presents findings from student artifacts, interviews, and observations. To better understand how students used addition strategies, the researcher observed and interviewed 28 first graders from two classes in the same school about the way they solved early addition problems. The researcher analyzed the student interviews, then analyzed data from classroom observations and artifacts collected during the observations. Results from all three data sources were combined to derive findings for Student Strategy Use. From the student strategy analysis, the researcher organized observations into categories: (a) Use of Conceptual Knowledge, (b) +1/-1 Strategies and Unitary Understanding, (c) Inventive vs. Algorithmic Thinking, (d) Retrieval and Memorization and Speed, and (e) Student Disposition.

In the second section, Part Two: Teachers' Beliefs and Practices, the researcher presents findings from teacher interviews and classroom observations and curriculum materials. To better understand how the teachers thought about addition strategies, the researcher conducted individual interviews with the two teachers in the study. To examine how the teachers taught addition, the researcher observed multiple math lessons. Results from interviews and observations were analyzed separately and then compared to find areas where evidence agreed or diverged. The results from classroom observation and interviews were organized into the following categories: (a) Description, (b) Awareness that Practices Diverge from Beliefs, (c) Mathematical Concepts, (d) Role of the Teacher, and (e) Memorization and Fluency. Areas of agreement or divergence between interviews and observations are discussed within those categories.

Setting

Southbridge School, a suburban Alabama K-5 elementary school, is the case and setting for this study. There are 10 other elementary schools within Southbridge's district. The school's population is approximately 565 students with approximately 32% receiving free or reduced lunch. There are four first-grade classrooms with fewer than 20 students each. Six ethnicities are represented in the student body with the largest ethnic group White, followed by Black (32.4%), Asian (6.8%), Hispanic (6.6 %), students of two or more ethnicities (0.4%), and other (0.2%). Sixty-one percent of students are achieving proficiency in math, which is higher than the state average.

An older school, Southbridge School has been updated and well-maintained. The overall atmosphere of the school is positive. Signs throughout the school offer encouragement to students and teachers, and student work is amply displayed in hallways. In addition to the many resources the school district provides for teachers and students, the principal and assistant principal are very visible in the school and heavily involved in curriculum and instructional decisions made within the school. Instructional leadership also includes a math coach who has worked at the school for over 10 years and served as the math coach for the past six years.

Field notes from observations revealed information about the classroom setting in the study. In both classrooms featured in the study, the rooms were neat and uncluttered, with minimal decorations. Number lines, hundred charts, and large numeral cards were displayed. Teachers were friendly and positive with the students yet maintained a no-nonsense atmosphere that communicated that work will be done and students will try hard. Neither room was silent, as students continued to work while whispering. Some students got extra support from teachers outside the classroom, such as ELL services, Title I, or intervention. Students entered and left the classroom regularly with little disruption to the lesson or the student work time.

The math instruction block was approximately an hour every morning and consisted of three parts, a warmup, an enVision lesson, and independent/small-group work. The first part of the lesson, a warm-up exercise, was not associated with the enVision lesson of the day. Mrs. Oakley used the number representing the day's date as well as the number representing the number of days in school to generate math discussion about even/odd and 10 more/10 less. Mrs. Fairfield started math with a short number talk using equations provided by the district. The problems were briefly discussed, and since the equations were much simpler than the lesson, they seemed to be primarily used for fluency practice. Sometimes students stayed in desks for the entire lesson, and sometimes students came to a carpeted area at the front. After the warm-up, the Envision lesson began with an open-ended problem related to the day's objective followed by a short video which was stopped at intervals for discussion. The video was projected on a large

SmartScreen where teachers could digitally manipulate models and write explanations when appropriate. Students then did independent work which consisted of workbook practice about the day's objective or practice on the i-Ready digital program which assessed and prescribed an individual learning path for each student. At the time of the study, the curriculum lessons were about place value strategies for adding two-digit numbers. Although the entire lesson was observed, the warm-up/number talk portion of the lesson was most informative for the study.

Student Participants

There are 28 students participating in the study, 19 students (100%) of Mrs. Oakley's students, and 9 (47%) of Mrs. Fairfield's 19 students. Two more of Mrs. Fairfield's students whose parents consented to the study choose not to participate. Within Ms. Oakley's [pseudonym] class, there are eleven girls and nine boys. There are eight White, five Asian, three Hispanic, and three Black. In Ms. Fairfield's [pseudonym] class, four participants are girls and seven are boys; six are White, three are Asian, and two are Black. Only very specific differences in the classes are noted. Student data is not discussed by class.

Teacher Participants

Both participating teachers have had multiple professional development opportunities. Both teachers have participated in district-level multiple-day professional development from trained presenters on additive reasoning. Ms. Oakley has a master's degree and has taught for 13 years, all in first grade, for a few years at a private school in another state, with the majority of her teaching experience at Southbridge Elementary. Ms. Fairfield has a master's degree and has taught for nine years, the past five years in first grade at Southbridge. Both teachers regularly meet with the school's math coach, participating in school-level professional development or planning for instruction.

Part One: Student Strategy Use

Even though the sample size is not large enough to make claims of statistical significance using accuracy rates, it seems important to note the high accuracy rate to add context to data for strategy use. For the accuracy rate of the students, 22 of the 28 students completing the interview (79%) had two or fewer errors (90 % correct). The researcher analyzed the student interviews, then analyzed data from classroom observations and artifacts collected during the observations. Results from all three data sources were combined to derive findings for Student Strategy Use. From the student strategy analysis, the researcher organized codes and themes into categories: Use of Conceptual Knowledge, +1/-1 Strategies and Unitary Understanding, Inventive vs Algorithmic Thinking, Retrieval and Memorization and Speed, and Student Disposition.

Table 2

Student Strategy Use: Organizational Chart for Categories

Student Strategy Use: Organizational Chart for Categories	
Category One: Use of Conceptual Knowledge (CK)	• Multiple Representations of the Same Number • Using What They Know • Combining Strategies • Knowledge of properties to simplify the problem. • Decomposing Strategies • Using Multiple Strategies • Errors in Conceptual Understanding
Category Two: +1/-1 Strategies and Unitary Understanding	• Counting All • Counting On • Counting on NOT one by one

	• Decomposing with +1/-1 Thinking • SOOOO many fingers- Where's the shame?--Use of fingers, finger configuration, Place Value Knowledge Using Fingers, Errors with finger use
Category Three: Inventive vs Algorithmic Thinking	• Use of Multiple or Combined Strategies. • *"Roll Over Ten Method."* • When is Counting on an Algorithm?
Category Four: Retrieval and Memorization and Speed	• Just Knew. • Memorized. • Speed and Fluency?
Category Five: Disposition	• Math is Hard • Persistence • Willing to Try, Struggle, and Engage

Category One: Use of Conceptual Knowledge (CK)

Using Hiebert's (1986) definition of conceptual knowledge, the researcher identified the use of conceptual knowledge exhibited in the ways that follow.

Multiple Representations of the Same Number

Students showed a variety of ways to "see" or think about a number. Knowledge of the network of relationships for a number allowed the same student to use the same number in multiple ways For example, 8 was broken up into 5 and 3 in one equation, 7 and 1 in another problem, and 2 away from 10 in another problem.

Using What They Know

Students showed an awareness that the equations were not just isolated information related in their ability to use what they knew from one equation to assist in solving another equation. Almost all of the strategies other than the counting-on strategies were based on what the student already knew such as a “friendly ten,” yet some students were unable to articulate how they “used” their previous knowledge. Students who said, “I used 7 + 3,” as the way they solved 7 + 2 would be coded “Used Known Fact” if no other explanation could be given. Students were coded as using a known fact to solve 15% of the problems. A known fact was used for at least one problem by 19 out of 28 students. If the child explained that they compensated or decomposed, then the problem was coded accordingly. Understanding that the problems were related, and that one problem could be used to solve another problem showed evidence of conceptual knowledge.

Somewhat like using a known fact, students would sometimes use a problem recently solved using a counting strategy during the interview to solve a new problem. Because the student counted to determine the original answer, it was likely the student would have solved the related problem with a counting strategy if the problem was presented in isolation.

Combining Strategies

Several of the strategies could have been considered a combination of strategies. The Making Ten strategy was a combination of decomposing an addend and using the known ten fact. There were seven students who combined strategies in an unexpected or novel way which showed an understanding of the network of relationships of strategies. For strategies that were a combination of strategies, the researcher coded for the strategy that was the most predominant or the more advanced strategy.

Knowledge of Properties to Simplify the Problem

Commutative Property. Many students demonstrated their conceptual knowledge in their ability to simplify problems using their knowledge of properties of operation. One of the earliest advances students have been shown to make in conceptual understanding is using commutative property to count on from the larger addend such as changing 2 + 19 to 19 + 2. Counting on from the larger addend was the most frequent strategy used by students in the study. Classroom observations also revealed that students who counted on were more likely to start with the larger addend regardless of the order the addends were presented.

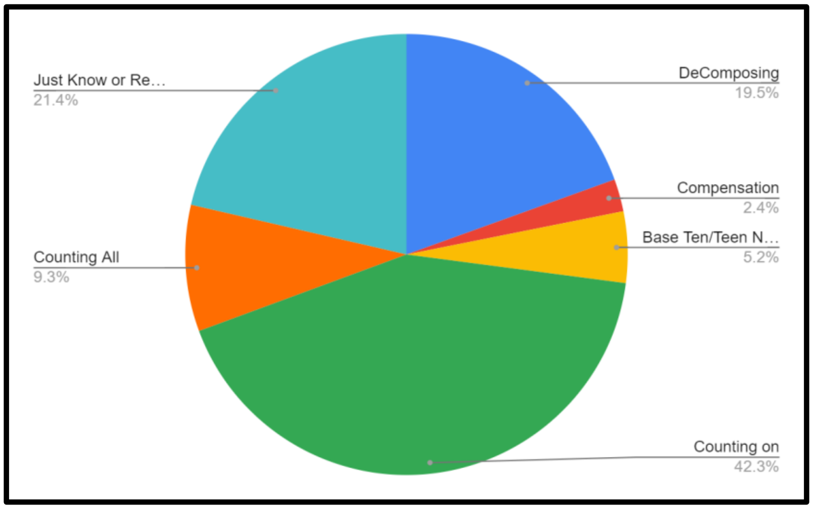
Associative Property. At a more advanced level, students utilized the associative property to change equations to make decomposing easier. Two notable examples, Levi changed 9 + 15 to 19 + 5 to decompose and make 20 and he changed 16 + 7 to 17 + 6 to add three and three more. For the students who combined operations or for strategies that were a combination of strategies and operations, the researcher coded for the strategy that was the most predominant or the more advanced strategy.

Decomposing Strategies

Strategies in which the student broke apart at least one addend in order to simplify the problem were considered to be the most advanced strategies. The chart (Figure 5) shows the strategy that each child used most frequently, also including a category for students who counted on exclusively. Figure 4 shows that 14 students (50%) used Advanced strategies and 11 (39%) used Simple strategies only. Two students used both Simple and Advanced strategies almost equally. One student had no strategies at all and only guessed.

Figure 4

Percentage of Students Using Each Strategy



Two specific strategies for decomposition, Make Ten and Compensation, were used by students in the study.

Decomposing to Make Ten. In the Make Ten strategy, a strategy name commonly used in curricular materials and research, the student is decomposing one addend very intentionally to "get to 10." An example of the Make Ten strategy: Anna explained her strategy for 9 + 5, "I put one piece from five and then made a ten, then four and 10 is 14." While some equations such as any number plus nine lend themselves to the Make Ten strategy, two of the students used making 10 consistently for almost all problems. Below are examples of ways students described the Make Ten strategy:

> Sam: Pump you up to 20
> Levi: (For 9 + 15) 15 + 5 + 4.
> Cody: (For 8 + 6) I know 10 + 4 and I know 8 + 2, so (take) 2 from 6 and add it to 8 (and that makes 10) and 4 more (which) equals 14.

Compensation. If the child clearly explained that they took an amount from one addend (decomposing) to give it to the other addend, then the strategy was coded as compensation. Students who used compensation showed knowledge of the associative property of addition.

Table 3 shows the ways that students used decomposition.

Table 3

Strategies involving decomposing

Strategy	**Description**	**Student Example:**
Compensation	The student intentionally decomposes one addend to make an easier equation. When adding 7 + 5, decomposing 7 into 6 + 1, grouping the 1 with the 5 to get 7+5 = 6+(1+5) = 6+6 =12	For 8 + 6, "If you put one on 6, it will be 7, and 7 + 7 = 14."
Decomposing to Make 10	The student intentionally decomposes one addend to make a ten from the other addend. When adding 8 + 5, decomposing 5 into 2 + 3. then you grouping the 2 with the 8 to get 8+5=(8+2)+3=10+3	Anna explains her strategy for 9 + 5: "I put one piece from five and then made a ten, then four and 10 is 14.
Decomposing a double	The student intentionally decomposes an addend into equal parts to add on in equal parts. When adding 9 + 6, decomposing 6 into 3 + 3, then counting on by threes to get 9+6 = (9+3) +3 = 15	For 14 + 8, Cody explains he added "four and four more."
Decomposing to use known fact	The student decomposes to use a fact that is a benchmark for that student. A student who knows 5 + 6 = 11 would use that fact to solve 5 + 8 to solve 5+8 = (5+6)+2= 13	Uziel knows 6 + 2 = 8 and uses that fact to solve 6 + 3.
Decomposing both addends	The student decomposes both addends to use a known fact. Strategy is often used with place value such as in the problem 11 + 11, a student decomposes each addend into 10 + 1 to solve 11+11 = (10+10) + (1+1) = 20+2 = 22	For 6 + 7, Daniel said "Seven and the six have five" and he added 5 + 5, and three more.

Using Multiple Strategies

Students showed a degree of selection in strategy use with 19 students (69 %) using more than one strategy. Of those 19, 14 students (50%) used more than two strategies. Like many of these students, Levi recognizes that he is using multiple strategies and choosing them intentionally. He says "I use that strategy for most of them" when speaking about thinking about number bonds to get to a double. He names the making 10 strategy. He can explain how his mother taught him a strategy and they named it the "roll over 10" strategy.

Error in Conceptual Understanding

Students were only noted for having an error in conceptual understanding if the answer or thinking was not reasonable. For example, giving an answer that was less than one of the addends would have been noted as an error in conceptual understanding. Simply not exhibiting or being unable to explain would not have been coded as an error in conceptual understanding. There were few errors in conceptual understanding. Lalani had the most unreasonable errors with three. Her main strategy was counting all using her fingers for all equations except one problem (4 + 3) where she counted on from the larger addend. Lalani's errors: 6 + 7 = 7, 5 + 8 = 8, and 4 + 7 = 6. For all three equations, Lalani used her fingers to count all. Because the sum was greater than her number of fingers, she could not keep track.

Category Two: +1/-1 Strategies and Unitary Understanding

In Geary's coding protocol, counting on (from either addend) was considered a simple strategy. Counting on strategies were used by almost every participant for at least one problem. Students using +1/-1 strategies exhibit unitary or one-by-one understanding. Figure 6 shows that of the 11 students who predominantly used simple strategies, nine (32 % of 28 student participants) counted on or counted all exclusively. Of the 14 Advanced Strategy users, nine (32 % of the 28 student participants) used counting on for at least one equation.

Counting All

Only four students Counted All for any equations, and only one of those students counted three times for one of the equations. Most students started counting with one addend and continued counting through. Of the four students, three used fingers to keep track. For all four students, challenges arose when the sum was larger than ten (number of fingers). Another challenge was when one addend was greater than five. Only Yali counted three times to count all. For 6 + 3, she counted six on her fingers, then 3 on her fingers, then miscounted the total.

Counting On

Regardless of whether counting from the lesser or greater addend, both would be considered a +1/-1 strategy because the student was only working one-by-one. For example, for 6 + 7, Mila put up 7 fingers, then put up six fingers and counted 7, 8, 9, 10, 11, 12, 13. Students used a variety of ways to count on by ones while counting on (Table 4).

Table 4

Examples of Counted on by Ones Strategies

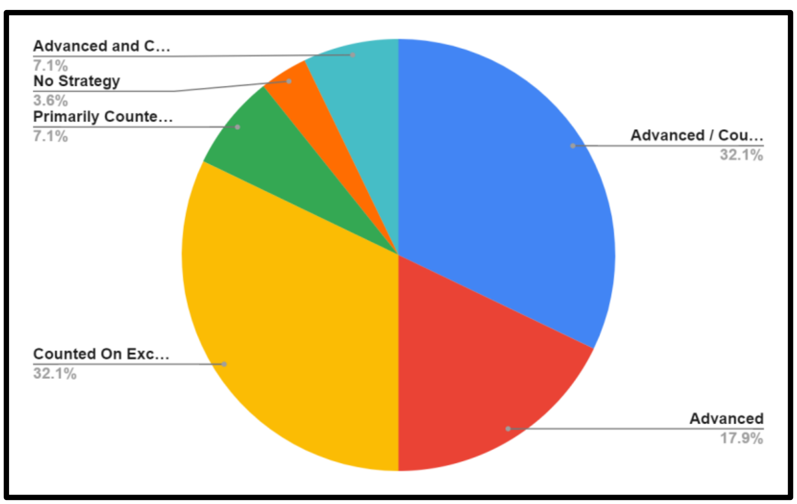
Description	Example
Used mental images to count	Student explained with a word problem--2 oranges and 3 oranges, mom made 5 pancakes and 4 more pancakes, “I love pancakes!”

Counted something in environment	Student used pictures on the wall, etc. or looking around for something to count
Partially adding in chunks, then moving to count by ones	In 4 + 7, Student saw 6 left, "used my mind for one more"
Using body (other than fingers) to keep track	Student used bobbing head, tapping on table, clicking tongue to represent +1 to keep track
Counting points on the numeral	Student counted the "sides of 4" like dots on a die (8+4--only one student)
Using number line	Student uses number line on wall or desk (begins with one addend and counts up the other addend)
Drawing number line	Student drew number line and counted by ones (begins with one addend and counts up the other addend)

Counting on was the most frequent strategy observed in the study. Of the 28 students, 13 of the students used Counting On for at least half of the problems, and nine of those used Counting On as their only strategy. Many students (19 of 28) used Counting On for at least one problem.

Figure 5

Most Frequent Strategy



Counting strategies were used for certain types of problems that did not lend themselves to other strategies such as Make Ten or using base ten knowledge or for equations in which students could utilize a known equation. For the problems 16 + 7, 9 +15, 17 +4, 6 + 19, and 14 + 8, approximately half of the students used a Counting On strategy. The only two-digit + one-digit problem that was not primarily solved by Counting On was 3 + 18, as many students were able to use the Make Ten strategy.

When Counting On Is NOT One by One

The counting on strategy is considered simple if the student counts by ones. However, several students counted on in more complex ways.

Counting On by ones in parts to make 106; For 6 + 7—decomposed 6 into 3 + 3 and counted on from 7 (8, 9, 10)(11, 12, 13)
Cooper: I did "7 + 3 + 3."
Making 10 and counting—For 8 + 6 = 8 + (2 + 2 + 2)
Counting on by something other than 1's—One student counted on by 2s or 3s.

Decomposing With +1/-1 Thinking.

Counting on is considered simple if the student is only counting by ones. If the same standard of +1/-1 thinking is applied to decomposing, then a new group of students, who only decompose by adding one or subtracting one is formed. For example, Jessilyn quickly knew that 9 + 5 would be 14 and explained her compensation using the former problem 8 + 6, she "took one more from the five (from 5 + 1) and gave it to the 8 (to make 9)." However, Jessilyn had solved 8 + 6 by "adding one more" from the problem before when she solved 6 + 7 by counting on from the 6. She then went on to count all for 5 + 8. Her advanced strategy use was inconsistent, and she is still only decomposing by ones.

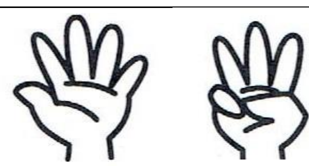
SOOOO Many fingers- Where's the Shame?

Finger Use. Any use of fingers was noted, and 18 out of 29 students (62%) used fingers to solve at least one equation. One student who did not use fingers used no strategies at all and only guessed (lowest level), and another of the students who did not use fingers to solve any equations used fingers to prove his answer. Using fingers was often used in conjunction with another strategy (see Table 4).

Finger Configuration. Multiple students explained that they knew 3 + 5 = 8 because that is the way it looks on their fingers. In this case, the student puts up one hand with three fingers without counting and the other hand with five fingers without counting and recognizes the configuration of the fingers is eight without any counting. This strategy was only named for the problems 2 + 3 (one student), 5 + 4 (one student), and 3 + 5 (three students), and was often only used to prove what the student "just knew."

Table 4

Ways Students Used Fingers to Solve Equations

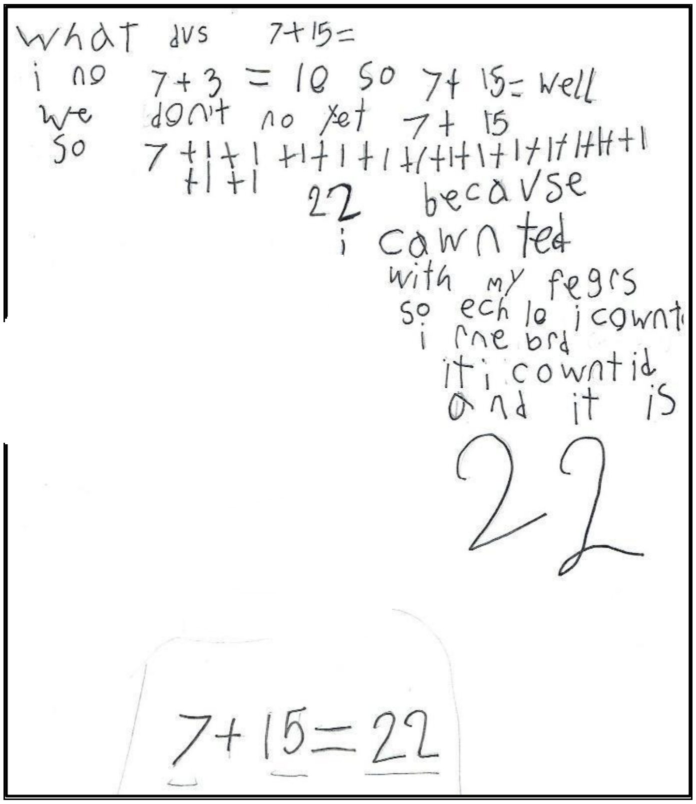
Ways Students Used Fingers to Solve Equations	
Explanation	Example
Finger Configuration Recognition	 Puts up 5 fingers Puts up 3 fingers Recognizes configuration of 8 fingers
Counting All	Student puts up all fingers, but only counts the second addend. For 5+4, student put up all fingers, and said 5.6,7,8,9.

Using fingers to count on	For 6 + 7, S put up 7, then put up six fingers and counted 7,8,9,10,11,12,13--most typical and often described as CO--if student counts from six or seven could potentially show a higher level of thinking Alina explains for 8 + 4, “ I know I don’t have 12 fingers, I started at eight, and counted four more.”
Using understanding that counting through all fingers will be ten and left over fingers will be a 10+ teen number	For 6 + 7, Student put up six fingers, then counted seven fingers beginning with one, knew the answer was thirteen because they had counted through all fingers on both hands and had three left over.
Making ten	Student uses fingers to determine the number needed to make ten. For 7 + 4, student would put up seven fingers up to determine that three were down, so seven and three more would be 10, and one more would be 11.
Place Value Using Fingers	Student keeping track of tens by counting how many times all ten fingers have been counted.

Place Value Knowledge Using Fingers. The students using this strategy were often not able to explain how they knew the answer. However, one student explained their thinking (Figure) as follows:

Figure 6

Student Thinking about 10 Fingers



"What does 7 + 15 = ?... I know 7 + 3 =10 so 7 + 15 = well, we don't know yet. 7 + 15…7 +1 +1 +1 +1 +1 +1 +1 +1+1 +1 +1 +1 +1 +1 +1 (=) 22 because I counted with my fingers so each 10 I counted I remembered that I counted and it is 22."

Errors With Finger Use. Use of fingers also revealed some common errors. The first problems on the protocol (2 + 3) can be solved by showing the number of fingers for one addend on one hand and the number of fingers for the second addend on the other hand. Beginning with the fourth problem, 6 + 3, students must use both hands to represent one of the addends. The eighth problem, 6 + 7, is the first sum greater than 10, where students must count past their number of fingers. One by one finger counters with no other strategies were not able to solve problems past the seventh equation, 7 + 2.

Another issue with counting with fingers is miscounting the fingers. Because the student is counting one by one with the last number as the answer, then a miscount of fingers is unlikely to be corrected. Similar to this error, students who held up the wrong number of fingers for one of the addends would have the incorrect sum even if counting correctly.

Category Three: Inventive vs Algorithmic Thinking

The researcher expected and specifically looked for evidence of step-by-step, algorithmic, or creative and inventive thinking in students' strategies.

Use of Multiple or Combined Strategies

Students' use of multiple and/or combined strategies not only shows conceptual knowledge, but also that their strategy use is not algorithmic as they use the strategies flexibly to choose the strategy that is most appropriate for the problem. Students showed a degree of selection in strategy use with 19 students (69%) using more than one strategy. Of those 19, 14 students (50%) used more than two strategies. There were seven students who combined strategies in an unexpected or novel way.

"Roll Over Ten Method"

Coded as Base Ten Knowledge, there were nine students who used this strategy at least once. There were four students who used a variety of strategies, but always chose the same strategy for the same type of problem. They predominantly just knew the solutions for problems with sums <10, used the Make Ten strategy for single-digit numbers with sums >10, and used the "Roll Over Ten Method" for the more complex problems involving a teen number. An example of the "Roll Over Ten Method" would be, for 4 + 8 =12, the student explains that if 4 + 8 = 12, then 14 + 8 = 22 because it would be 10 more. However, not all students were able to explain how the strategy worked. Wesley only attempted this strategy once and Counted On for all of the other two-digit + one-digit problems. For 17 + 4,

Wesley: 7 + 4 = 11 so 17 + 4 = 21
Interviewer: Can you explain why that is true?
Wesley: Because there could be two even numbers or 2 odd numbers.

No one except the first student seemed to understand why the method worked and were not able to generalize the base-ten thinking. In fact, on further questioning, responses included the following:

Levi: (For 16 + 7) My mom teached me this strategy-you just roll it over with 10.
Interviewer: Why do you think that works?
Levi: It's what my mom said.
Interviewer: Where did the ten come from?
Levi: Right there with the 16. Roll over with one ten.
Interviewer: Could you do it with more than one ten? Like 9 + 35
Levi: You would roll over with the nine. 45?

Anna: (For 16 + 7) We've been doing it in class. 7 + 6 =13. If it was 7 + 16, then it will equal 23.
Interviewer: Why does that work?
Anna: It's like the same, but……?
Anna goes on to do all of the problems with a teen number addend plus a single digit using the same method. Yet, she counts on by ones when adding a single digit to 27.
Interviewer: 27 + 7 (Anna counts on from 26 to 33 without hesitation).

Anna also offered this strategy in a classroom discussion.

Anna: I know that 9 + 2 = 11, so 2 + 19 = 21
Teacher (explaining strategy to class): So, you looked at that and said inside of 19,just like Ashu you broke up 19 into 10 and 9, and I know that 9 + 2 = 11, so there were 10 more inside that 19, so you added that 10."
Anna:(nods)

There was no way to determine if the student understood or if the teacher was just explaining the strategy for the benefit of the other students.

When is Counting On an Algorithm?

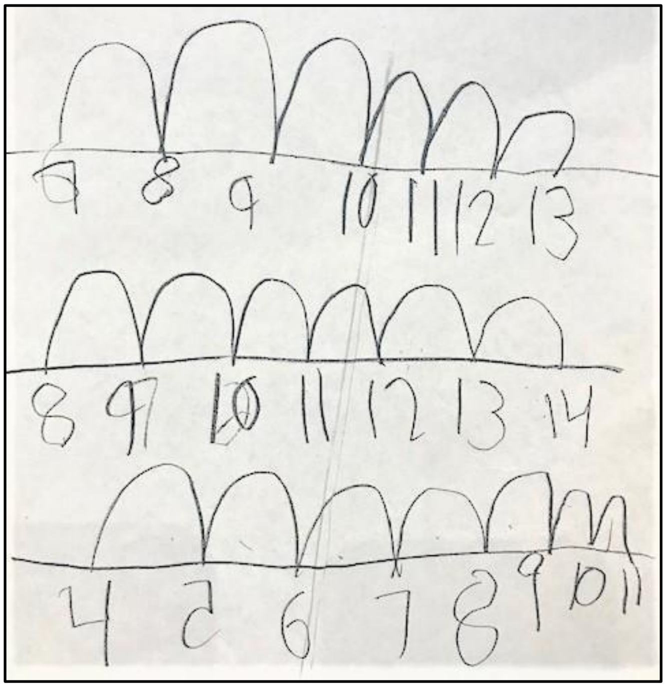
All of the one-strategy-only students used to count on as their strategy. As a group, students counted on in many different ways, but individual students were consistent in their way of counting, most commonly finger counting. There were no overt signs that students thought counting on was the only way to solve the problems. However, students seemed reluctant to break away from the way that they could confidently solve a problem. Specifically in the case of counting on using a number line. A typical interaction for using number line to count on is as follows:

Teacher: How did you solve the problem?
Sam: Started on the number line and…
Teacher: So, you used the number line starting with 6, and then what did you do?
Sam: I hopted 6 times.
Teacher: You jumped six times and where did you land?
Sam: It would be 12.

In Adeline's interview, she was unable to solve the problems without the number line (which is taped to her desk), so she asked to draw the number line to solve. She redrew the number line with each equation even after being asked if she could use the same number line she had already drawn (Figure 7).

Figure 7

Adeline Redrawing the Number Line



The use of the number line for the Counting On strategy revealed several points of interest, including the following:

- Some students used the number line as only one of several strategies.
- Cooper said he saw the number line in his head as he decomposed and made jumps.
- One student said, "I numberline it" as an explanation of how he knew the sum.
- The number line strategy was never used in a novel way.
- Only the number line and counting all fingers were the only strategies used exclusively by any students.
- Student who drew a new number line for each equation showed no evidence of a network of relationships.

Category Four: Retrieval and Memorization and Speed

In this study, many students (25 of the 29, 86%) answered "just know" for at least one equation in the interview. Follow-up questions helped to clarify how the student "knew" the answer. The researcher probed with questions such as the following:

- How do you just know?
- What if I said it was (incorrect answer)?
- It looks like you are thinking. What are you thinking about?

The follow-up questions showed differences in the ways that students "just know." One way to "just know" is about remembering the sum. For the equation 2 + 3, 25 of the 28 students said they "just knew" with no further explanation. It was not possible to determine whether these first problems were memorized based on the students' responses.

Memorized

After further questioning of a "just know" response, some students offered the way they "just knew." Table 6 shows ways the students explained how they remembered. The most common way that students remembered was Xtra Math, an online math-fact-practice that students used at home and at school.

Table 6

Ways Students Describe "Just Knowing" and "Remembering"

Ways that Students describe "Just Know"	Ways that Students describe "Remembering"
Just know	"I already knew it because….Extra math"
"I thinked and I knew"	It's on my math thing…Extra math….IReady

What fingers look like/feel like--"I know what 8 feels like"	Teacher told me
"Mental math"	"I practice it"
"I have done it"	"Remember from worksheets"
	"We do cards at home"
	"Easy! It's on my math thing."
	"I remember it."
	"Racing the teacher on Extra Math"
	"I usually know, but I forgot." "I already know it, but I forgot."
	(I see you thinking? What are you thinking about? "I remember it.")

Students who said they just knew and answered the follow-up questions with mathematical proof showed some understanding of mathematical concepts. Below are examples of student responses that showed mathematical proof for what they "just know:"

- Dice configuration,
- Fingers,
- Counting on,
- It's a flip flop, and
- Like another problem.

Another observation is that students who "just knew" solutions to easier problems were different than the students who relied only on memorization. Students who move from solving using a strategy to remembering the sum are different and more capable than students who only know from memorization.

Speed and Fluency?

Speed is often equated with accuracy and fluency, and in the case for Geary's protocol it is a sign of advanced strategy use. However, computational fluency is described as the ability to compute with accuracy, flexibility, and efficiency (Adding It Up, 2001). Fluency is not simply speed and accuracy, but also the flexibility to use relationships in novel situations with speed and accuracy. In the original interview protocol used in multiple studies by Geary (2014), response times (less than 3 seconds) are recorded as a measure of automatic retrieval (Geary, 2014). Using Geary's coding system, automatic retrieval would be coded as an advanced strategy. The researcher for this study chose to explore the strategy used in all problems regardless of speed.

Follow-up questions showed that speed was not necessarily a sign of advanced strategy use. For example, Maggie was a fast counter. She answered 16 of the 20 equations in less than three seconds. However, she reported that she counted on, and she visibly used her fingers to do so. Though Maggie solves quickly, she was utilizing a counting strategy, which was not considered advanced.

In a more dramatic example, there was a student who relied heavily on memorization and did not know what to do when she did not remember. She had no way of trying to figure it out. Her response was either "just knew" or "don't know."

Category Five: Disposition

Math is Hard

Few students voiced any objections or made any comments about their feelings about math during the interview. Hudson said, "Math is hard. I am bad at math." When questioned further, he revealed that he felt like math was hard because he did not know all the problems. Marley had similar feelings mostly because she got problems wrong on her online math practice. However, neither showed reluctance to share strategies with the whole class during discussions.

Several students noticed that the problems were getting harder. Facial expressions and gasps were not uncommon when the equations changed from sums <10 at the equation 6 + 7. Additionally, when the equations changed to two-digit + one-digit at the equation 16 + 7, students reacted similarly. Alina commented on the ninth equation, 8 + 6, “It’s gonna get hard. I think the last one is gonna be hard.” Again, on 3 + 8, she said, “It’s a little hard,” and 9 + 15, she said, “That’s hard.” However, she solved all the problems correctly.

Persistence

No one asked to stop. Only six interviews were incomplete, and all were stopped by the interviewer. Two of the six incomplete interviews were stopped because the student was obviously guessing on at least three problems in a row. Two more of the six were stopped because the student’s primary strategy was counting all, and no new information would be gained by asking the student to continue counting all. One of the six, Alina, was stopped because of her primary strategy of redrawing the number line. Continuing the interview would have been too time and effort intensive for the student. One student completed all but three of the problems before having to leave the interview to rejoin her class. Even when they knew it was hard, they solved it. Students gasped, sighed, raised eyebrows as the problems got harder, but no one asked to stop.

There were a number of students willing to work on a problem for over 30 seconds. Of the 15 students who needed > 30 seconds for any problems, 12 worked on at least one equation for more than 30 seconds. Of the remaining students, 13 did not need that long to solve any of the equations, one student only guessed, one student only solved equations that were solved quickly, and one student may have rushed given that he had by far the greatest percentage of errors per attempt (13/20).

Willingness to Try, Struggle, and Engage

One way that students' willingness to try was exhibited was the lack of random guesses. Guessing was not common. Only five students guessed on any problems at all. One student did nothing other than guess, and the interview was stopped after the first three problems. Another student guessed on two problems. Three students guessed on one problem only. Students were also willing to struggle. There were very few occurrences of "don't know," and every problem asked was attempted. Students exhibited a willingness to engage in the struggle. Observations of students during classroom discussions revealed a willingness to engage in all aspects of discussion. Almost every student shared strategies. Students were willing to defend even when questioned. In interviews and in the classroom observations, students did not change answers when they were questioned. Students were also willing to listen to classmates share thinking.

Summary of Results for Student Strategy Use

Many students in this study showed evidence of conceptual knowledge in multiple ways. However, there were issues specifically in the areas of unitary one-by-one thinking. Students showed knowledge of number relationships through creative and inventive thinking, yet evidence showed that some students were using algorithmic thinking by following procedures with little understanding. Retrieval was not considered an advanced strategy in this study, and some students who were fast and accurate showed little understanding. Students were positively disposed towards math even when challenged.

Part Two: Teachers' Practices and Beliefs

The two teachers were interviewed, and the two classrooms were observed. There was very little difference in the specific instruction of addition strategies as both teachers utilized a number talk protocol and were required to use the enVisions curriculum. Comparison was not the purpose of the study. However, there were a few differences in the classes in regard to the math lessons which are noted throughout this section.

The interview results and observations were analyzed separately and then compared to find areas where evidence agreed or diverged. Teachers named several barriers that prohibited practices they believed were best to complement how students learn or their goals for students. This section begins with a description of the classrooms followed by the ways in which teachers were aware that their beliefs did not match their practices in multiple ways for well described reasons. The results from classroom observation and interviews will follow, organized in the following categories: Description, Awareness that Practices Diverge from Beliefs, Mathematical Concepts, Role of the Teacher, and Memorization and Fluency. Areas of agreement and divergence between teacher interviews and observations will be discussed within those categories. Table 7 outlines the order in which Teachers' Beliefs and Practices findings are presented and how they were coded along with evidence and its source.

Table 7

Teachers' Beliefs and Practices: Organizational Chart for Categories

Teachers' Beliefs and Practices : Organizational Chart for Categories	
Category One: Classroom Description	● Whole-group vs. Small-group instruction ● Lessons driven by enVision ● Number Talks ● Manipulative Use ● Use of Individual Devices ● Answer-Driven
Category Two: Awareness that Practices Diverge from Beliefs	● Barriers--Covid, new curriculum, student issues ● Specific Practices Not Happening--Small groups (peer and teacher led), "Hands-on"
Category Three: Mathematical Concepts	● What is number sense? ● Part-Part-Whole ● Flexibility with Number Understanding ● Magnitude on Number Line ● Commutative Property ● Must you count to Know How Many? ● Number Line Encouraged
Category Four: Role of the Teacher	● Teacher as Encourager ● Teacher as Interpreter/Translator ● Teacher as Skeptic, not Judge ● Teacher as Conduit-Connecting Strategies ● Teacher as Teacher
Category Five: Memorization and Fluency	● Efficiency ● On-line Fluency Practice

Category One: Classroom Description

Whole-Group vs. Small-Group Instruction

For almost every observed lesson, the format was whole-group instruction followed by individual independent practice. The math block began by assembling the group, engaging students in some type of number talk warm-up, which was followed by the enVision lesson format (problem to solve, video to watch, and guided practice), and then independent practice. On some days the students had to complete selected workbook pages independently before going to IReady or other programs on individual devices, and on other days the students would have the workbook pages as one “station” in a rotation of stations including workbook pages, games, and individual devices.

Small-group instruction was completed with the teacher during the independent work time most often with students needing extra help. Students did occasionally work together in small groups independently to play games, but this was not observed every day and was sometimes reserved for students who finished the other tasks.

Lessons Driven by enVision

Teachers did not make major decisions about the content of the lessons. Each day, the enVision lesson opened with a word problem. Then, the class watched a video, the visual part of enVisions, in which characters explicitly explain a concept or strategy. There are intentional stopping points in the video. Both teachers utilized these stopping points to ask students questions and elicit responses. Teachers did make choices about the number talk. The district provided a suggested pace and content, and teachers were able to make choices based on the needs of students. For instance, Mrs. Oakley did three days in a row of number talks about using what you know about ten to solve problems with sums over 20. Her number talks lasted longer than the usual fifteen-minute warm-up, and she said she felt “okay about doing that” because she said she “was a little ahead” (in the curriculum) of the other teachers in first grade.

Number Talks

Number conversations outside the textbook lesson, most commonly in the number talk format (Parrish, 2010), were the most informative for this study. Students were encouraged and allowed to talk about their strategies. While the whole-group problem-solving portion of enVision lessons also allowed for student strategy choice and opportunities for questions and justification, the problems were always limited to the lesson for the day. At the time of the study, the whole-group lessons were about place value and the whole-group problem was generally about adding and subtracting by tens and ones.

Manipulative Use

Students did not use manipulatives other than fingers to “build” a number even with the place value problems. EnVision lesson videos showed ways that a number could be built using manipulatives and models such as base-ten blocks, tens-frames, sticks and dots, and equations. Teachers would demonstrate with demonstration-sized manipulatives. Student use of manipulatives and models was not encouraged although students referenced manipulatives indicating they had been used at some point. Students did not seek out manipulatives other than fingers.

Use of Individual Devices

Students were observed using individual devices at some point in every lesson. During independent work time, there was always at least one group on their devices if not the entire class as their workbook pages were completed. During math time the students could use two programs, IReady or Xtra Math. Information from the respective websites is shown below.

I-Ready. From the i-Ready website: "i-Ready is an online program for reading and/or mathematics that will help your student's teacher(s) determine your student's needs, personalize their learning, and monitor progress throughout the school year. i-Ready allows your teacher(s) to meet your student exactly where they are and provides data to increase your student's learning gains. i-Ready consists of two parts: Diagnostic and Personalized Instruction."

I-Ready Personalized Instruction provides students with lessons based on their individual skill level and needs, so your student can learn at a pace that is just right for them. These lessons are fun and interactive to keep your student engaged as they learn.

Xtra Math. From the Xtra website: XtraMath is an online math fact fluency program that helps students develop quick recall and automaticity of basic math facts. Students with a strong foundation have greater confidence and success learning more advanced math like fractions and algebra.

Answer-Driven

Number talks and lessons were very answer-motivated. There were no open-ended investigations. Follow-up questions during or after a student strategy explanation were "and that equaled?" There was one correct answer, and the focus of the lesson/talk was accurate answers. Workbook pages were numbered problems, usually equations or word problems, with one answer or limited answers. While there might be more than one way to think about it or strategy for solving, the objective was the one right answer. Mrs.

Oakley said, “You may have shown your strategy and got the wrong answer…” to justify asking the students to write their thinking instead of using mental math. The language the teachers used showed the awareness that the right answer was the destination. When talking with students about number line use, Mrs. Oakley often used the phrase, “ and where did you land?” The teachers asked, “How did you get there?” to multiple students several times for the same problem, encouraging the idea that there was more than one way to visualize and/or derive the answer. “Some of us got there [to the answer] in a different way. Raise your hand if you got there a different way.” Teachers encouraged students to think in different ways, but the resources used offered little opportunity for more than one answer.

Category Two: Awareness that practices diverge from beliefs

Comparison of the teacher interviews and observations revealed multiple areas that teachers’ beliefs about practices did not align with what was actually happening in the classroom. However, in the interviews, teachers voiced awareness of the divergence and offered several reasons. Evidence from observations confirmed their concerns that small groups were not being implemented, there was little student collaboration independent of the teacher, students had little opportunity for games, and “hands-on” activities were not used. It is not in the scope of this study to justify the barriers or their impact.

Barriers

COVID-19. The study was conducted in Spring 2022. Southbridge School began learning remotely in Spring 2020. The school year prior to the study, teachers and students started the school year working remotely, then students came on alternating

days, and finally all students came for in-person learning in late fall 2020. Students had to socially distance and no manipulatives were shared. Teachers, parents, and students became accustomed to students working independently on individual devices. While the 2021-2022 school year was not as disrupted as 2020-2021, the year did start with social-distancing, mask-wearing, and home quarantines for students with or exposed to COVID-19. Mid-year, mask-wearing and social-distancing was no longer required. However, routines had been established, and students were spending a great deal of time working independently in workbooks and devices without manipulatives.
Informal conversations with teachers in debriefings as well as formal interviews revealed that Covid was still very present in the minds of the teachers.

New Curriculum. The 2021-2022 school year was the first year with enVision Mathematics curriculum. Because of gaps made apparent during the pandemic, the school adopted a program that had a comprehensive virtual component. In interviews, teachers made multiple references to the issues presented by the adoption of a new math program. To address the need for a consistent curriculum and to ensure that teachers tried and learned the new curriculum, the district required fidelity to the pacing and certain parts of the new program. Teachers were required to do three components of the program: solve and share (the daily problem), Visual Learning (a 2-3 minute video illustrating the concept of the day), and independent practice (workbook pages). Teachers expressed concern that learning the program required teachers to spend so much time that no time was left for using other resources such as games or hands-on materials. They needed to just "work through the book," Mrs. Oakley said. "Next year I hope to move a little away from the fidelity because nothing is going to be the end all be all." However, teachers had some choice in the number talk and modified that part of the lesson based on the needs of the students.

Student Issues. Teachers also mentioned several student barriers to practices they would like to be implementing. Mrs. Oakley mentioned that even when she was allowed to have her students "come to the circle" (sit on the front carpet as a group instead of at desks) she did not start that practice. She was concerned that time would be wasted because students were not accustomed to that practice since COVID protocols, and there would be behavior issues.

Mrs. Fairfield felt that the diametric levels of the students in her class made it challenging for her to teach at her best: "It's a class with lots of lows and I feel like my other kids-- I have some who are pretty much on grade level-- so it's almost like half and half--on grade level and those who were just way below. It makes it really hard behavior-wise to give everybody what they need. At the beginning of the year, I had so many that had nothing to build on." She had multiple students who were not able to access the required curriculum.

Specific Practices Not Happening

During interviews, teachers mentioned practices that they felt were good or even essential, but they were not able to do in their current circumstance because of barriers listed above.

Small Groups: Meaningful Math Talk Between Students. Both teachers mentioned their desire to teach more in small groups. Teachers said that students benefited from small group instruction which was stated as a way to differentiate instruction. Games and activities done in independent small groups were also noted as a way to offer students agency in their learning by giving choice and voice. Mrs. Oakley said that students had more control of the situation in small groups. She felt that students had more meaningful talk in small groups such as game play when students negotiated and defended their thinking. More students had the opportunity to share their thoughts than in the whole group.

Hands-On. Both teachers talked about the desire to do more "hands-on." The only specific example was dropping sticks for place value. Mrs. Oakley talked about looking forward to "being able to touch things again." These comments suggested that teachers wanted the students to be able to use manipulatives and games more. However, it was not clear how the manipulatives would be used. Both teachers talked about the lack of games they were using with their students. Mrs. Oakley said that she wished she could "break down EnVision and have a math game that went along with the lesson." She mentioned the value of having kids teach other kids.

Category Three: Mathematical Concepts

What Is Number Sense?

Both teachers said that number sense was the ultimate goal for their students. Number sense was described by Mrs. Oakley as "a real understanding of what a number really means." For addition, students could exhibit number sense by being able to get started and to pick a strategy that "is not drawing 37 things and 64 things." Mrs. Fairfield felt that it was important that students were able to explain their solutions. Mrs. Oakley illustrated with the example that "students [should be] familiar with 'Why do you think that? Can you prove how you knew that?'"

Part-Part-Whole

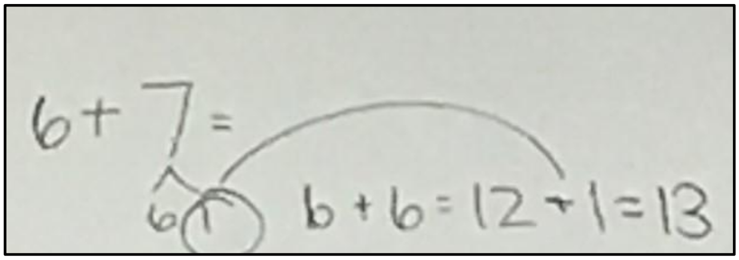
Both teachers named and described part-part-whole understanding as a goal for students. In discussing what they hoped for their students to be able to do by the end of first grade, Mrs. Fairfield said that she wanted her students to have an “understanding of a number--the concrete aspect, but also breaking it apart.” She wanted them to understand “adding two numbers is taking two parts and making a larger whole.” Mrs. Oakley illustrated with her examples that the student would understand “you can really break the number apart. When you are taking a number away, you are taking a part away” and that students will be able to “see 8 + 3 and it's only 3 more, and you can break that number into a two and a one” to facilitate addition.

Part-Part-Whole understanding is an essential understanding for using a decomposition strategy. Teachers in both rooms encouraged decomposition strategies. One way that decomposition was modeled was through the use of the number bond. Number bonds provided a representation for the part-part-whole concept. Figure 8 shows a representation of what Mrs. Oakley wrote for the following conversation about 6 + 7:

> Carson: I knew that 6 + 6 =12, so 6 + 7 = 13.
> Teacher: So, Carson actually used the doubles fact that he knew, and this is a doubles plus one more. Carson said that he knew that 6 + 6 = 12, so he said 6 = 6 = 12, but I have to add one more, so 12 + 1 = ?
> Class: 13.

Figure 8

Use of Number Bond



Flexibility With Number Understanding

It was not only important to teachers that students understood the part-part-whole concept, but also that the students could use that understanding flexibly. For example, Mrs. Oakley explained that “knowing that 63 is 6 tens and 3 ones, and if you also know there is one ten and 3 ones and if you took five (tens) away you've got 13 left,” was an essential understanding. She further elaborated students need to be flexible in their understanding in “knowing that 17 is 7 greater than 10, one ten and 7 ones, 15 + 2, 3 less than 20 like 7 is 3 less than 10.”

Magnitude on Number Line

Mrs. Oakley explained that a student with number sense can say this number is more than 10 less than this is because of where it falls on the number line or 100 more. In her explanation of number sense, she included that number sense is “movement and place on the number line.”

Commutative Property

Both teachers mentioned the need for students to understand and use the commutative property of addition in order to have number sense and also be efficient with strategies. Teachers used a more first-grade friendly term, flip flop, to discuss the use of the commutative property for addition. The term flip flop was used multiple times daily in strategy explanation. Teachers emphasized the use of commutative property in aiding students to count on from the larger addend when counting on. Mrs. Oakley encouraged students to use commutative property to make the equation 2 + 19 easier to solve.

> So, our parts in addition can be moved around. 2 + 19 is the same as 19 + 2. You are going to get the same sum. I am glad you guys are not saying that you are planning to start with 2, and I'm going to add 19. 2, 3, 4, 5, ... 21. Instead, you are saying 19, 20, 21.

Below is an excerpt from Mrs. Fairfield's class about "flip flops."

> Teacher: So, S said they have the same numbers. This one the 2 is first, and this one we started with the 9 and added the 2. What do we call those when we are adding and we change the order of the numbers we add together?
> George: Flip flops.
> Teacher: We call them flip flops, because when you are adding, you can flip flop the order. So [student] quickly knew that if 2 + 9 is 11, then we know that 9 + 2 is 11.

Other Concepts Mentioned

The following concepts were also mentioned as important to have, but not illustrated or explained.

- less than/greater than,
- fact families (number partners are always going to equal the number),
- unitizing (especially with a group of 10), and
- number paths instead of number lines.

Must You Count to Know How Many?

While the answer was the goal, teachers did not emphasize counting as the way to "get there." The video lesson from enVisions often asked the students to prove by counting with some form of “Let’s Count to Find Out.” However, teachers asked students about their way of knowing without emphasizing counting by asking “How did you see it?” instead of offering counting as a way to prove. The emphasis was on how to determine how many, not on counting to determine how many. Teachers let students offer explanations for how to determine how many. An excerpt from Mrs. Fairfield is as follows:

> Teacher: What have we been working on in math? With those tens and ones?
> Levi: We’re trying to use strategies on them and putting numbers together, so like 10 + 10 = 20.
> Teacher: So, I heard Levi say something about combining and putting numbers together. What are you doing to those numbers?
> Cody: Adding.

At one point in one of Mrs. Fairfield’s enVision lessons, the students were obviously not understanding a challenging equation adding two 2-digit numbers. This was the only time either teacher was heard to specifically ask students to prove the answer by counting.

Number Line Encouraged

In Mrs. Oakley’s class, students had number lines on their desks, and Mrs. Oakley scribed the number line strategy in multiple observations. Mrs. Oakley’s number talks almost always included at least one student using a number-line strategy (counting on by ones). She seemed to have some sense of the limitations of counting on using the number line strategy as she mentioned in both her interview and a lesson that students would be at a disadvantage the following year if the number line was not available (on their desks). Below is a typical interaction for using number line to count on for the equation 7 + 5.

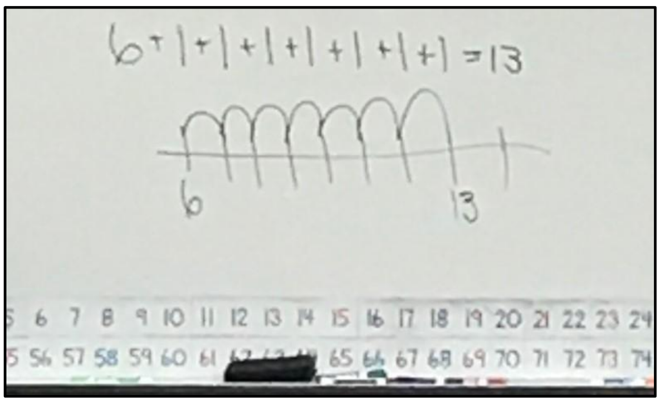
Teacher: How did you solve the problem?

Maggie: I used the number line.
Teacher: So, you used the number line. How many times did you go?
Maggie: 5.
Teacher: Where did you land?
Maggie: On 12.

Figure 10 shows how the strategy was recorded on the number line for 6 + 7.

Figure 9

Recording on Number Line for 6 + 7



Mrs. Fairfield mentioned number lines often when offering strategy options, “Sometimes we use number lines.” and “We mentioned some strategies that help us [add]: number line, mental math. Levi said he knew 10 + 10 in his head. We have used pictures to help us. We have used all sorts of strategies,” and “If you would like to use a mode to explain, a number line, and strategy is fine.”

Category Four: Role of the Teacher

The teacher’s many roles were exhibited in the classroom observations as well as interviews.

Teacher as Encourager

Mrs. Oakley hoped her practices were “pulling them on to the next place” as she spoke to the role of teacher as encourager. Both teachers also mentioned their desire for their students to try or to not give up. Teachers encouraged students to try and continue even if they did not know the answer right away. Both teachers encouraged students through their questioning and expectation--by directly asking them to think. Further, teachers conveyed to students that they valued their thinking by allowing them to share and discuss. Mrs. Oakley talked about what was happening “in your brain” or “in your head.”

At the end of the number talk, Mrs. Oakley encouraged her students in the following way:

> Okay, can we all get the same answer? Did people do different strategies? Is it okay to use different strategies? Is there only one way to solve the problem? No, there’s lots of different ways to solve a problem. Use what you already know to do that. You guys just used a lot of brain power to solve a problem that is even higher than 20. It was a little bit tricky, but you used what you knew from this problem and from this problem to solve this problem.

The teachers encouraged multiple strategies. Mrs. Oakley drew attention to all the strategies by saying, “Let’s look and see the different strategies we used: number lines, we counted on, we used facts we knew and added to that, and some of us just… knew.” At one point, Mrs. Fairfield changed lessons to allow for other strategies because she knew that students could not do the strategy required in the enVision lesson, and she encouraged them to try any strategy if they could not do the one suggested.

Teachers encouraged their students through their high expectations**.** In both classrooms all students were expected to participate. Teachers asked questions with varying degrees of difficulty to engage as many students as possible. Teachers called on multiple students in every portion of the lesson. Students volunteered to answer, and the participation rate was higher than 85% in all lessons. The expectation that students would think, and consequently learn, permeated the math lesson. The question was never “did you think?” but “What did you think?”

Both teachers expressed concern about reaching their goals for their students. Mrs. Fairfield worried that the make-up of her class would impede her ability to "reach" all of her children. About her students reaching her number sense goals, Mrs. Oakley said, "I can name three students who can do that" then asked rhetorically, "Do I think all are gonna get there?' and added, "Some are farther along than others." The researcher observed nothing in the way the teachers interacted with individual students in the classroom that revealed their doubt.

Teacher as Interpreter/Translator

Mrs. Oakley also alluded to her role as an interpreter in the comment about flexibility of number when she said that "a lot of them sometimes think like that, but they don't know how to articulate it." Both teachers acted as interpreters by rephrasing and recording student thinking to be more mathematically correct and/or accessible to all the students. A typical interaction (for 8 + 7 from class number talk) illustrates how Mrs. Oakley interpreted and translated one child's thinking about a strategy for making 10 to be accessible to the class.

> Hudson: 8 + 2 + 5. If you add 2, you wouldn't want to put the 2 with 7, you put the 2 with the 8 , then you need to add up …
> Teacher: So, are you saying you broke the 7 up into what?
> Hudson: 2 and 5.
> Teacher: So, Hudson says he put the 8 and the 2 together , because 8 and 2 is 10, then 5 more make 15.

Sometimes this looked like stretching out a student strategy:

> Teacher: How did you solve it?
> Mason: In my head.
> Teacher: How did you do that in your head?
> Mason: I used my fingers.
> Teacher: Talk me through how you used your fingers.
> Mason: I did 10 and 2.
> Teacher: But you didn't have 10. What did you do with your fingers?

Mason: I started with 7.
Teacher: You have 7, then what did you do?
Mason: 3 more.
Teacher: You added three more and you got to?
Mason: 10.
Teacher: And you are adding how many more?
Mason: 2.
Teacher: 2 more, and what answer did you get?
Mason: 12.
Teacher: So, Mason used his fingers to do 7 + 3, because he knows that 7 + 3 = what?
Class: 10.
Teacher: And then he added 2 more because he didn't have 2 fingers, he added 2 more to that because he knows that 3 + 2 = what?
Class: 5.
Teacher: 3 + 2 = 5, and he kinda broke that apart, and used the parts to solve that problem.

Sometimes, the teacher interpreted and added to what the child said. Adeline is solving the problem 7 + 15.

Adeline: 7 + 5 = 12, so 7 + 15 must equal 22.
Teacher: So, she says 7 + 5…?
Adeline: Equals 12.
Teacher: 7 + 5 = 12, we know that, alright,..
Adeline: So, 7 minus 15 = 22.
Teacher: Minus?
Adeline: Plus.
Teacher: So, you are saying that 7 + 15 must equal what?
Adeline: 22.
Teacher: 22, so she is using what she knows about her 10s to say, well, if I know that 7 + 5 = 12, then I know that 7 + 15 = 22 because there's an extra 10 in there.

Teacher as Skeptic, Not Judge

Questions asked by the teacher during the lesson were not scripted. Teachers were responsive to the answers students offered. When a student offered an answer, the teacher did not tell the child if the answer was correct. Instead, the teacher acted as skeptic, and asked the student to verify how he derived the answer. For example, for 5 + 8, the conversation went as follows:

Daniel: 5 + 5 = 10, 3 more equal 13.

Teacher: Where did the 5 and 5 come from? (asking the student to name the parts)
Teacher to class: If you have 5, what is the missing part of 8?

For equation 12 + 9, Mrs. Oakley says, “Where did you pull the one from?” Then, Mrs. Oakley helps the child explain, “You actually broke the 12 down into 1 and 11. Then, you used the one to make a friendly 10.”

***Teacher as Conduit-Connecting Strategies*.**

Through questioning and examples teachers often connected one strategy to another, more advanced strategy. Even during enVision videos, teachers stopped repeatedly to help students make connections by asking relevant questions. As shown in the example of Mason above, Mrs. Oakley used what the child did to help connect his strategy to the Making Ten strategy.

Teacher: So, Mason used his fingers to do 7 + 3, because he knows that 7 + 3 = what?
Mason: 10.
Teacher: And then he added 2 more because he didn’t have 2 fingers, he added 2 more to that because he knows that 3 + 2 = what?
Class: 5.
Teacher: 3 + 2 = 5, and he kind of broke that apart, and used the parts to solve that problem.

Another example shows the teacher connecting a strategy to what the student already knew.

Teacher: 9 + 1, Quiet thumbs up. That’s a quick one. Hudson?”
Hudson: 10
Teacher: 10, so we used 9 + 1 to help us solve which one that we have already done? The 9 + 2, so these ten friends are really helping us.

Teacher as Teacher

Both teachers used the number talks and lessons as a time to teach directly. In some cases, the teacher offered strategies and asked, “Did anyone say, I know that 7 + 3

= 10, like Mason did on his fingers, you already know that and then I know that I need 2 more to go with that 3, so then 10 plus 2 more equal 12."

In another example, Mrs. Oakley makes one students' thinking available to other students. She explained, "Do you see how he broke apart the 5 into 2 parts, the four and the one, right? So, he knew that 7 + 4 = 11, then he added one more."

There were multiple examples of teachers naming the strategies that students used. To add 35 + 8, Valent said, "I recognized that I had two fives, and then I added… and I got 43." The teacher said, "So, Valent used the make-a-ten strategy."

Naming Strategies

The teacher said,

> These are the strategies I saw the most. I saw a lot of people counting on ....with breaking up that 8 and adding the 5 and adding the 3. I saw a lot of friends making a ten with a picture. We talked about how we could have used a number line. Could you have drawn all the ones, then added 8 more ones?

Category Five: Memorization vs. Fluency

Memorization or fluency was never discussed with Mrs. Fairfield. Mrs. Oakley talked about fluency in her interview and then made the distinction between fluency and memorization ("kind of a no-no"). She also mentioned the "repetitive fluency practice" in the daily calendar routine when students determine ten more and ten less from the daily number. She states that she does not want to "drill, drill, drill," yet she encouraged her students to use a timed online fluency program.

Efficiency

Teachers did not always ask students to justify their answer on simpler problems, such as combinations to 10 and doubles. For these problems, the students could say they just knew the answer. Often, the teacher would ask the student why it was an easy problem or how they just knew (e.g., friendly ten, doubles, dot configuration on dice).

Ten Friends

The teacher said, " 7 + 3. Put a quiet thumb up if you know 7 + 3. What is 7 + 3, Levi?" "10," said Levi. "10. This is one of those ten friends that we just know. So, we are using those to help us find others," the teacher said.

The teacher also pushed efficiency explicitly and asked, "Is that a long way to solve the problem?" Another example was as follows:

> That is a much more efficient strategy starting with the larger number (2 + 19). Is that the most efficient strategy? It kinda takes a little bit, but I do like how you started with the 19 instead of the 12. You flip-flopped them.

Mrs. Oakley closed her lesson one day by saying,

> Did we all come up with the same answer? The same sum? Did we use different strategies to solve that? Would drawing 12 things and 9 things be the most efficient strategy? No, it would take me a pretty long time to do that. And then I would have to go back and count them all. So, think about what you know. Think about what you know from previous problems….Think about how you break numbers down to best help you solve these problems. Okay? That will help your addition and subtraction, and it will make you more efficient.

Online Fluency Practice

Mrs. Oakley's class was assigned math fluency practice on a computer-based program in addition to I-Ready. Students were enrolled on the program so that the program could track their progress and adjust the difficulty of the problems. Students moved through levels of increasing difficulty based on accurate completion of each level. The program showed a timer. The program did not move to the next equation until the correct answer was chosen. For example, presented with the equation 7 + 4, if the child chose the answer 10 from the choices provided, the program would not go to the next problem. The student could continue making choices until the student chose the correct answer of 11.

Summary of Results for Teachers' Beliefs and Practices

In analysis of teacher interviews and observations, the researcher organized data into the following categories: (a) Description, (b) Awareness that Practices Diverge from Beliefs, (c) Mathematical Concepts, (d) Role of the Teacher, and (e) Memorization and Fluency. The classroom environment was described revealed an engaging classroom that was predominantly teacher-led and answer-driven. However, the teacher led students with discussions about numbers where students were active participants. Students were encouraged to think independently, make choices about strategies, and defend their strategy choices. Teachers showed their knowledge and skill in encouraging conceptual knowledge through their many roles. Teachers understood and valued number sense, and encouraged it through modeling, interpreting, and connecting strategies. However, practices that encouraged use of procedures and memorization before or without understanding were still implemented. Students held high expectations for their students yet encouraged and supported them to be successful.

Relationship Between Practices and Beliefs and Student Strategy

In comparison of the categories and codes within categories for student strategy use and Teacher Beliefs and Practices, the researcher identified the following four ways in which the two parts were related: (a) Teaching and Learning Conceptual Knowledge, (b) Issues with Conceptual Knowledge, (c) Retrieval vs. Fluency vs. Memorization, and (d) Disposition (Table 8). In the following chapter, the researcher will discuss the relationship between Student Strategy Use and Teachers' Beliefs and Practices.

Table 8

Comparison of Student Strategy Use and Teachers' Beliefs and Practices

Teaching and Learning Conceptual Knowledge	
Student Strategy Use	**Teacher Beliefs and Practices**
50% of students using adv strategies at some point	Number talks
Multiple representation of the same number	Number sense as goal
Using what they know	Values part-part-whole
Combining strategies	Modeled decomposition w/#bonds
Using knowledge of properties	Models and names properties
Decomposing a variety of ways	Does not overemphasize counting to determine how many
Using a variety of strategies	Asked students to think
Choosing strategies	Asked students to visualize
Very few "nonsense" errors--"Errors in Conceptual K.	Encourages multiple strategies
	Demonstrates, names, models, connects, interprets, and translates strategies
	Asks for proof/Questions thinking
	Shares new information
Issues with Conceptual Knowledge	
50% using counting on strategies at some point and 9 students exclusively	Answer-driven
One strategy only	No hands on
+1/-1 thinking	No talk b/w students
Algorithmic--Counting on number line	Over emphasis on # line: aware that students are using as a crutch

Algorithmic--teen number + single digit, 7 + 7 = 14, 17 + 7 = 24 with no understanding	
Fluency vs. Memorization vs. Retrieval	
Just Know with no ability to explain	Answer-driven
Explaining with various explanations of ways they had memorized	Xtra Math
Remembering considered a way to justify	Accepting of Just know on easy problems, but questioned in decomposing
Lots of reference to Xtra math	Did not overemphasize speed as goal
Fast counters	
Disposition	
Willing to Engage	Classroom Attitude: math is a puzzle, high participation rate
Persistence	High Expectations
Math is Hard	Encouraged students to try
Willing to Try	Created safe environment
Of note, but no obvious category and nothing in common	
Finger use--not always unitary	enVisions
	i-Ready

Conclusion

Analysis of all participant data and the zoomed-in lens of case study analysis exemplified the complex nature of the relationship between student strategy use and teachers' practices and beliefs.

Analysis of student interviews, classroom observations, and student artifacts revealed that students in this study showed evidence of conceptual knowledge in multiple ways. However, there were issues specifically in the areas of unitary one-by-one thinking. Students showed knowledge of number relationships through creative and inventive thinking, yet evidence showed that some students were using algorithmic thinking by following procedures with little understanding. Retrieval was not considered an advanced strategy in this study, and some students who were fast and accurate showed little understanding. Students were positively disposed towards math even when challenged.

From analysis of teacher interviews and observations, the researcher found that teachers showed their knowledge and skill in encouraging conceptual knowledge through their many roles. Teachers understood and valued number sense and encouraged students to make connections between strategies. However, memorization and use of procedures without understanding were observed. Teachers held high expectations for their students yet encouraged and supported them to be successful.

In comparing findings from student strategy use and teachers' beliefs and practices, the researcher found that teachers' beliefs and practices were reflected in student strategy use in specific areas. Teachers' concept of conceptual knowledge was visible in the way that students used strategies. Positively, teachers emphasized building a network of numbers and connecting strategies. Negatively, teachers overemphasized memorization and procedures in very specific incidences. Student disposition toward math reflected their teachers' belief that they would succeed.

CHAPTER V

DISCUSSION AND RESEARCH IMPLICATIONS

"So, it is a strategy, but it may not be the best strategy."

—Mrs. Oakley to class

Statement of Problem

The problem this study addressed was that many students continue to use immature strategies for solving early addition problems past the point "it is advantageous to do so" (Hopkins, 2020). Prior to this study, little research had been done concerning the instructional practices at the crucial time that students transition to more mature strategies. This qualitative study was designed to fill a gap in the literature regarding instructional practices that can potentially encourage or inhibit students' use of mature addition strategies. This was examined through a case study of 28 first-graders and their teachers. Students' addition strategies were compared to their teachers' beliefs and practices. Looking at the influences of the learning environment allowed the researcher to look for evidence of the relationship between teacher practices and student strategies. The exploratory nature of this study allowed the researcher to be open to unforeseeable possibilities or factors that encouraged or inhibited advanced addition strategies.

Chapter V includes research questions, a summary of results including the three areas of interest, the significance of the study, and possible implications. The researcher concludes the chapter with recommendations for further research and recommendations to improve this study.

Research Questions

Central Research Question

How do teachers' beliefs and practices influence student strategy use for addition in first grade?

Sub-Questions

- What addition strategies do students use to solve early addition?
- Is addition treated as a procedure or as an understanding?
- What conceptual knowledge about addition do students exhibit?
- What are teachers' beliefs about the role and impact of conceptual knowledge in addition strategies?
- What curricular practices are teachers enacting to encourage or inhibit students' conceptual understanding needed for advanced addition strategies?
- How do teachers' practices align or diverge from their stated beliefs?

What addition strategies do students use to solve early addition?

Results showed that students used a variety of strategies with the level of student's strategy generally aligning with the level of student's conceptual knowledge. There was not one predominant strategy. However, most of the students used Counting On strategies at some point, and Counting On was the most frequent strategy. All students used a variety of strategies. Individual students chose strategies based on problem-type, their level of understanding, and the numbers in the problem. There was evidence that some students over-relied on the rules of certain strategies such as counting on the number line and using base-ten knowledge.

Most of the students used Counting On as a strategy for at least one problem. Geary (2004) classified counting on as a simple strategy. Evidence from this study showed that more important than the idea of counting on is the idea of counting by ones, or a unitary understanding of number. Students in the study exhibited one-by-one counting in a variety of, most notably the number line. However, four students used only +1/-1 thinking even in their decomposition strategies.

Almost all students used fingers to solve equations at some point. Fingers were used in a variety of ways from the simplest strategy of Counting All to strategies showing evidence of Conceptual knowledge of place value. Use of manipulatives, including fingers, can be a sign of immature counting strategies, as fingers can encourage unitary, or one-by-one counting, which might delay advancing to part-part-whole understanding (Murata, 2004; Murata & Fuson, 2006).

Is addition treated as a procedure or as an understanding?

Students showed evidence of understanding addition as described below, but also a procedural view of addition. The National Research Council (2001) defined procedural fluency as "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently." Students sometimes only show knowledge of the procedure and accuracy, without flexibility or efficiency. This lack of flexibility and efficiency is characteristic of an algorithm which Webster describes as "a step-by-step procedure for solving a problem or accomplishing some end." Students who were using the number-line strategy, even said "I numberline it," showing the inability to demonstrate the strategy on another model such as fingers. The need to redraw the number line every time also showed that the student was not flexible and was only following a step-by-step procedure to solve the problem. Even with the highest level of student thinking in the study, procedural thinking was evident when students could only use the Base-ten-knowledge strategy for a very specific type of problem and could not explain or generalize the procedure.

Most students in the study used a variety of strategies. The ability to choose when a strategy was appropriate shows evidence of procedural fluency. Although most students used immature strategies at some point, many students used the strategies selectively for more difficult problems.

The multiple and inventive ways that students counted on and decoded showed they were able to use the useful part of the procedure in a novel way. The words students used to describe their strategies were also creative, in contrast to the student who had a name for a strategy given by his mother, and he was not able to generalize to another situation.

What conceptual knowledge about addition do students exhibit?

Students' strategy use revealed how they think about number. Students showed their ability to use the integrated network of relationships to make sense of and solve the equations. Research supports that as children develop, they will "use multiple strategies and choose among these strategies adaptively" (Siegler, 2006). Students who understand the network and relationships can choose and use strategies flexibly to best satisfy the situation. Many of the students went beyond simply understanding these concepts and used them in novel ways.

Students who used unitary counters (such as fingers or a number line) had procedural fluency because they used their one-by-one understanding to solve the equations accurately and efficiently, flexibly using their knowledge of +1/-1 thinking. However, these students did not show evidence of part-part-whole understanding needed for advanced strategy use., These four students who could use decomposition strategies with +1/-1 revealed a gap in understanding worth further investigation.

What are Teachers' Beliefs About the Role and Impact of Conceptual Knowledge in Addition Strategies?

Teachers clearly stated in their interviews that developing number sense in children was their goal. For them, number sense included mathematical concepts such as part-part-whole, more/less, correct placement on a number line, and the ability to use number concepts confidently and flexibly. While the lessons in the classroom were not open-ended and were answer-driven, the teachers were able to promote number sense in alignment with their stated goals through the way they implemented the lessons and included number talks.

Both in interviews and in practice, teachers showed that they valued conceptual knowledge. Teachers specifically stated number sense as the goal for their students and further described number sense with important concepts such as part-part-whole, multiple representations of a number, and to "know what a number really is."

What Curricular Practices are Teachers Enacting to Encourage or Inhibit Students' Conceptual Understanding Needed For Advanced Addition Strategies?

Teachers had high expectations for their students. Students were encouraged to share answers, answer questions, defend their thinking, and listen to and evaluate others' strategies. They asked students, “Why?” and “How do you know?” to encourage students to verify and prove their answers. They often used these questions although they were not part of the scripted lessons. Teachers named, demonstrated, connected, and explained strategies. Even during enVisions videos, teachers stopped repeatedly to help students make connections by asking relevant questions.

Even though teachers did not overemphasize counting to determine how many, there was still an overemphasis on +1/-1 thinking with the use of the number line and an emphasis on getting to the answer. On-line fluency practice may also have had a role in students’ memorization of simple facts without being able to use what they knew flexibly for strategies.

How Do Teachers’ Practices Align or Diverge From Their Stated Beliefs?

Teachers were aware that some of their current practices did not align with stated beliefs, and they were able to name specific barriers. The barriers named were almost all situational due to the recent COVID-19 protocols and the adoption of new curriculum. Teachers were aware that the lack of voice and choice for their students, as well as for themselves, did not align with their beliefs. Analysis of interviews and observations revealed a great deal of agreement between teachers’ beliefs about early number concepts and how number concepts were taught. Teachers wanted the students to be confident in number sense and confident in fluency. Specifically, teachers wanted students to be able to decompose numbers in multiple and useful ways. Teachers modeled, named, and encouraged decomposition. Observations also showed that teachers connected simpler strategies to advanced strategies intentionally. There was little direct divergence from beliefs and practices. However, teachers may have inadvertently discouraged part-part-whole understanding with practices that encourage one-by-one thinking.

How Do Teachers' Beliefs and Practices Influence Student Strategy Use for Addition in First Grade?

The data collected through teacher and student interviews, classroom observations, lesson plans, and student work evidenced the relationship between what the teacher believed and said and what the students exhibited in their addition strategy use. Results showed that teachers' practices and beliefs were reflected in student strategy use specifically in these areas: Conceptual knowledge, Retrieval vs. Fluency vs. Memorization, and Disposition.

Conceptual Knowledge

Building a Network. In order to use strategies flexibly, students must construct a network of numerical relationships. Teachers enacted many strategies to facilitate the construction of such a network. Teachers continually talked about decomposing numbers offering multiple models for ways to think about numbers, and many students were able to use what they learned to solve problems flexibly with multiple strategies. Almost all students were making efforts to make sense of the equations. Even though some students showed limited understanding or issues with conceptual knowledge, the overall commitment to the construction of conceptual understanding was evident in teacher and student data.

Possible Barriers to Building the Network. The use of and teaching of the number line strategy can be helpful as an open number line strategy is very useful in later addition and subtraction. However, the evidence of overreliance on that strategy to the exclusion of any other strategies was problematic. Likewise, the Counting On strategy is a milestone in conceptual understanding requiring fundamental requisites skills, and yet it proved problematic when students were exclusively using counting on and not incorporating other strategies.

Certain teacher beliefs and practices could be contributing to students' inability to move past counting strategies to decomposition strategies. Students who are still using counting strategies show a view of number as a collection of ones. Even the students who decompose with +1/-1 understanding could be showing that their conception of a number is a collection of ones that can be decomposed into one and the rest. Rittle-Johnson et al. (2001) proposed a model of iterative development, where gains in one type of knowledge positively affect gains in the other type of knowledge which in turn lead to more gains in the first--a more reciprocal/symbiotic relationship like NAP's intertwined model (2001). In order to build this understanding, students need to work with these numbers. These students would likely benefit from opportunities to compose and decompose smaller (<10) numbers. The teachers were aware of the lack of practice their students had as they admitted that they would have liked to do more hands-on games and activities. Another effective practice offered by Clements (2020) is the use of different problem types. The Envision curriculum did use different problem types, but the quantities in the problems in the enVisions lessons had moved to tens and ones, and there was limited opportunity for the students to work with composing and decomposing smaller numbers.

It is important to note that algorithms are necessary for efficient mathematical computation. However, for multi-digit addition, many have accepted that the introduction (and overuse) of algorithms without regard for conceptual understanding can be harmful and have long-lasting negative effects (Kamii & Dominick, 1998). The same may be said for strategies for early addition problems. It is not that algorithms should not be utilized, it is a question of when algorithms should be introduced and encouraged, who is in charge of the thinking, and if understanding is encouraged. For the students in this study, certain strategies seem to have become algorithmic in that very little understanding accompanied the procedure. Students seemed to stop trying to make sense of the problem when they had an answer. Student invention should also be encouraged before algorithms are taught (Clements, 2021). Teachers seemed open to invention in the number talk portion of the lesson, yet a large part of the math time was used for the Envision video in which the strategy was generally demonstrated and then practiced. Student choice came after all strategies had been tried.

Memorization vs. Fluency vs Retrieval

Computational fluency is described as the ability to compute with accuracy, flexibility, and efficiency (Adding It Up, 2001). Fluency is not just speed and accuracy, but also the flexibility to use relationships in novel situations with speed and accuracy. In many studies, including the Geary (2004) study in which the interview was originally used, retrieval was considered an advanced strategy, and a response was considered retrieval if the student answered within three seconds. In this study, retrieval was not counted as a strategy. Instead, the interviewer continued to ask questions seeking to determine if the child had the conceptual knowledge to derive the fact and is now fluent or had simply memorized the combination as a fact.

When students were Counting On quickly, they could get the right answer and did not need to transition to more advanced strategies. In fact, being able to “get to” the answer quickly provides little motivation to transition to a more mature strategy that might be more time-consuming in the short-term.

The Xtra Math online fluency practice left more than one student with negative feelings about their math ability. Further, students could not explain their thinking when citing Xtra Math as the place they learned the answer.

Disposition

Students’ positive disposition towards math reflected the overall disposition of the classroom. Even when they recognized that the problems were getting harder, students seemed excited to continue. Students did not shy away when asked, even by a relative stranger, to defend their thinking. The students were accustomed to being encouraged by the teacher and the classroom environment in which they felt their thinking was valued. Students were willing to work on a problem when they did not know the answer right away, and they did not give up. Their persistence reflected the high expectations of their teachers that they would engage deeply in math and succeed.

Significance of the Study

Over the past two decades, research has shown that early math matters for future success in school and beyond (McCoy et al., 2017; Ritchie & Bates, 2013; Watts et al., 2014). Research has shown early math skills as a predictor for later math achievement as well as educational achievement in general (Duncan et al., 2007; Geary, 2013; Jordan et al., 2010; Nguyen, 2016). Numerous studies have looked at how to identify and address the needs of students with difficulties, but few studies look at the reasons a typical student might hold on to counting strategies instead of moving on to more advanced strategies. Instructional practices affect learning, and researchers are calling for more observational studies of actual teacher practices (Ball & Forzani, 2011)

Looking at all the influences of the learning environment (e.g., teacher, curricula, students) allowed the researcher to look beyond the practice to the evidence of the relationship between the strategy use and practice. The more holistic approach illuminated the interaction of students and practices. This exploratory nature of the study provided an opportunity to generate questions for later empirical research but does not allow for causal relationships to be drawn between student understanding and classroom practice.

While the current study is not generalizable to other populations and settings, the findings are important in providing a better understanding of how students' conceptual knowledge is revealed in strategy use and how teacher beliefs and practices can affect the development of conceptual knowledge. The most noteworthy finding in this study was that teachers' beliefs and actions were directly reflected in student strategy use in conceptual understanding, fluency, and disposition. Teachers facilitated the construction of a network of relationships for students, and a number of students exhibited a strong network of relationships in their variety and level of strategy use. A number of students revealed a lack of a conceptual network which could be related to an overemphasis on specific strategies for getting the answer or practices that encourage memorization. The positive and productive disposition of the students mirrored the classroom atmosphere the teachers created.

Recently in Alabama, the legislature passed the Alabama Numeracy Act (SB171, 2022) which outlines a plan for evaluation and intervention for schools whose students show low performance in math. The bill makes special mention of conceptual understanding as "the ability to reason in settings involving careful application of concept definitions, relations, or representations of either" (p. 2). The bill further describes fluency as "the ability of students to choose flexibly among methods and strategies to solve contextual and mathematical problems, to understand and explain their approaches, and to produce accurate answers efficiently" (p. 2). The bill calls for math coaches and teachers to evaluate students and to learn and enact practices that would encourage conceptual understanding and fluency. Identifying practices that promote the type of thinking exhibited by students in this study would be beneficial for those who hope to lead these efforts.

Implications

The importance of conceptual knowledge to success in mathematics is widely recognized in the literature (Rittle-Johnson & Schneider, 2015). Benefits of conceptual knowledge include flexible problem-solving, facility in procedure selection and novel situations, reasonableness of solutions as well as long-term benefits such as deeper and longer-lasting mathematical understanding. A deeper understanding of conceptual knowledge will be of value for researchers "espousing a range of theoretical perspectives who are interested in mathematical thinking, learning, and instruction" (Crooks & Alibali, 2014, p. 346). Teaching students to use strategies goes far beyond teaching students how to get the correct answer. While students are figuring out strategies, they are constructing conceptual knowledge. Practices that encourage advanced strategy use and the conceptual knowledge that advanced strategy use reveals should be further studied and implemented.

Stake (1995) stated that “conclusions arrived at through personal experience so well constructed that the person feels as if it happened to themselves” (p. 95) are naturalistic generalizations. He explained that in a case study, this same kind of generalization might not be generalizable to a larger population, but it can be relatable and applicable to the lives of others. Based on what the researcher observed about this school, the reader may draw from the implications to transfer elsewhere.

Implications for Government Institutions

The Alabama Numeracy Act has a heavy emphasis on testing. Often, these large-scale evaluations are the best means for young children to adequately exhibit their mathematical thinking. Curriculum developers, administrators, and teachers are increasingly influenced by the emphasis on raising test scores. In such an environment, undue pressure is felt at every level for students to do well on standardized assessments. While there are some open-ended assessment prompts and attempts to measure conceptual understanding, unfortunately many high-stakes tests can be predominantly multiple choice and answer-driven. Teachers become driven by how to facilitate performance on assessments instead of conceptual understanding. The emphasis on answers forces many students to become adept at quickly counting by ones, and teachers have little motivation to ask their students to try to understand when the goal is to get the right answers on the test. Less pressure for test scores and more emphasis on a learning progression that teaches students how to build connections can only come from the top.

The Alabama Numeracy Act offers many supports for improvement in mathematics including school-level coaching, quality professional development, and timely identification of students and schools in need. However, only the neediest schools will benefit from these resources. Benefits need to be expanded to all schools to ensure mathematical proficiency for all Alabama children.

Implications for School Administrators

The administration of the school in this study had ensured that many structures were in place including dedicated planning time for grade-level teams, collaboration time with and without the instructional coach, support for and protection of instructional time, purposeful and intentional analysis of student data with an emphasis on moving students through increasing understanding--not tricks for higher scores. School administrators received training on how to cultivate an atmosphere of rigor with understanding instead of just answers to harder problems.

Administration valued teachers and collaborated with experienced teachers who understand where and when concepts need to be emphasized. Administrators provided coaching and feedback with an emphasis on data that informed instruction with particular attention to conceptual understanding. School administrators offered on-going professional development for teachers to increase teachers' math content knowledge and appreciation of the conceptual understanding needed to have continued success in mathematics.

Implications for Curriculum Leaders and Instructional Coaches

Among the recommendations for the Alabama Numeracy Act, Curriculum and Instructional leaders are asked to identify and support effective mathematics teaching practices and student practices, develop the ability to identify effective instructional practices in early childhood classrooms, and to improve numeracy. Numeracy is defined in the bill as the "ability to work with and understand numbers" (p. 4). As this study illustrates, an instructional coach can have a substantial impact. Teachers in the study mentioned the role the math coach played in guiding and supporting their planning, the implementation of a new curriculum, and their in-the-trenches professional development. In this study, grade-level teams met with the math coach at least once per month. Teachers brought student work to be assessed and compared. With the help of the math coach, the team considered the needs of the students exhibited in the student work, along with upcoming standards to be addressed in planning instruction. The math coach was knowledgeable about the grade-level standards, the curriculum, and the needs of the students. The math coach also supported the teachers in the classrooms by modeling and co-teaching lessons. Working with the math coach was not viewed as a weakness as it was expected for every teacher who taught math. The math coach was very present in the school, and teachers often stopped to ask questions or share what their students were doing.

Implications for Classroom Teachers

In the current atmosphere where teachers are allowed very little autonomy in choosing materials or what concepts to teach and when, teachers can use effective practices to foster conceptual knowledge. Teachers can control the how--the practices used. Teachers can ask questions focused on facilitating the students' construction of a network of relationships.

If students can begin to form the network of mathematical relationships in the early years, then connections can be made to what they know as opposed to having to learn new rules with every new concept. The network of ways of thinking about number is built when students connect, demonstrate, interpret, and use strategies.

The Alabama Numeracy Act (2022) is explicit about what teachers are expected to accomplish. Among the 14 items that each elementary school teacher, with the full support of their principal, shall do are the following:

- Build fluency with procedures on a foundation of conceptual understanding, strategic reasoning, and problem solving over time.
- Provide students access to tools that will support mathematical thinking.
- Provide a learning environment that promotes student reasoning, student discourse, and student questioning and critiquing the reasoning of their peers.

Further, "an elementary school teacher may not engage in any practice that minimizes sense making and understanding of mathematics concepts." While these expectations are clear, how to accomplish the tasks or what practices will lead to these outcomes is yet to be determined.

Findings from the current study suggest that teachers can encourage a "foundation of conceptual understanding." Students were given time and opportunity to think and talk about numbers and mathematical relationships. Teachers facilitated the construction of those relationships by asking questions, modeling, and connecting strategies, asking students to explain and defend. Teachers expected students to engage and achieve. Teacher's ability to respond to students in a way that facilitated the construction of numerical relationships was made possible by teachers' knowledge of those relationships and knowledge of how students progress in early mathematics.

Recommendations to Improve Study

One of the limitations of this study is that it was implemented with a small population of only two first-grade classes. The teachers in this school had resources such as an up-to-date standards-based curriculum, supportive and knowledgeable administration, and professional development and coaching which explained and encouraged conceptual understanding. The findings from this school would not necessarily generalize to other elementary school populations.

A further limitation of this study was the low student participation rate in one of the classes. Because of the low number of students participating in one of the classes, the researcher was unable to compare the two classes. Cross-case comparison might have illuminated differences worth studying.

Another factor in this study was the lasting influence of COVID-19. Teachers who declined to participate or dropped out before the study began specifically mentioned the effects of the stressful year attributed to the virus. Effects from COVID-19 included student absences and virtual learning which could have affected first-graders' performance. Additionally, COVID-19 protocols changed teacher practices, and the teachers were still not back to using pre-COVID-19 practices.

Recommendations for Future Research

The study has the potential to provide needed clarification about what is happening in the classroom to connect research to practice. Additionally, findings in this study may be beneficial in identifying variables for future intervention studies. The purpose of this study was to explore how strategy was taught and how students solved addition problems in a natural environment without the addition of an intervention. This study was conducted in an optimal setting where teachers and students were well supported with training, funding, and structures. Any one of those areas could be explored to tease out possible variables for further study. In the area of training, quality professional development and follow-up in this district was intentional and well-planned. Entire grade levels attended training together along with math coaches. Teachers in this case had training in instructional practice and also using formative assessment to guide instruction. While formative assessment was not observed in this study, the training may have advanced teachers' knowledge of what to look for in student thinking making them better able to respond to students in the moment. In the area of funding, the district has chosen to fund a mathematics coach for every school. Teachers got support from an instructional coach who was knowledgeable about early additive reasoning and was dedicated to mathematics at this school. Another avenue worth investigating specifically could be exploring or comparing coaching knowledge as math coaches are often more knowledgeable about upper elementary or higher. In the area of structures, teachers were given dedicated planning time with and without the instructional coach, collaboration was expected and encouraged, and instructional time was protected. Most of these structures are generally known to be advantageous, yet not specifically in the area of early mathematics.

Many valid reasons discouraged cross-case analysis for this study. However, a cross-case analysis of students and teachers in diverse settings or with diverse beliefs and practices might allow for clearer conclusions to be drawn. Of special interest would be the comparison of student strategy use in a similar demographic setting espousing a more traditional approach where demonstration is more common than allowing students to choose conceptual strategies.

Even in this optimal setting, many students exhibited unitary counting strategies. Several issues were raised regarding student strategy that could be examined in a comparison study such as the use of the number line as an addition strategy, online fluency practice, or direct teaching of counting on strategies.

Thisstudy could only show one point in time, while a longitudinal study could determine if 1-1 unitary thinking persists in subtraction in 1st grade, and even into second and third grade and beyond. As the major focus in second grade is place value understanding, it may be unlikely that students have the opportunity in later grades to move to more advanced strategies if unfinished first-grade learning is not addressed. As this study has shown, what the teacher does in the mathematics class matters, and a look at how addition strategies are encouraged in later grades would also be informative.

REFERENCES

Agodini, R., & Harris, B. (2016). How teacher and classroom characteristics moderate the effects of four elementary math curricula. *The Elementary School Journal*, *117*(2), 216–236. https://doi.org/10.1086/688927

Agodini, R., Harris, B., Seftor, N., Remillard, J., & Thomas, M. (2013). *After two years, three elementary math curricula outperform a fourth (NCEE 2013-4019).* National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. http://files.eric.ed.gov/fulltext/ED544185.pdf

Alabama Numeracy Act, Alabama Senate Bill 171, Regular Session. (2022). http://alisondb.legislature.state.al.us/ALISON/SearchableInstruments/2022RS/PrintFiles/SB171-int.pdf

Bachman, H. J., Votruba-Drzal, E., El Nokali, N. E., & Castle Heatly, M. (2015). Opportunities for learning math in elementary school: Implications for SES disparities in procedural and conceptual math skills. *American Educational Research Journal*, *52*(5), 894–923.

Ball, D. L., & Forzani, F. M. (2011). Building a common core for learning to teach: And connecting professional learning to practice. *American Educator*, *35*(2), 17. https://eric.ed.gov/?id=EJ931211

Baroody, A. J., & Gannon, K. E. (1984). The development of the commutativity principle and economical addition strategies. *Cognition and Instruction*, *1*(3), 321-339.

Baroody, A. J., Lai, M. L., & Mix, K. S. (2006). *The development of young children's early number and operation sense and its implications for early childhood education.* In *Handbook of research on the education of young children* (pp. 205-240). Routledge.

Bay-Williams, J., & Kling, G. (2019). *Math fact fluency: 60+ games and assessment tools to support learning and retention*. ASCD.

Bisanz, J., & LeFevre, J. A. (1990). Strategic and nonstrategic processing in the development of mathematical cognition. Akt: DF Bjorklund (Ed.), *Children's strategies: Contemporary views of cognitive development*, (213–244).

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education*, *33*(4), 239–258. https://doi.org/10.2307/749740

Bodrova, E., & Leong, D. J. (1996). *Tools of the mind: The Vygotskian approach to early childhood education.* Merrill Prentice Hall.

Business, D., & Skills, I. (2011). *Innovation and research strategy for growth* (Vol. 8239). The Stationery Office.

Canobi, K. H. (2009). Concept-procedure interactions in children's addition and subtraction. *Journal of Experimental Child Psychology*, *102*, 131–149. https://doi.org/10.1016/j.jecp.2008.07.008.

Canobi, K. H., Reeve, R. A., & Pattison, P. E. (1998). The role of conceptual understanding in children's addition problem solving. *Developmental Psychology*, *34*(5), 882–891. http://dx.doi.org/10.1037/0012-1649.34.5.882

Carpenter, T. P., Franke, M. L., Jacobs, V. R., Fennema, E., & Empson, S. B. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education*, *29*(1), 3-20.

Carr, M., & Alexeev, N. (2011). Fluency, accuracy, and gender predict developmental trajectories of arithmetic strategies. *Journal of Educational Psychology*, *103*(3), 617.

Cheng, Z. J. (2012). Teaching young children decomposition strategies to solve addition problems: An experimental study. *The Journal of Mathematical Behavior*, *31*(1), 29-47.

Cheng, Z. J., & Chan, L. K. S. (2005). Chinese number-naming advantages? Analyses of Chinese pre-schoolers' computational strategies and errors. *International Journal of Early Years Education*, *13*(2), 179-192.

Clements, D. H., Dumas, D., Dong, Y., Banse, H. W., Sarama, J., & Day-Hess, C. A. (2020). Strategy diversity in early mathematics classrooms. *Contemporary Educational Psychology*, *60*, 101834.

Clements, D. H., & Sarama, J. (2020). *Learning and teaching early math: The learning trajectories approach*. Routledge.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education*, *23*(1), 2-33.

Contreras, J. N. (2002). Preservice secondary mathematics teachers' modeling strategies to solve problematic subtraction and addition word problems involving ordinal numbers and their interpretations of solutions. In: Proceedings of the Annual Meeting [of the] North American Chapter of the International Group for the Psychology of Mathematics Education (24th, Athens, GA, October 26-29, 2002). Volumes 1-4

Creswell, J. W., & Plano Clark, V. L. (2018). *Designing and conducting mixed methods research.* Sage publications.

Crooks, N. M., & Alibali, M. W. (2014). Defining and measuring conceptual knowledge in mathematics. *Developmental Review*, *34*(4), 344–377. https://doi.org/10.1016/j.dr.2014.10.001

Curriculum Associates. (2022, October 24). *i-Ready Central Resources.* https://i-readycentral.com/familycenter/what-is-i-ready/

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., Pagani, L. S., Feinstein, L., Engel, M., Brooks-Gunn, J., Sexton, H., Duckworth, K., & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, *43*(6), 1428–1446. https://doi.org/10.1037/0012-1649.43.6.1428

Engel, M., Claessens, A., & Finch, M. A. (2013). Teaching students what they already know? The (mis)alignment between mathematics instructional content and student knowledge in kindergarten. *Educational Evaluation and Policy Analysis*, *35*(2), 157–178. https://doi.org/10.3102/0162373712461850

Fishman, B. J., Marx, R. W., Best, S., & Tal, R. T. (2003). Linking teacher and student learning to improve professional development in systemic reform. *Teaching and Teacher Education*, *19*(6), 643-658.

Foundations for Success: The Final Report of the National Mathematics Advisory Panel. (2008). *U.S. Department of Education*. U.S. Department of Education. https://eric.ed.gov/?id=ED500486

Geary, D. C. (2013). Early foundations for mathematics learning and their relations to learning disabilities. *Current Directions in Psychological Science*, *22*(1), 23–27.

Geary, D. C., Brown, S. C., & Samaranayake, V. A. (1991). Cognitive addition: A short longitudinal study of strategy choice and speed-of-processing differences in normal and mathematically disabled children. *Developmental Psychology*, *27*(5), 787.

Geary, D. C., Hoard, M. K., & Bailey, D. H. (2012). Fact retrieval deficits in low achieving children and children with mathematical learning disability. *Journal* *of Learning Disabilities*, *45*(4), 291–307. https://doi.org/10.1177/0022219410392046

Geary, D. C., Hoard, M. K., Byrd-Craven, J., & Catherine DeSoto, M. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of Experimental Child Psychology*, *88*(2), 121–151. https://doi.org/10.1016/j.jecp.2004.03.002

Geary, D. C., Hoard, M. K., Nugent, L., & D. H. Bailey. (2013). *Adolescents' functional numeracy is predicted by their school entry number system knowledge*. *PloS one*, *8*(1), e54651.

Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *The Journal of Special Education*, *33*(1), 18–28.

Gervasoni, A. (2018). The impact and challenges of early mathematics intervention in an Australian context. *Invited Lectures from the 13th International Congress on Mathematical Education*, 115–133.

Gilmore, C., & Cragg, L. (2018). Chapter 14—The Role of Executive Function Skills in the Development of Children's Mathematical Competencies. In A. Henik & W. Fias (Eds.), *Heterogeneity of Function in Numerical Cognition* (pp. 263–286). Academic Press. https://doi.org/10.1016/B978-0-12-811529-9.00014-5

Ginsburg, H. (1997). *Entering the child's mind: The clinical interview in psychological research and practice*. Cambridge University Press.

Gordon, R. J., Kane, T. J., & Staiger, D. (2006). *Identifying effective teachers using performance on the job* (pp. 2006-01). Brookings Institution.

Hamre, B. K., & Pianta, R. C. (2007). Learning opportunities in preschool and early elementary classrooms. In R. C. Pianta, M. J. Cox, & K. L. Snow (Eds.), *School readiness and the transition to kindergarten in the era of accountability* (pp. 49–83). Paul H Brookes Publishing.

Hiebert, J. (1986). *Conceptual and procedural knowledge: The case of mathematics*. Erlbaum.

Hiebert, J. (2013). *Conceptual and procedural knowledge: The case of mathematics*. Routledge.

Hopkins, S., Russo, J., & Siegler, R. (2020). Is counting hindering learning? An investigation into children's proficiency with simple addition and their flexibility with mental computation strategies. *Mathematical Thinking and Learning*, *24*(1), 52-69.

Johnson, R. B., & Christensen, L. B. (2020). *Educational research: Quantitative, qualitative, and mixed approaches* (7th ed.). Sage.

Jordan, N. C., Glutting, J., & Ramineni, C. (2010a). The importance of number sense to mathematics achievement in first and third grades. *Perspectives on Math Difficulty and Disability in Children*, *20*(2), 82–88. https://doi.org/10.1016/j.lindif.2009.07.004

Jordan, N. C., Glutting, J., Ramineni, C., & Watkins, M. W. (2010b). Validating a number sense screening tool for use in kindergarten and first grade: Prediction of mathematics proficiency in third grade. *School Psychology Review*, *39*(2), 181–195. https://doi.org/10.1080/02796015.2010.12087772

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development, 74*(3), 834–850. https://doi.org/10.1111/1467-8624.00571

Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research & Practice*, *22*(1), 36–46.

Kamii, C., & Dominick, A. (1998). The harmful effects of algorithms in grades 1-4. *The Teaching and Learning of Algorithms in School Mathematics*, *19*, 130-140.

Kamii, C., & Ewing, J. K. (1996). Basing teaching on Piaget's constructivism. *Childhood Education*, *72*(5), 260-264. https://doi.org/10.1080/00094056.1996.10521862

Knapp, M. S. (1997). Between systemic reforms and the mathematics and science classroom: The dynamics of innovation, implementation, and professional learning. *Review of Educational Research*, *67*(2), 227-266.

Kuhn, T. S. (1996). *The structure of scientific revolutions* (3rd ed.). University of Chicago Press.

Laski, E. V., Ermakova, A., & Vasilyeva, M. (2014). Early use of decomposition for addition and its relation to base-10 knowledge. *Journal of Applied Developmental Psychology*, *35*(5), 444-454.

Lê, M.-L., & Noël, M.-P. (2021). Preschoolers' mastery of advanced counting: The best predictor of addition skills 2 years later. *Journal of Experimental Child Psychology*, *212*, 105252. https://doi.org/10.1016/j.jecp.2021.105252

Lee, J. S., & Ginsburg, H. P. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australasian Journal of Early Childhood*, *34*(4), 37–45. https://doi.org/10.1177/183693910903400406

Lincoln, Y., & Guba, E. (1985). *Naturalistic inquiry.* Sage Publications.

Lobato, J., & Walters, C. D. (2017). A taxonomy of approaches to learning trajectories and progressions. *Compendium for Research in Mathematics Education*, 74-101.

McCoy, D. C., Yoshikawa, H., Ziol-Guest, K. M., Duncan, G. J., Schindler, H. S., Magnuson, K., Yang, R., Koepp, A., & Shonkoff, J. P. (2017). Impacts of early childhood education on medium- and long-term educational outcomes. *Educational Researcher*, *46*(8), 474–487. https://doi.org/10.3102/0013189X17737739

McNeil, N. M., Chesney, D. L., Matthews, P. G., Fyfe, E. R., Petersen, L. A., Dunwiddie, A. E., & Wheeler, M. C. (2012). It pays to be organized: Organizing arithmetic practice around equivalent values facilitates understanding of math equivalence. *Journal of Educational Psychology*, *104*(4), 1109. doi:10.1037/a0028997

Merriam, S. B. (2002). Introduction to qualitative research. *Qualitative Research in Practice: Examples for Discussion and Analysis*, *1*(1), 1-17.

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation.* Jossey-Bass.

Mishra, S. (2021). Dissecting the case study research: Stake and Merriam approaches. In A. K. Dey (Ed.), *Case method for digital natives: Teaching and research* (1st ed., pp. 265-293). Bloomsbury.

Morgan, P. L., Farkas, G., Wang, Y., Hillemeier, M. M., Oh, Y., & Maczuga, S. (2019). Executive function deficits in kindergarten predict repeated academic difficulties across elementary school. *Early Childhood Research Quarterly*, *46*, 20-32.

Morse, J. (1991). Approaches to qualitative–quantitative methodological triangulation. *Nursing Research*, *40*, 120-123.

Murata, A. (2004). Paths to learning ten-structured understanding of teen sums: Addition solution methods of Japanese Grade 1 students. *Cognition and Instruction*, *22*, 185–218.

Murata, A., & Fuson, K. (2006). Teaching as assisting individual constructive paths within an interdependent class learning zone: Japanese first graders learning to add using 10. *Journal for Research in Mathematics Education*, *37*(5), 421–456.

Murata, A., Bofferding, L., Pothen, B., Taylor, Megan & Wischnia, S. (2012). Making connections among student learning, content, and teaching: Teacher talk paths in elementary mathematics lesson study. *Journal for Research in Mathematics Education. 43*, 616-650.

Myers, M., Sztajn, P., Wilson, P. H., & Edgington, C. (2015). From implicit to explicit: Articulating equitable learning trajectories based instruction. *Journal of Urban Mathematics Education*, *8*(2), 11-22.

McCoy, D. C., Yoshikawa, H., Ziol-Guest, K. M., Duncan, G. J., Schindler, H. S., Magnuson, K., Yang, R., Koepp, A., & Shonkoff, J. P. (2017). Impacts of early childhood education on medium- and long-term educational outcomes. *Educational Researcher*, *46*(8), 474–487. https://doi.org/10.3102/0013189X17737739

Ostad, S. A., & Sorensen, P. M. (2007). Private speech and strategy-use patterns: Bidirectional comparisons of children with and without mathematical difficulties in a developmental perspective. *Journal of Learning Disabilities*, *40*(1), 2–14. https://doi.org/10.1177/00222194070400010101

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Washington, DC: Authors.

National Mathematics Advisory Panel NMAP (2008). *Foundations for Success: The Final Report of the National Mathematics Advisory Panel.* U.S. Department of Education.

National Research Council. (2001). *Adding It Up: Helping Children Learn Mathematics* (J. Kilpatrick, J. Swafford, & B. Findell, Eds.). The National Academies Press. https://doi.org/10.17226/9822

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J. S., Wolfe, C., & Spitler, M. E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement? *Early Childhood Research Quarterly*, *36*, 550–560. https://doi.org/10.1016/j.ecresq.2016.02.003

OECD. (2013). *OECD Skills Outlook 2013 First Results from the Survey of Adult Skills* https://escholarship.org/uc/item/4b75m4tj

Parsons, S., & Bynner, J. (2005). Measuring basic skills for longitudinal study: The design and development of instruments for use with cohort members in the age 34 follow-up in the 1970 British cohort study (BCS70). *Literacy and Numeracy Studies*, *14*(2), 7–30.

Principals to Action: Ensuring mathematical success for all. (2014). National Council for Teachers of Mathematics.

Reardon, S. F. (2013). The widening income achievement gap. *Educational Leadership, 70*(8), 10–16.

Rhodes, K. T., Lukowski, S., Branum-Martin, L., Opfer, J., Geary, D. C., & Petrill, S. A. (2019). Individual differences in addition strategy choice: A psychometric evaluation. *Journal of Educational Psychology*, *111*(3), 414.

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science*, *24*(7), 1301–1308. https://doi.org/10.1177/0956797612466268

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology*, *91*(1), 175.

Rittle-Johnson, B., & Schneider, M. (2015). Developing conceptual and procedural knowledge of mathematics. In *Oxford handbook of numerical cognition* (pp. 1118–1134). Oxford University Press.

Sarama, J., & Clements, D. H. Why Early Math? Can't We Just Wait?.University of Denver.

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge. https://doi.org/10.4324/9780203883785

Sarama, J., Clements, D. H., Baroody, A. J., Kutaka, T. S., Chernyavskiy, P., Shi, J., & Cong, M. (2021). Testing a theoretical assumption of a learning-trajectories approach in teaching length measurement to kindergartners. *AERA ,7* (1), 1–15. https://doi.org/10.1177/23328584211026657

Sarama, J., DiBiase, A. M., Clements, D. H., & Spitler, M. E. (2004). The professional development challenge in preschool mathematics. *Engaging young children in mathematics: Standards for early childhood mathematics education*, 415-446.

Savvos Learning Company (2022). Envisions Mathematics, Alabama.

Saxton, M., & Cakir, K. (2006). Counting-on, trading and partitioning: Effects of training and prior knowledge on performance on base-10 tasks. *Child Development*, *77*(3), 767–785. https://doi.org/10.1111/j.1467-8624.2006.00902.x

Schneider, M., Rittle-Johnson, B., & Star, J. R. (2011). Relations among conceptual knowledge, procedural knowledge, and procedural flexibility in two samples differing in prior knowledge. *Developmental Psychology*, *47*(6), 1525.

Schwandt, T. (2000). Three epistemological stances for qualitative inquiry. Interpretivism, Hermeneutics and Social Constructionism. In N. K. Denzin & Y. S. Lincoln (Eds) *Handbook of qualitative 4esearch* (2nd ed., pp. 189–213). Sage.

Siegler, R. S. (2006). Microgenetic analyses of learning. In D. Kuhn, R. S. Siegler, W. Damon, & R. M. Lerner (Eds.), *Handbook of child psychology: Cognition, perception, and language* (pp. 464–510). John Wiley & Sons Inc.

Siegler, R. S. (1987). Some general conclusions about children's strategy choice procedures. *International Journal of Psychology*, *22*(5-6), 729-749.

Stake, R. E. (1995). *The art of case study research*. Sage.

Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education*, *36*(5), 404–411.

Stake, R. E., Denzin, N. K., & Lincoln, Y. S. (1994). *Handbook of qualitative research*. N. K. Denzin & Y.S. Lincoln (Eds.), 236-247. Sage.

Schwandt, T. A. (2000). Three epistemological stances for qualitative inquiry: Interpretivism, hermeneutics, and social constructionism. In *Handbook of qualitative research* (pp. 189-213). Sage.

Tashakkori, A., Johnson, R. B., & Teddlie, C. (2020). *Foundations of mixed methods research: Integrating quantitative and qualitative approaches in the social and behavioral sciences*. Sage publications.

Tashakkori, A., & Teddlie, C. (Eds.). (2003) *Handbook of mixed methods in social and behavioural research,* Sage.

U.S. Department of Education, Institute of Education Sciences, What Works Clearinghouse (2016, June). *Primary Mathematics Intervention Report: enVisionMATH.* http://whatworks.ed.gov

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Harvard University Press.

Walkowiak, T. A., Berry, R. Q., Meyer, J. P., Rimm-Kaufman, S. E., & Ottmar, E. R. (2014). Introducing an observational measure of standards-based mathematics teaching practices: Evidence of validity and score reliability. *Educational Studies in Mathematics*, *85*(1), 109–128.

Watts, T. W., Duncan, G. J., Siegler, R. S., & Davis-Kean, P. E. (2014). What's past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher*, *43*(7), 352–360.

Wu, S. S., Meyer, M. L., Maeda, U., Salimpoor, V., Tomiyama, S., Geary, D. C., & Menon, V. (2008). Standardized assessment of strategy use and working memory in early mental arithmetic performance. *Developmental Neuropsychology*, *33*(3), 365-393.

Yin, R. (2009). *Case study research: Design and methods* (4th Ed.). Sage.

Xtramath. (2022, October 24). *XtraMath-ten minutes a day for math fact fluency*. home.xtramath.org. https://home.xtramath.org/

APPENDIX A

GATE KEEPER LETTER AND CONSENT

Gate Keeper Letter and Consent

Consent Form to be Part of a Research Study

Title of Research: A Mixed-Methods Multiple Case Study of Classroom Practices for Addition Strategies in First Grade

UAB IRB Protocol #: IRB-300008439

Principal Investigator: Lori StClair Rhodes

Sponsor: UAB School of Education, Department of Curriculum and Instruction

Purpose of the Research
The purpose of this research is to understand how advanced addition strategies are related to instructional practice. This study will add to the body of knowledge to help educators understand how to encourage best practices for teaching and learning addition in the first-grade classroom.

Explanation of the Procedures
First grade teachers and their students will be invited to participate in this mixed-methods study that utilizes multiple case studies. Teachers will be interviewed for 30 minutes about their beliefs and practices concerning addition instruction. Students will be interviewed individually for 10 minutes about addition strategies in first grade. Math lessons will be observed and video-recorded for up to 10 days, and the corresponding math lesson plans and curriculum materials will be collected.

Risks and Discomforts
There are no known risks or discomforts associated with this study.

Confidentiality
Information obtained about participants for this study will be kept confidential to the extent allowed by law. However, research information that identifies participants may be shared with people or organizations for quality assurance or data analysis, or with those responsible for ensuring compliance with laws and regulations related to research. They include:

- the UAB Institutional Review Board (IRB). An IRB is a group that reviews the study to protect the rights and welfare of research participants.
- the Office for Human Research Protection (OHRP)
- The information from the research may be published for scientific purposes, however, identifiers will not be given out. Identities of participants, school, or system

will not be revealed in any reports, any professional presentations or journal articles, or any discussions that result from the observation.

Voluntary Participation and Withdrawal

Whether or not you take part in this study is your choice. There will be no penalty if you decide not to be in the study. If you decide not to be in the study, you will not lose any benefits you are otherwise owed. You are free to withdraw from the research study at any time. Your choice to leave the study will not affect your relationship with the institution.

Cost of participation

There will be no cost to you for taking part in this study.

Payment for Participation in Research

You will not be compensated for participating in this study.

Questions

If you have any questions, concerns, or complaints about the research or research-related injury including available treatments, please contact the principal investigator. You may contact Mrs. Lori Rhodes at 205.451.8737 or lstclair@uab.edu.

If you have questions about your rights as a research participant, or concerns or complaints about the research, you may contact the UAB Office of the IRB (OIRB) at (205) 934-3789 or toll free at 1-855-860-3789. Regular hours for OIRB are 8:00 a.m. to 5:00 p.m. CT, Monday through Friday.

Legal Rights

You are not waiving any of your legal rights by signing this consent form.

Signatures

Your signature below indicates that you have read (or been read) the information provided above and agree to:

- I voluntarily agree to help facilitate this research study.
- I understand that even if I agree to help now, I can withdraw at any time without any consequences of any kind.
- I have had the purpose and nature of the study explained to me in writing and I have had the opportunity to ask questions about the study.
- I understand that in signing this, I am allowing the study to take place within [redacted]
- I understand that all data collected in this study is confidential and anonymous.
- I understand that I am free to contact any of the people involved in the research to seek further clarification and information.

Signature of gate keepers

[redacted]

[redacted]

[redacted] [redacted]

March 8, 2022

To Whom It May Concern:

Lori Rhodes has permission to conduct her research at [redacted] Elementary School in [redacted] City Schools in the Spring Semester 2022.

Respectfully,

[redacted]

[redacted], Chief Learning Officer
[redacted]

APPENDIX B

INFORMED CONSENT FOR TEACHER PARTICIPANTS

Informed Consent Form for Teacher Participants

Consent Form to be Part of a Research Study

Title of Research: A Mixed-Methods Multiple Case Study of Classroom Practices for Addition Strategies in First Grade

UAB IRB Protocol #: IRB-300008439

Principal Investigator: Lori StClair Rhodes

Sponsor: UAB School of Education, Department of Curriculum and Instruction

General Information	You are being asked to take part in a research study. This research study is voluntary, meaning you do not have to take part in it and may end at any time. The procedures, risks, and benefits are fully described further in the consent form.
Purpose	The purpose of this research is to understand how advanced additional strategies are related to instructional practice. This will help educators understand how to encourage these strategies in the classroom.
Duration and Visits	You will participate in a 30-minute interview about your thoughts about addition in first grade. Your classroom will also be observed during math lessons up to 10 times.

Overview of Procedures	You will participate in a 30-minute interview about your thoughts about addition in first grade. Your classroom will be observed during math lessons up to 10 times. Classroom observations will be video recorded and transcribed. Although I will know your identity and contact information, I will keep this information separate from your video recordings and interview responses, and I will destroy this information as soon as it is no longer needed.
Risks	The most common risks include loss of confidentiality.
Benefits	In participating in this study, you may not directly benefit from the research. There is a potential to grow the body of research about addition instruction and help teachers understand how to encourage students' advanced strategy use in the classroom.
Alternatives	If you do not want to take part in the study, your alternative is not to participate.

Section 1.01

We are asking you to take part in a research study. The purpose of this research is to understand how advanced addition strategies are related to instructional practice. This will help educators understand how to encourage best practices in the classroom. You are being asked to participate because you teach in a school with a diverse population, and your school is very supportive of best practices in mathematics. This study plans to enroll 3 teachers.

Section 1.02

If you agree to participate in this study you will be interviewed for 30 minutes on your thoughts about addition instruction in first grade. Your math lesson will also be observed and video recorded for up to 10 consecutive days.

The private information collected as part of the research will not be used or distributed for future research studies even if identifiers are removed.

Section 1.03 Risks and Discomforts

The only risk associated with this study is breach of confidentiality.

There may also be risks that are unknown at this time. You will be given more information if other risks are found.

Section 1.04 Benefits

You may not benefit directly from taking part in this study. There is a great potential to grow the body of research about instructional practices and advanced addition strategies in first grade which can encourage best practice in the classroom.

Section 1.05 Alternatives

Your alternative is to not participate in this study.

Confidentiality and Authorization to Use and Disclose Information for Research Purposes

Federal regulations give you certain rights related to your personal information. These include the right to know who will be able to get the information ad why they may be able to get it. The principal investigator must get you authorization (permission) to use or give out any personal information that might identify you

What information may be used and/or given to others?

All identifying information will be removed before data is shared with others. Those that may have access to data include the principal investigator and the principal investigator's dissertation committee. This information may include information shared with interviews of observations.

Who may use and give out information about you?

Your personal information will only be shared by the principal investigator to those involved in supporting the study, including the principal investigator's dissertation committee.

Who might get this information?

This information may be shared with the above-mentioned people.

Why will this information be used and/or given to others?

This information may be shared to facilitate the completion of this study, including the guidance of the principal investigator's dissertation committee.

What if I decided not to give permission to use and give out my personal information?

By signing this consent form, you are giving permission to use and give out the information listed above for the purposes described above. If you refuse to give permission, you will not be able to be in this research.

May I review or copy the information obtained from me or created about me?

You have the right to review and copy the information obtained in this study. However, if you decide to be in the study and sign this permission form, you will not be allowed to look at or copy your information until after the research is complete.

May I withdraw or revoke my permission?

Yes, but this permission will not stop automatically. The use of your personal information will continue until you cancel your permission.

You may withdraw or take away your permission to use and disclose your information at any time. You do this by sending written notice to the principal investigator. If you withdraw your permission, you will not be able to continue being in the study.

When you withdraw your permission, no new information which might identify you will be gathered after that date. Information that has already been gathered may still be used and given to others. This would be done if it were necessary for the research to be reliable.

Section 1.06

Whether or not you take part in this study is your choice. There will be no penalty if you decide not to be in it.

You are free to withdraw from this study at any time. Your choice to leave the study will not affect your relationship with this institution. Please contact the principal investigator if you wish to withdraw from the study.

You may be removed from the study without your consent if the sponsor ends the study, if the principal investigator believes it is not in your best interests to continue, or you are not following study rules.

Section 1.07 Cost of Participation

There will be no cost to you for participating in the study.

Section 1.08 Payment for Participation

There is not compensation for participating in this study.

Section 1.09 New Findings

You will only be told by the principal investigator if new information becomes available that might affect your choice to stay in the study.

Section 1.10 Questions

If you have any questions, concerns, or complaints about the research or research-related injury including available treatments, please contact the principal investigator. You may contact Mrs. Lori Rhodes at 205.451.8737 or lstclair@uab.edu.

If you have questions about your rights as a research participant, or concerns or complaints about the research, you may contact the UAB Office of the IRB (OIRB) at (205) 934-3789 or toll free at 1-855-860-3789. Regular hours for OIRB are 8:00 a.m. to 5:00 p.m. CT, Monday through Friday.

Section 1.11 Legal Rights

You are not waiving any of your legal rights by signing this consent form.

Your signature below indicates that you have read (or been read) the information provided above and agree to participate in this study. You will receive a copy of this signed consent form.

____

Signature of Participant Date

____

Signature of Person Obtaining Consent Date

APPENDIX C

CONSENT TO BE A RESEARCH SUBJECT AND HIPAA AUTHORIZATION

Consent to be a Research Subject and HIPAA Authorization

Title: “A Mixed-Methods Multiple Case Study of Classroom Practices for Addition Strategies in First Grade”

UAB IRB Protocol #: IRB-300008439

Principal Investigator: Lori StClair Rhodes

Sponsor: UAB School of Education, Department of Curriculum &Instruction

Introduction

You are being asked to be in a research study. This form is designed to tell you everything you need to think about before you decide to consent (agree) to be in the study or not to be in the study. **It is entirely your choice. If you decide to take part, you can change your mind later on and withdraw from the research study.** The decision to join or not join the research study will not cause you to lose any benefits.

Before making your decision:

- Please carefully read this form or have it read to you
- Please ask questions about anything that is not clear

For Children (persons under 18 years of age) participating in this study, the term “You” addresses both the participant ("you") and the parent or legally authorized representative ("your child").

Purpose of the Study

The purpose of this research is to better understand how instructional beliefs and practices are related to strategies for addition. This can help teachers and others in the field support students with addition strategies.

Duration of the Study

Your child’s daily math lesson will be observed 5-10 times. Your child’s work from the corresponding math lessons will be collected and analyzed. Your child will participate in a brief (<10 minute) interview about how he/she solves addition problems.

Study Participation and Procedures

To participate in the study, your child will continue his/her regular participation in the daily math lesson while being observed. The observations will be video recorded for use by the researcher. Your child will also participate in a brief (<10 minute) interview about how he/she solves addition problems.

Risks and Discomforts
The only risk to participating in this study is breach of confidentiality. However, all study data will be locked. Your child will be given a pseudonym and his/her name will not be disclosed on any study data forms. This is how your child's confidentiality will be protected. There are no additional risks or discomforts from participating in this study.

Benefits
There will be no direct benefit to you or your child from taking part in this study. However, there is potential to grow the body of research knowledge in supporting addition strategy use. Knowledge may be gained that will benefit all current and future

Alternatives
Your alternative is to not participate in the study.

Confidentiality And Authorization to Use and Discolse Information for Research Purposes
Federal regulations give you certain rights related to your personal information. These include the right to know who will be able to get information and why they may be able to get it. The principal investigator must get your authorization (permission to use or give out any personal information that might identify you.

What information may be used and/or given to others?
All identifying information will be removed before data is shared with clothes. Those that may have access to data include the principal investigator, the principal investigator's dissertation committee. This information may include information shared with interviews or observations.

Who may use and give out information about you?
Your personal information will only be shared by the principal investigator to those involved in supporting the study, including the principal investigator's dissertation committee.

You may be removed from the study without your consent if the sponsor ends the study, if the principal investigator believes it is not in your best interests to continue, or you are not following study rules.

Cost of Participation
There will be no cost to you for taking part in this study.

Payment for Participation
There is no compensation for participation in this study.

New Findings
You will be told by the principal investigator or the study staff if new information becomes available that might affect your choice to stay in the study.

Questions
If you have questions, concerns, or complaints about the research or a research-related injury including available treatments, please contact the principal investigator. **You may contact Lori StClair Rhodes at 205-451-8737 or lstclair@uab.edu.**

If you have questions about your rights as a research participant, or concerns or complaints about the research, you may contact the UAB Office of the IRB (OIRB) at 205-934-3789 or toll free at 1-855-860-3789. Regular hours for the OIRB are 8:00 a.m. to 5:00 p.m. CT, Monday through Friday.

Legal Rights

You are not waiving any of your legal rights by signing this consent form.

Consent

Please, print your name and sign below if you agree to be in this study. By signing this consent form, you will not give up any of your legal rights. We will give you a copy of the signed consent, to keep.

Name of Subject

Signature of Legally Authorized Representative Date
Time

Authority of Legally Authorized Representative or Relationship to Subject

Name of Person Conducting Informed Consent Discussion

Signature of Person Conducting Informed Consent Discussion Date
Time

APPENDIX D

CLASS DESCRIPTION

Class Description

Class: A B C

Information about Teacher:
Gender: __________
Race/Ethnicity ___________
Grade Level ___________
Highest Degree ___________
Years teaching ___________ Teaching current grade ___________
Mathematics Professional Development:

Information about Curriculum:
Name of adopted Curriculum: _____________________________
Description:

Supplemental Materials:___________________________________
Description:

Information about Students:
Total number in class: Boys_______ Girls __________
Racial/Ethnicity information:
Emergent bi/multi-linguals (describe):___________________________________

Students with IEPs/504s (describe):_____________________________________

Other special needs (describe):____

Describe ability level of class as whole compared to student population of school:

Describe family involvement:
Home:
With the school:

Other information about the class that might influence classroom decisions:

APPENDIX E

IRB APPROVAL LETTER

IRB Approval Letter

THE UNIVERSITY OF ALABAMA AT BIRMINGHAM
Office of the Institutional Review Board for Human Use

470 Administration Building
701 20th Street South
Birmingham, AL 35294-0104
205.934.3789 | Fax 205.934.1301 | irb@uab.edu

APPROVAL LETTER

TO: Rhodes, Lori A

FROM: University of Alabama at Birmingham Institutional Review Board
Federalwide Assurance # FWA00005960
IORG Registration # IRB00000196 (IRB 01)
IORG Registration # IRB00000726 (IRB 02)
IORG Registration # IRB00012550 (IRB 03)

DATE: 17-Mar-2022

RE: IRB-300008439
IRB-300008439-002
A Mixed-Methods Multiple Case Study of Classroom Practices for Addition Strategies in First Grade

---

The IRB reviewed and approved the Initial Application submitted on 16-Mar-2022 for the above referenced project. The review was conducted in accordance with UAB's Assurance of Compliance approved by the Department of Health and Human Services.

Type of Review: Expedited
Expedited Categories: 7
Determination: Approved
Approval Date: 17-Mar-2022
Approval Period: Expedited Status Update (ESU)
Expiration Date: 16-Mar-2025

Although annual continuing review is not required for this project, the principal investigator is still responsible for (1) obtaining IRB approval for any modifications before implementing those changes except when necessary to eliminate apparent immediate hazards to the subject, and (2) submitting reportable problems to the IRB. Please see the IRB Guidebook for more information on these topics.

Documents Included in Review:

- CONSENT.CLEAN(PARENT).220317
- CONSENT.CLEAN(TEACHER).220317
- ASSENT.CLEAN.220317
- IRB EPORTFOLIO

To access stamped consent/assent forms (full and expedited protocols only) and/or other approved documents:

1. Open your protocol in IRAP.
2. On the Submissions page, open the submission corresponding to this approval letter. NOTE: The Determination for the submission will be "Approved."
3. In the list of documents, select and download the desired approved documents. The stamped consent/assent form(s) will be listed with a category of Consent/Assent Document (CF, AF, Info Sheet, Phone Script, etc.)

APPENDIX F

ADDITION STRATEGY PROTOCOL (Geary et.al, 2004)

Addtition Strategy Protocol (Geary et.al, 2004)

Total problems	20 problems
Single-digit problems	fourteen simple addition problems* (integers two through nine with half of the problems summing >10, half of the problems presenting the smaller integer in the first position, and no doubles)
Complex problems	six complex problems (16 + 7, 3 + 18, 9 + 15, 17 + 4, 6 + 19, and 14 + 8).
Instructions	The problems are presented horizontally on paper as well as verbally. Students are asked to solve the problem mentally as quickly as possible using any strategy of their choice. The child is asked to say the answer and then explain the strategy used to solve the equation.

*Each digit is presented two to four times, and one half of the problems sum to 10 or less. The larger-valued integer was presented in the first position for one half of the problems.

Addition Strategy Assessment

S = Sum   MIN= Counted on from larger MAX =Counted on from smaller ✓=correct answer

	✓	Notes--see key	Time	Code
2 + 3				
5 + 4				
3 + 5				
6 + 3				
2 + 6				
4 + 3				
7 + 2				
6 + 7				
8 + 6				
9 + 5				
5 + 8				
4 + 7				
3 + 8				
8 + 4				

16 + 7				
3 + 18				
9 + 15				
17 + 4				
6 + 19				
14 + 8				

(Geary et al., 2004)

APPENDIX G

CODE CHART BASED ON GEARY'S (2004) CODES

Code Chart Based on Geary's (2004)

Strategy	Description	Code
Counting All	counts both addends	SUM**
Counting three times	counting out both addends, then counting all, i.e., in adding 3 + 4, the student will count 1,2,3, then count 1,2,3,4, then count 1,2,3,4,5,6,7)	SUM 3x**
Counting two times	counting from one, i.e., in adding 3 + 4, the student will count 1,2,3 and then count 4,5,6,7	SUM 2x**
Counting ON	states one addend and then counts a number of times equal to the second addend	CO**
Counting on from smaller addend	starts with the larger addend (4, then 5,6,7	COS**
Counting on from larger addend	starts with the smaller addend (3, then 4,5,6,7	COL**
Decomposition	one or both addends are decomposed to facilitate solving the problem—decomposed 4 into 3 + 1 in order to get 3 + 3 = 6, then added one more.	DEC
Other	Strategies that do not fit in any of the preceding categories	Specific strategy will be recorded

**Use of fingers or other manipulatives, head bobbing, etc. was recorded.

APPENDIX H

CLASSROOM LESSON PROTOCOL

Classroom Lesson Protocol

(Inspired by Inside the Classroom: Observation and Analytic Protocol, Horizon Research, Inc, 2000)

Observation Date: ______________________

Time: ________ to ________

Teacher A B C

Part One: Overview

Number of students present: Boys: _________ Girls: _________
Other Adults in the Room
____

Comments about teacher or class specific to this lesson:

Information from lesson materials: about Lesson Standards, Focus, and/or Purpose:

Comments from teacher about Lesson Standards, Focus, or Purpose:

Description of Lesson (synopsis to be completed after the lesson)

Observer comments about Lesson Overall:

Part Two: Observation:

During observation, write notes as detailed as possible about what is happening during the lesson.

Part Three: Observation of recorded lesson

What do you notice about the topics below in relation to addition:

Planning:

Resources:

Use of time (whole-group/small groups/individual, interruptions, housekeeping)**:**

Access, equity, diversity (cooperative learning, language-appropriate strategies/materials):

Student-Teacher Interaction:

Student-Student Interaction:

Mathematical Content:

Teacher Confidence/Accuracy with Content:

Classroom Culture (participation, respect, encouragement of conjectures, proof, justification)

Degree of Sense-Making:

Activities (games, centers, worksheets, etc.):

Use of Manipulatives:

Other: (Use back if necessary)

Part Four: Teacher Debrief of Lesson (if possible)

Tell Me Your Thoughts about the Lesson:

Possible prompts (if needed):
How did you decide on that lesson?
What went well?
What would you do differently?
What do you think about the student's thinking?
Where will you go next?
What do you want to know about what the students are thinking?

APPENDIX I

TEACHER INTERVIEW

Teacher Interview

Class: A B C

Information about Teacher:
Gender: __________
Race/Ethnicity ___________
Grade Level ___________
Highest Degree ___________
Years teaching ___________ Teaching current grade ___________
Mathematics Professional Development:

Tell me about your math background and/or your background in teaching math?

What experience has most affected your math instruction (college class, PD, coworker, etc.)?

Who makes the decisions about how you teach math? What degree of autonomy do you have?
Is there anything you wish you could do differently?

What do you want your students to be able to do when they go to the next grade?

What understandings do you want your students to have concerning addition/subtraction?

APPENDIX J

ASSENT PROTOCOL FOR STUDENTS

Assent Protocol for Students

"Today we are going to talk about how you solve some math problems. This will help me understand how to help students learn about adding number together. It will also help teachers, too. Anything that you tell me is just between us. You do not have to talk to me if you don't want to. Are you okay if we talk a bit about how you add numbers?"

______ Child assented to talk.

______ Child did not want to participate.

APPENDIX K

EXPANDED CODES

Expanded Codes

Strategy	Description	Code
Error in conceptual understanding	Error in conceptual understanding resulting in an unreasonable answer	NO!
Guessing	No apparent strategy or attempt at accuracy	G
Counting All (count 3x)	Counting all without counting on by counting three times	CA3
Counting All	Counting on from one--not counting 3 times (1,2,3,4,5,6,7,8, 9,10,11,12,13,14)	CA→
Finger Configuration	Student uses the configuration of fingers to determine the amount without counting either addend or the sum	FC
"Just Know"	Student indicates automatic recall	JK
Counted on Larger	Counts on from the greater addend	COL
Counted on smaller	Counts on from the lesser addend	COS
Remembered	Remembered from another source	REM
Commutative Property	Specifically mentions "flip-flop"	COMM
Compensated	Adjusting quantity of addends to simplify the equation	COMP
Former Problem	Using a former problem	UFP
Using known fact	Using known fact (and can explain)	UKF
Teen Number Knowledge	Student counts all first addend on fingers, then begins counting the second addend from 1 on the remaining fingers and determines the sum by how many fingers are left when the 10 fingers are all counted.	TNK
Base Ten Knowledge	The student uses knowledge of the base-ten number system to determine the sum without counting as was done in TNK	BTK
Decomposed	Decomposing in some way other than	DEC

	one of the specific strategies listed	

**Use of fingers or other manipulatives, head bobbing, etc. was recorded.

APPENDIX L

INVITATION LETTER

Invitation Letter

Date:

Dear Families,

My name is Lori Rhodes, and I am working on my doctorate degree at UAB. To complete my degree, I am conducting a research study investigating math instructional practices and addition strategies in first grade. I am asking for your permission to include your child in this research study. The purpose of this study is to better understand how instructional practices are related to addition strategies. If you choose to participate, your child will take part in a 10-minute interview about his/her addition strategies. I will also be observing in their classroom during math lessons.

Please read the attached informed consent document. If you agree for your child to participate in this study, please sign it and return to your child's teacher. If you have questions, please call or email me. I would be happy to talk with you about the study.

Lori StClair Rhodes
[redacted]
lstclair@uab.edu
UAB