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Computational Modeling of Agile Tire Slippage

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COMPUTATIONAL MODELING OF AGILE TIRE SLIPPAGE

by

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MECHANICAL ENGINEERING

ABSTRACT

Tire slippage dynamics is based on steady state analysis. This poses limits on advances in traction control systems and other vehicle control systems that can improve the safety of a vehicle. Agile tire slippage dynamics is a newer method that uses a smaller time step and time frame. It is typically applied to analyze the transient mode of a tire, which occurs within the first few milliseconds of a torque being applied to a wheel. During this time, the tire is not yet rotating, which can be a potential cause for safety hazards. Using agile tire slippage dynamics to analyze a tire's behavior at this point can allow for vehicle control systems to be activated sooner resulting in an overall improvement to vehicle safety.

Keywords: tire dynamics, tire slippage dynamics, agile tire dynamics, agile tire slippage dynamics, computational and mathematical tire model, and computer simulation

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C_s is the suspension damping

C_t is the damping on the tire

D_i is the current distance traveled

D_{i-2} is the distance traveled for two test runs before the current test

D_0 is the initial distance that was traveled

F_a is D'Alambert's force, it can also be referred to as the inertia force on the tire

$F_{frame.x}$ is a force acting on a wheel from the vehicle frame in the x-direction (frame force)

F_x is the circumferential force on the tire

f_1 is the initial rolling resistance coefficient for both the minimum and maximum values

f_2 is the final rolling resistance coefficient for both the minimum and maximum values

I_w is the moment of inertia of a wheel about the axis of rotation

K_s is the suspension stiffness

K_t is the tangential stiffness

k is the empirical exponential factor

k_1 is the initial empirical exponential factor for the minimum and maximum values

k_2 is the final empirical exponential factor for the minimum and maximum values

M is the gross mass of the vehicle

m_s is the sprung mass of a rear wheel

m_u is the unsprung mass of a rear wheel

$mass$ is the total mass of a rear wheel

n and n_i are the number of wheel revolutions in both the driven and driving modes

n_{i-2} is the number of wheel revolutions for two test runs before the current test

n_0 is the initial number of wheel revolutions in both the driven and driving modes

p_w is the internal pressure of the tire

R_x is the rolling resistance force of a wheel, it can also be referred to as the longitudinal reaction force

R_z is the normal reaction of the wheel

R_z^{static} is the static normal reaction of the wheel, and is equal to W_w

r is the unloaded tire radius

r_w is the rolling radius

${r_w}^0$ is the rolling radius in the driven mode on asphalt

$r_{w_{cum_i}}$ is the cumulative rolling radius in the driving mode

$r^0_{w_{cum_i}}$ is the cumulative rolling radius in the driven mode

$r_{w_{inst_i}}$ is the instantaneous rolling radius in the driving mode

$r^0_{w_{inst_i}}$ is the instantaneous rolling radius in the driven mode

S_δ is the tire slippage

T_w is the wheel drive torque

V_t is the theoretical velocity at zero slip

V_x is the actual velocity of the center of the wheel

W_w is the normal load of the wheel

z_r is equal to the road profile, h

z_u is the normal reaction force for the unsprung mass as a function of the distance, D

ε_w is the angular acceleration of the wheel

μ_{px} is the peak friction coefficient

ν_1 and ν_2 are empirical factors

ω_w is the angular velocity of the driving tire, which is loaded with a torque

CHAPTER 1

TECHNICAL BACKGROUND, AND PROBLEM FORMULATION

1.1 Introduction

Agile tire slippage is a category within agile tire dynamics or agile tire slippage dynamics. Agile tire slippage dynamics is usually defined as the study of the interactions between a tire and the soil it comes in contact with before the end of the relaxation time [1]. Agile tire slippage dynamics is a fairly new area of research for “tire-surface interactions” [1] that is an advancement on tire slippage dynamics. One benefit of this would be applications that use traction control systems, which apply brakes to a wheel after it starts to spin. The use of agile tire dynamics would enable traction control systems to be activated sooner. Therefore, the main goal of this thesis is to design and model the agile slippage of a tire, and to show how agile tire dynamics is an advancement over conventional or steady state tire dynamics. This thesis will focus specifically on agile tire slippage when the tire is in a transient mode, and when a steady motion is reached. This transient mode occurs during the relaxation time of the tire, which is mentioned to be “approximately 60-80 ms” [1]. Completing this goal would allow for further advancements in tire and vehicle design that will minimize the adverse effects of tire slippage and maximize the additional safety that a tire can provide to the vehicle. The following is an example of how tire slippage can be dangerous even if it only lasts for a few seconds or milliseconds, further illustrating the importance in being able to improve overall vehicle safety by improving the design of both vehicle control systems and tires.

If a tire is spinning on ice, but the vehicle has not started to move forward yet, then the traction control will kick in, which will automatically slow down the vehicle. The traction control should shut off once the vehicle begins to move in the desired direction. This scenario poses a safety risk for both military and passenger vehicles. In the case of military vehicles, the safety risk comes into effect while the vehicle is moving more slowly due to the traction control, which is causing them to be an easy target. In the case of passenger vehicles, the decreased speed caused by the traction control poses a safety risk to both the passenger vehicle and other vehicles on the road. The limitations to traction control described in the dangerous scenarios explained above could be minimized or avoided with the aid of agile tire slippage dynamics. Sections 1.2 and 1.3 describe both tire slippage dynamics and agile tire slippage dynamics in more detail.

1.2 Tire Slippage Dynamics

This section will briefly explain tire slippage dynamics in order to set the foundation for understanding agile tire slippage dynamics. In simple terms, tire slippage dynamics studies “tire kinematic and force parameters and characteristics” [2]. There are two common approaches for doing this. The two approaches are as follows: Terramechanics and Wheel Dynamics based approaches [2]. Both of these approaches will be described briefly in the following paragraph. An analysis for tire slippage dynamics is typically done by using a steady state analysis.

As mentioned above, there are two main approaches to tire slippage dynamics. The Terramechanics approach focuses on “tire-soil normal and shear deflection-stress relationships which make presuppositions and create foundations for determining, through an integration process, the tire normal load and longitudinal force” [2]. In Terramechanics, the normal load and the longitudinal force “are formed by the tire-soil deflection-stress relationship under a pre-specified numerical level of the tire slippage” [2]. However, this poses a problem because the tire slippage is actually a result of the “force interaction between the tire and ground that is determined by normal wheel dynamics”, “torsional wheel dynamics”, “mechanical properties of the tire-soil system and physical properties of soil” [2]. However, in the Wheel Dynamics based approach, the tire slippage is not a pre-specified value. Instead, it adds to the Terramechanics approach “by creating an analytical basis for developing algorithms and systems to control the tire slippage process” [2].

The use of steady state analysis is effective for gathering data on a tire’s general behavior; however, the accuracy and reliability of these results have limitations due to the fact that most applications are not well suited for steady state analysis. Another issue is that these limitations for steady state analysis will limit the advances that can be developed for vehicle control systems, ultimately restricting improvements to vehicle handling and safety. The use of agile tire slippage dynamics is a possible solution for this problem and is further discussed in section 1.3.

1.3 Agile Tire Slippage Dynamics

Agile tire slippage dynamics is still considered to be relatively new. There are a few key differences that separate it from tire slippage dynamics. As previously stated in section 1.2, an analysis for tire slippage dynamics is typically done by using a steady state analysis. In contrast, agile tire slippage dynamics uses non-steady and non-constant values such as the instantaneous rolling radius. These changes require the data to be recorded in the tens of milliseconds instead of seconds, which is common for tire slippage dynamics. Having data recorded in a smaller time frame and with a smaller time step makes it possible to mathematically observe a tire's behavior before it begins to move. Whenever a torque is applied to a wheel, there is a short period of time when only the wheel is rotating. During this time, the tire is deflecting as a result of the applied torque; this period is usually referred to as the transient mode of the tire. After a few milliseconds, the tire begins to rotate with the wheel. Modern vehicles typically have several built-in safety systems with fast response times with some as short as 0.100 seconds [3]. "These really small time intervals provide fast vehicle-driver-environment interactions, but the actual control of the interactions occurs after the vehicle has got to a critical motion situation" [3]. Once a vehicle has reached this type of situation, it poses a safety hazard to the passengers and other nearby vehicles. Agile tire dynamics makes it possible to analyze a tire's behaver sooner, which could help prevent vehicles from reaching such a critical point in their motion and mobility. Knowing a tire's behavior during the transient mode will allow for vehicles to be controlled sooner, ultimately leading to better tire control mechanisms such as traction control systems and safer vehicles.

The next step was to find a program that is capable of accurately modeling a tire using both conventional and agile tire dynamics. In addition to this requirement, the ideal program would be capable of running the tire model on different types of terrain including asphalt or deformable surfaces. The following programs were considered: Adams Tire, FTire, MATLAB, and LabVIEW. Each of these programs are discussed further in the following paragraphs.

1.4.1 Adams Tire

Adams Tire is built into the main Adams programs such as “Adams Car, Adams Chassis, Adams Solver, or Adams View” [4]. As the name implies, Adams Tire makes it possible to add tires to a mechanical model; it can be used “to simulate maneuvers such as braking, steering, acceleration, free-rolling, or skidding” [4]. FTire is another program that can be used with Adams Tire. In this case, FTire cosin/tools is the version of the FTire software that is compatible with Adams.

1.4.2 FTire

Some basic information on FTire can be found on the Cosin Scientific Software website. FTire stands for “Flexible Structure Tire Model,” and it is “based on physics-oriented modeling” [5]. The website describes in detail some of the features and key components of FTire. The following is a brief summary of some of these features and key components: it is designed to “explain complex tire phenomena on a strictly mechanical, tribological, and thermodynamic basis” and “provides a large variety of analysis and visualization tools, as well as detailed output for further user-specific post-processing" [5]. Figure 1.1 is an example of a tire model built using FTire.

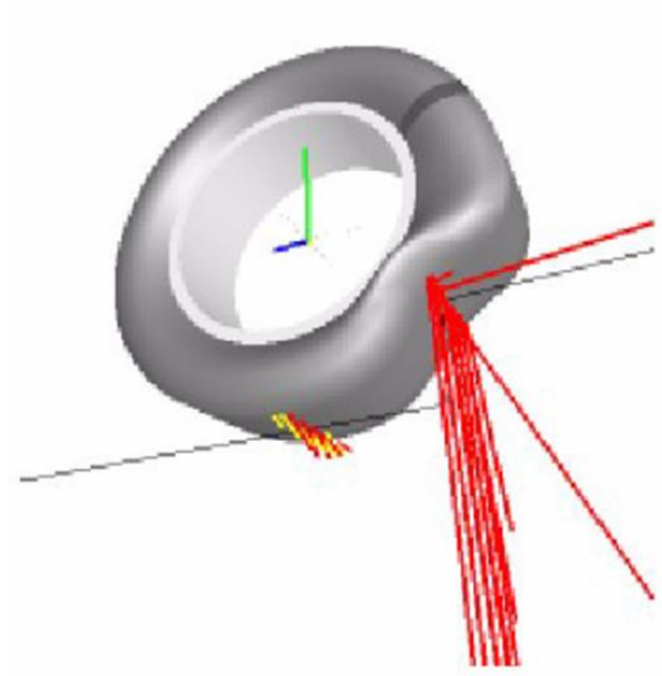

Figure 1.1. Example of a FTire Tire Model [4].

1.4.3 MATLAB

MATLAB is another program that could be used to develop a computational and mathematical model of a tire capable of using agile tire dynamics. In addition to the following information about some built in ODE solvers, FTire can be used through MATLAB or Adams as a third-party software depending on how the FTire license is set up. According to an article written by David Houcque, MATLAB uses the Runge-Kutta method for solving ODEs in addition to having a built-in solver called ODE45 [6].

ODE45 uses an “explicit Runge-Kutta (4,5) formula, Dormand-Prince pair” [6]. The 4 and 5 mean that MATLAB uses a combination of a fourth and a fifth order method. Both methods are comparable to the more traditional classical fourth order Runge-Kutta method. Because this method uses a varying step size by “choosing the step size at each step in an attempt to achieve the desired accuracy,” the ode45 is appropriate for a broad range of applications. It is especially useful when the ODE problem is being solved for the first time [6].

There are a few possible tire modeling programs provided by MathWorks. The potential programs are as follows: Tire (Friction Parameterized), Tire (Magic Formula), Tire (Simple), and Vehicle Body. These programs are available as Simscape Blocks [7]. These blocks can be accessed through Simulink, which is included with the MATLAB software.

1.4.4 LabVIEW

Depending on which modeling program is chosen, LabVIEW may not be needed; LabVIEW and Simulink both work in similar ways by using a block-based coding method. If LabVIEW is used in addition to one of the modeling programs, the information provided on the National Instruments website will aid in learning how to properly set up the program for both a conventional and agile tire dynamics slippage simulation.

Upon the completion of an initial analysis of the different software programs that were available, it was decided that FTire would be the most appropriate program to use.

Even though MATLAB was not designed specifically for tires, it could be used in addition to FTire or as an alternative program. FTire can be set up to run through MATLAB or Adams Tire. A detailed software analysis will be discussed in section 1.5 to finalize the decision on which program will be the best suited to create a computational and mathematical model of a tire capable of simulating agile tire slippage.

1.5 Detailed Software Analysis

Sections 1.6.1 and 1.6.2 are a detailed software analysis of FTire and MATLAB respectively. One software program will be selected in section 1.6.

1.5.1 FTire

As previously stated in section 1.4.2, FTire is a commercial software for modeling tires and running simulations on the tire model. FTire (cosin/tools) is the version of FTire that works through Adams. Table 1.1 shows the tire characteristics obtained from the Continental website for the Continental MPT 81 – 365/80 R 20 tire, which was chosen to create a tire model.

Table 1.1. Tire Characteristics and Parameters for the Continental MPT 81 Tire [8, 9]

Continental MPT 81 - 365/80 R 20 - (14.5R20)		
Load Index	152	
Speed Index	K	
Height	42.87"	1089 mm
Width	14.88"	378 mm
Weight	154.2 lbs.	70 kg
Overall Diameter	42.87"	1089 mm
Rim Width	11"	280 mm
Speed Rating	68 MPH	110 km/h
Inflation Pressure	88PSI	6 bar
Max Load Rating	7826 lbs. (34812 N)	3550 kg
Rolling Circumference	129"	3275 mm
Tread Width	14.80"	376 mm
Tread Depth	0.8"	20 mm

As part of the analysis on the tire model, it was desired to be able to calculate the slippage and longitudinal deflection using FTire (cosin/tools); however, these characteristics are only offered as an input rather than an output. This conflicts with the main goal of developing a computational and mathematical model of a tire capable of simulating agile tire slippage.

1.5.2 MATLAB

Based on the initial analysis for MATLAB from section 1.4.3, MATLAB is a good candidate for developing a computational and mathematical model of a tire using both conventional and agile tire dynamics. MATLAB and Simulink have the flexibility to design the computational and mathematical tire model in such a way that allows for a broad range of analysis options. One key advantage that MATLAB and Simulink have over FTire is that both allow for easy additions or adaptions to the tire model as needed. This is important in order to compare the tire slippage results from both the conventional and agile methods in an organized and easy to understand format. Because Simulink and LabVIEW work in similar ways and because Simulink is already part of the MATLAB software, a detailed software analysis was not necessary for LabVIEW.

1.6 Software Selection

FTire and MATLAB were two possible programs that could be used to develop a computational and mathematical model of a tire. In section 1.5.1, it was concluded that FTire would not work for this thesis because the tire slippage and longitudinal deflection are only offered as inputs. In section 1.5.2, it was concluded that MATLAB and Simulink could be a possible software choice for this thesis based on the overall flexibility and ease of making adaptions to the model offered by both programs. As a result, out of the programs considered, it was decided that MATLAB and Simulink are the best option for this thesis for the development of a computational and mathematical model of a tire.

1.7 Main Goal and Objectives

The main goal is to facilitate improvements of the overall safety of a vehicle by modeling agile tire slippage dynamics in a computer simulation. Knowing the agile tire slippage will allow for traction control systems to be safer and quicker resulting in an overall improvement to vehicle safety. Based on the above-formulated goal of the thesis work, the objectives of this research study come out as follows:

1. Develop a mathematical model capable of performing agile tire slippage computer simulations
2. Use the computational results to analytically prove that the agile tire slippage dynamics approach provides faster and more precise data in modeling tire slippage as compared to the conventional approach to tire slippage analysis dynamics.

CHAPTER 2

COMPUTATIONAL AND MATHEMATICAL MODEL

2.1 Mathematical Model for Agile Tire Dynamics

Equations (2.1 – 2.3) are normally used for a steady state analysis; however, these equations can be used in an agile tire dynamic analysis if the instantaneous rolling radius is used. Tire slippage is commonly calculated from Equation (2.1) [2].

$$S_\delta = \frac{V_t - V_x}{V_t} \tag{2.1}$$

V_t is the theoretical velocity at zero slip

V_x is the actual velocity of the center of the wheel

Because ${r_w}^0$ can be used in place of r_{we}, the effective rolling radius, in the case of a tire operating "on both firm and deformable surfaces," Equation (2.2) is another equation that can be used to calculate the tire slippage [2].

$$S_\delta = 1 - \frac{V_x}{\omega_w r_w^0} \tag{2.2}$$

ω_w is the angular velocity of the driving tire, which is loaded with a torque

${r_w}^0$ is the rolling radius in the driven mode on asphalt

In addition, the rolling radius can be calculated from Equation (2.3) [2].

$$r_w = \frac{V_x}{\omega_w} \tag{2.3}$$

In order to analyze the agile tire slippage dynamics of a tire, the rolling radius is replaced with the instantaneous rolling radius. In addition, the equations for tire slippage and the rolling radius take a different form, see Equations (2.4 and 2.6 – 2.9). Equation (2.4) is the form of the slippage equation that is used when a vehicle is being tested as long as the condition of the wheel travel, D, is the same in both modes [2].

$$S_\delta = 1 - \frac{n_{driven}}{n_{drive}} \tag{2.4}$$

n_{driven} is the number of wheel revolutions in the driven mode

n_{drive} is the number of wheel revolutions in the driving mode

Equation (2.5) is another way to express Equation (2.4) [2].

$$S_\delta = 1 - \frac{r_w}{r_w^0} \tag{2.5}$$

Equations (2.6 – 2.7) are for the instantaneous rolling radius in the driven and driving modes respectively [2]. In the MATLAB code, which includes Simulink, and the plots the instantaneous rolling radius is written as r_w^0 instead of how it is displayed in Equation (2.6).

$$r_{w\,inst_i}^0 = \frac{D_i - D_{i-2}}{2\pi(n_{driven_i} - n_{driven_{i-2}})} \tag{2.6}$$

$r_{w\,inst_i}^0$ is the instantaneous rolling radius in the driven mode

D_i is the current distance traveled

n_i is the number of wheel revolutions in both the driven and driving modes

D_{i-2} is the distance traveled for two test runs before the current test

n_{i-2} is the number of wheel revolutions for two test runs before the current test

$$r_{w_{inst_i}} = \frac{D_i - D_{i-2}}{2\pi(n_{drive_i} - n_{drive_{i-2}})} \tag{2.7}$$

$r_{w_{inst_i}}$ is the instantaneous rolling radius in the driving mode

Equations (2.8 and 2.9) are for the cumulative rolling radii in the driven and driving modes respectively [2].

$$r^0_{w_{cum_i}} = \frac{D_i - D_0}{2\pi(n_{driven_i} - n_{driven_0})} \tag{2.8}$$

$r^0_{w_{cum_i}}$ is the cumulative rolling radius in the driven mode

D_0 is the initial distance that was traveled

n_0 is the initial number of wheel revolutions in both the driven and driving modes

$$r_{w_{cum_i}} = \frac{D_i - D_0}{2\pi(n_{drive_i} - n_{drive_0})} \tag{2.9}$$

$r_{w_{cum_i}}$ is the cumulative rolling radius in the driving mode

Equation (2.10) is used to mathematically approximate the relationship between the tire slippage and the circumferential force on the tire [2]. This equation is commonly rearranged to solve for the tire slippage.

$$F_x = \mu_{px} R_z (1 - e^{-ks_\delta}) \tag{2.10}$$

F_x is the circumferential force on the tire

μ_{px} is the peak friction coefficient

R_z is the normal reaction of the wheel

k is the empirical exponential factor

Equation (2.11) is used to find the torque when the circumferential force on the tire is known. This equation was based on an equation in chapter 1.3 of the *Driveline System of Ground Vehicles* textbook in equation (1.35) on page 71 [10].

$$T_w = F_x \cdot r_w^0 \tag{2.11}$$

T_w is the wheel drive torque

Equation (2.12) is used to find the rolling radius in the driven mode when the vehicle is on asphalt [11], and Equation (2.13) is used to find the normal reaction [12].

$$r_w^0 = r\frac{rp_w+v_1W_w}{rp_w+v_2W_w} \tag{2.12}$$

r is the unloaded tire radius

p_w is the internal pressure of the tire

W_w is the normal load of the wheel

v_1 and v_2 are empirical factors

$$R_z = R_z^{static} + K_t(z_r - z_u) + C_t(\dot{z}_r - \dot{z}_u) \tag{2.13}$$

R_z^{static} is the static normal reaction of the wheel, and is equal to W_w

C_t is the damping on the tire

K_t is the tangential stiffness

z_r is equal to the road profile, h

z_u is the normal reaction force for the unsprung mass as a function of the distance, D

Where h is the height or the road profile of the road as a function of D.

Equation (2.12) can be rewritten in the form of Equation (2.14) because W_w is equal to R_z^{static}.

$$r_w^0 = r\frac{rp_w + \nu_1 R_z^{static}}{rp_w + \nu_2 R_z^{static}} \tag{2.14}$$

The circumferential force was calculated in Simulink based on equation (1.42) on page 74 of the *Driveline System of Ground Vehicles* textbook [10], see Equation (2.15). The definitions for each variable are based on the ones listed in textbook [10].

$$T_w = \left(R_x + F_{frame.x} + F_a\right)r_w^0 + I_w \varepsilon_w \tag{2.15}$$

R_x is the rolling resistance force of a wheel

$F_{frame.x}$ is a force acting on a wheel from the vehicle frame in the x-direction (frame force)

F_a is D'Alambert's force

I_w is the moment of inertia of a wheel about the axis of rotation

ε_w is the angular acceleration of the wheel

Note, R_x, can also be referred to as the longitudinal reaction force and F_a, can also be referred to as the inertia force on the tire.

2.2 Further Explanation on the Mathematical Model

Tire characteristics and behaviors such as the conventional and agile tire slippage, the circumferential force on the tire, and the velocity of the tire will be modeled mathematically via the equations in section 2.1 and graphically using plots. These plots will show their respective tire characteristic or behavior vs time. Plotting the data in the time domain will allow for an easy comparison of the steady state results vs the agile dynamic results; these results are shown and discussed in section 3.1. Finally, the conventional and agile approaches for determining the tire slippage will be compared including any advantages and disadvantages for both conventional tire slippage dynamics and agile tire slippage dynamics.

Based on the results of the detailed software analysis from section 1.6, MATLAB was chosen to be the program used for the computational and mathematical model. For this thesis, a preexisting code was used to form the basis for the computational and mathematical model of the tire. A few adjustments were made that are explained further in section 2.3 including, some additions and changes to the existing Simulink system and subsystems. In some cases, these preexisting subsystems were changed or added to in order to gather the desired data from the simulation. These changes and additions to the Simulink system are further explained in section 2.3 as they pertain to each method.

2.3 Computational Algorithm

Below, Figures 2.1 through 2.3 show a block diagram to illustrate the three methods used for determining the slippage of a tire: actual, conventional, and agile respectively. First, it is important to note that excluding the title block, all three methods share the same computational algorithm for the first 9 blocks with the block for F_x being the 9th block. The first step in the computational algorithm is to select the terrain type. In this thesis, asphalt, meadow, and soil are the three different terrains that were used. Once the terrain type has been selected, then the main code file is ready to run the first part of the simulation. This first part of the simulation uses various tire and vehicle parameters needed to create the computational and mathematical model of the tire; this lays the framework for the second part of the simulation that will generate data for both the conventional and agile tire dynamics based methods using Simulink. Once the first stage of the simulation is complete, figures of the road profile height, h; the rolling resistance coefficient, f; the empirical factor, k; and the peak friction coefficient, μ_{px} are plotted as a function of the distance traveled, D. In this case, the distance traveled is in increments of 1 meter up to 1,000 meters. These figures are shown in the Appendix in Figures A.1 through A.12. Next, a Simulink file that is part of the MATLAB code is run; this generates the data needed for comparing the results from conventional and agile tire dynamics. The Simulink file is the second part of the computational simulation. For this thesis, a 10 second simulation was used. Once the Simulink simulation is complete, it sends the data from its results to MATLAB. This data is then stored in MATLAB until the next simulation run, which replaces the old data with data from the most recent run of the computational simulation. The next step was to create a script in MATLAB that would generate the plots needed to illustrate the results from the data in both parts of the computational simulation of the tire. These plots are shown in Figures 3.1 through 3.39 and show the results for each terrain type. After running the script, the process was repeated for each type of terrain.

During the Simulink portion of the simulation, the velocity, V_x, which is a function of time, is integrated to obtain the distance traveled at a specific time step. This process produces values for $h, f, k,$ and μ_{px} at the same time step. The next block, z_r, is equal to the road profile height, h. The code for MATLAB and Simulink is set up to solve for z_u, which is an ODE. This value is used in the next step to help solve for the normal reaction force, R_z, which was done using Equation (2.13). This equation was incorporated into a subsystem in Simulink designed to calculate the normal reaction force. The other values needed to determine the normal reaction force are shown in Tables 2.2 and 2.3. The longitudinal reaction force, R_x, is calculated in Simulink as well by multiplying the rolling resistance coefficient, f, by the normal reaction force, R_z. Finally, the circumferential force, F_x, was calculated in Simulink based on Equation (2.15); this equation was simplified based on the conditions of the simulation and some of the variables were substituted with an equivalent variable.

As previously stated, the tire slippage was found using three different methods. The actual method uses Equation (2.10), which can be rearranged to determine the tire slippage; this is the typical way to calculate the tire slippage. In order to approximate the tire slippage on a tire as it is rotating, the conventional method was used; this method uses the rolling radius in both the driven and driving modes, see Equation (2.5) for further details. Finally, the agile method uses agile tire dynamics while in contrast, the conventional method used tire dynamics. Both the conventional and agile methods use Equation (2.5) to find the slippage. In both methods, the rolling radii have been replaced with a different set of rolling radii that correspond to each method. For the conventional method, the rolling radii are replaced with the cumulative rolling radii, and for the agile method, the rolling radii are replaced with the instantaneous rolling radii. The cumulative rolling radii were calculated based on Equations (2.8 and 2.9), and the instantaneous rolling radii were calculated based on Equations (2.7 and 2.14). Because Equation (2.14) was used to determine the instantaneous rolling radius in the driven mode, Equation (2.6) was rearranged in Simulink to solve for n_{driven}. This is a valid approach based on equation (21) on page 29 of the *Agile Tire Slippage Dynamics for Radical Enhancement of Vehicle Mobility* article [13]. The results from the computational simulation are shown and discussed in section 3.1.

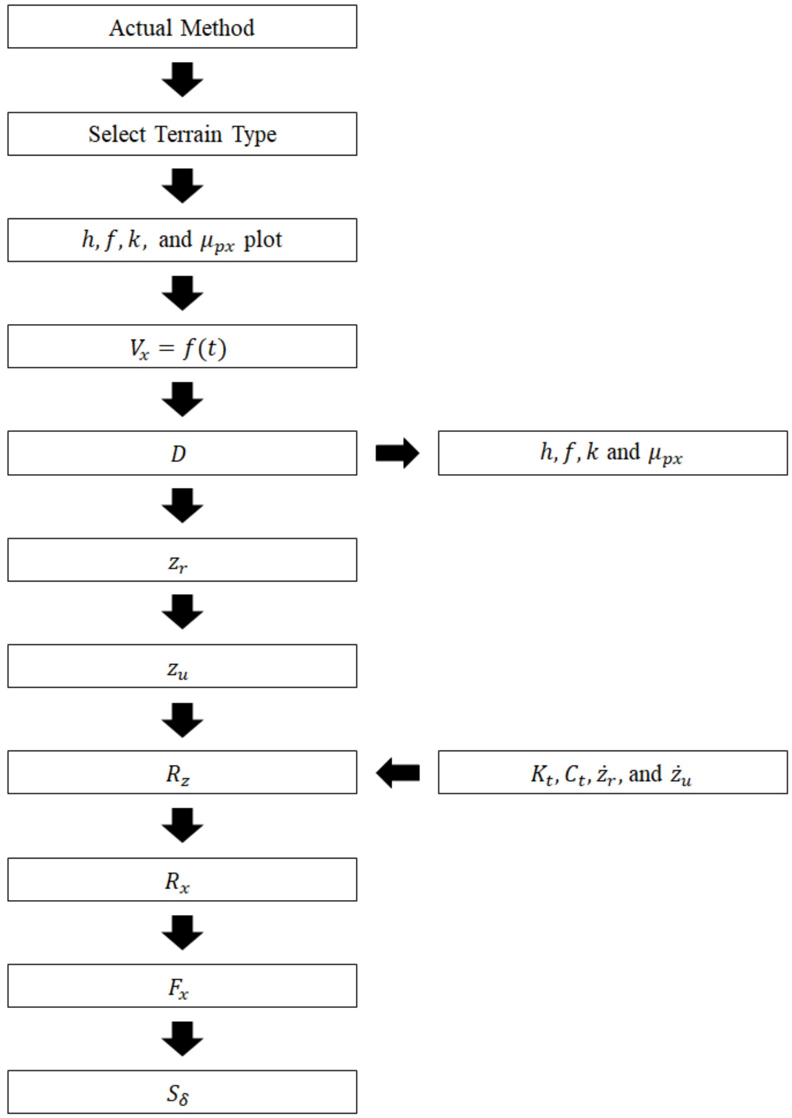

Figure 2.1 Actual Method Block Diagram

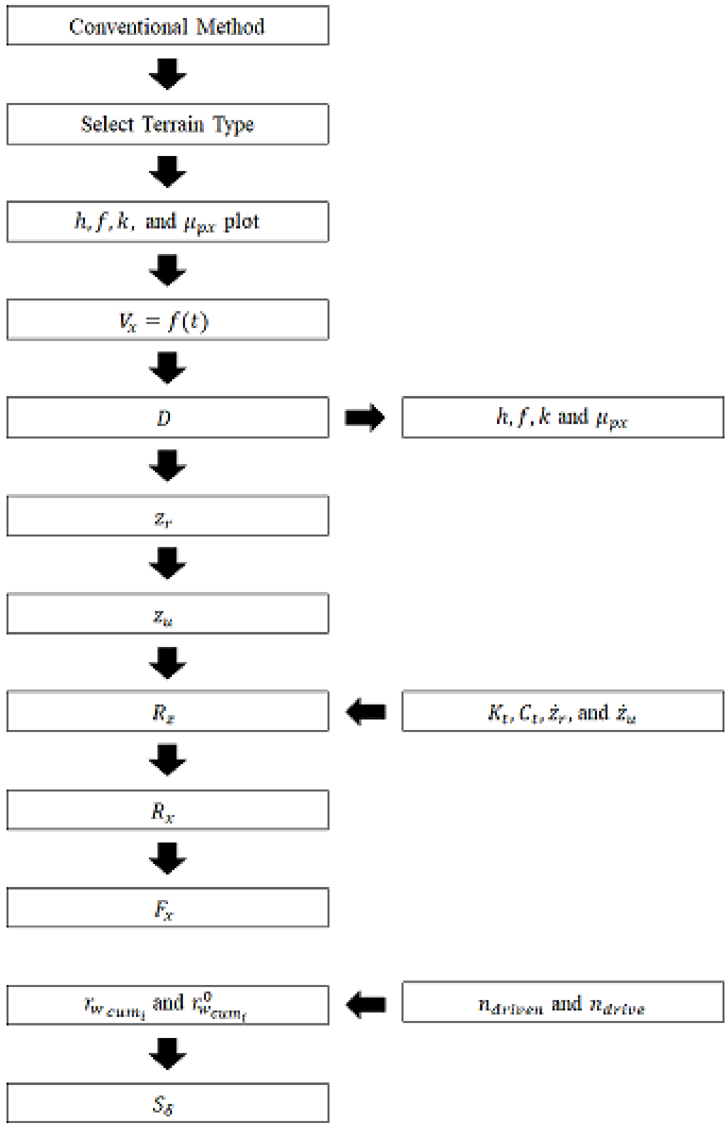

Figure 2.2 Conventional Method Block Diagram

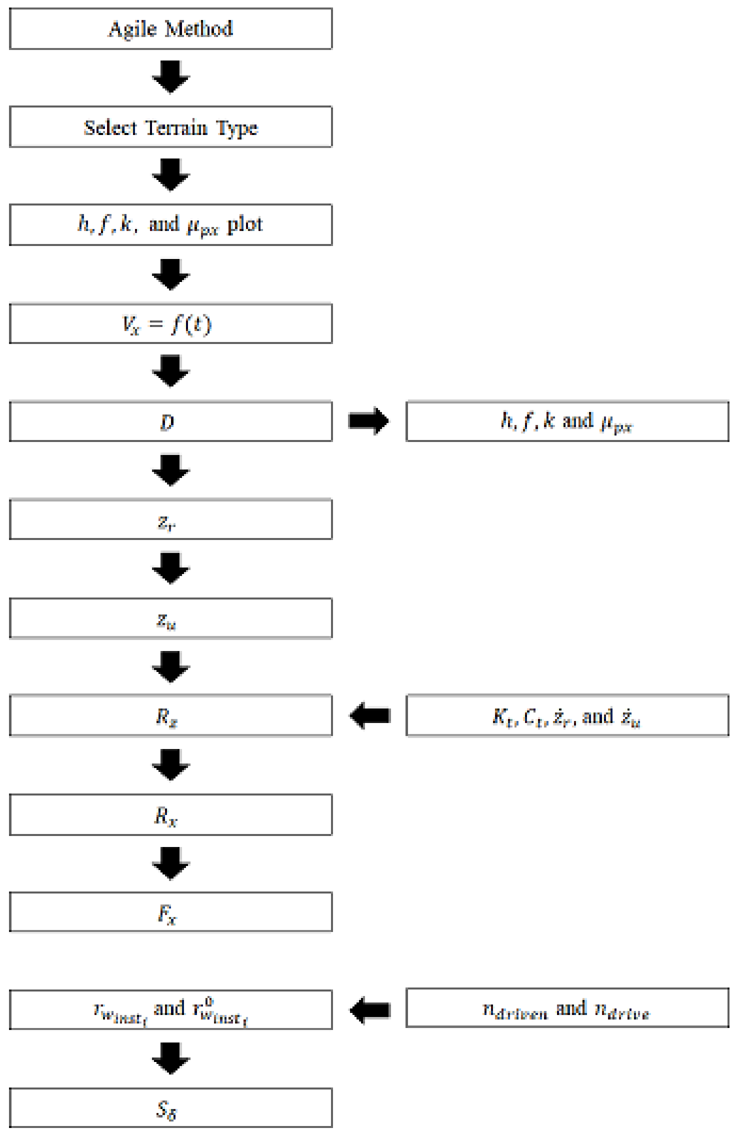

Figure 2.3 Agile Method Block Diagram

The values and parameters displayed in Tables 2.1 through 2.3 are used in the main MATLAB code, which is used for the first part of the computational tire simulation. These values were part of the original code and were based on values from vehicles and tires that are similarly sized to the Continental MPT 81 – 365/80 R 20 tire because experimental values could not be recorded. Table 2.1 shows the conversions that were used, Table 2.2 lists the terrain characteristics for the three types of terrain that are used in this thesis, and Table 2.3 lists the initial parameters and constants. Note, in MATLAB, W_w, is the weight of a rear wheel.

Table 2.1. Conversions

RPM	$30/\pi$
MPH	2.236936
DEG	$180/\pi$
RAD	$\pi/180$

Table 2.2. Terrain Characteristics

	Asphalt	**Meadow**	**Soil**
$f_{1\,min}$	0.016	0.086	0.095
$f_{1\,max}$	0.048	0.223	0.242
$f_{2\,min}$	0.016	0.0640	0.0707
$f_{2\,max}$	0.048	0.1659	0.1801
$\mu_{1\,min}$	0.65	0.425	0.285
$\mu_{1\,max}$	0.85	0.65	0.407
$\mu_{2\,min}$	0.65	0.48	0.308
$\mu_{2\,max}$	0.85	0.72	0.378
$k_{1\,min}$	6.9	6	3.8
$k_{1\,max}$	8.8	7	5.2
$k_{2\,min}$	6.9	6.8	4.9
$k_{2\,max}$	8.8	8	6.4
K_t	96,130 N/m	79,209 N/m	25,479 N/m

Table 2.3. Initial Parameters and Constants

M	9,525 kg
m_s	2,354 kg
m_u	336 kg
$mass$	2,690 kg
W_w	26,389 N
C_s	13,000 N*sec/m
K_s	50,000 N/m
C_t	1,740 N*sec/m

The next section will discuss in greater detail how the MATLAB simulation was set up and used to produce the results for chapter 3.

2.4 MATLAB Simulation

As previously mentioned in the Acknowledgements section, the computational and mathematical tire model for this thesis were created by adding to a preexisting code for a tire model. The original tire model had a main code that is referred to as the first part of the simulation in section 2.3. In addition to the main code, there were additional codes for generating the road profile or the height and some codes needed for the Simulink portion of the simulation to run properly. The main code used the data shown in Tables 2.1 through 2.3 in addition to setting up the initial parameters and characteristics for the computational and mathematical model. The Simulink portion of the code consists of one main block system that is organized into several subsystems. Some of the subsystems create the necessary background data for the file to run as well as creating the data needed for the majority of the system. Most of the system is focused on vehicle dynamics. Some of the following subsystems within this portion of the main system are designed to calculate the following: the circumferential force, the linear velocity, the normal reaction, and the slippage. The data for these parameters in addition to a few others that are shown in section 3.1, is sent back to MATLAB after the Simulink portion of the simulation is complete. As previously mentioned in section 2.3, a script was created to develop all of the figures shown in section 3.1 and the Appendix. In addition to creating the script, a few additions or changes were made to the Simulink file. These changes were made in order to determine the agile tire slippage of the tire based on agile tire dynamics.

The first objective was to develop a mathematical model capable of performing agile tire slippage computer simulations. As previously described in section 2.3 and in the previous paragraph for this section, MATLAB and Simulink were used in conjunction with agile tire slippage dynamics to develop a mathematical model that is capable of performing agile tire slippage computer simulations. The computational results of this computer simulation are shown and discussed in section 3.1.

CHAPTER 3

COMPUTER SIMULATION RESULTS, ANALYSIS, AND DISCUSSION

3.1 Computational Results and Discussion

The following figures show the results from a 10 second simulation with three different types of terrain. The terrain types used are asphalt, meadow, and soil. The figures are organized to allow for an easy comparison of how each type of terrain affects a given tire characteristic or set of gathered data. Additional figures for some of the initial parameters such as the road height and the peak friction coefficient are shown in the Appendix. It is important to note, that this type of simulation would normally be run approximately 20 times or so, and then the data would be averaged together; however, this paper is written more as a proof of concept. As a result, the following figures only show data from one simulation for each type of terrain.

Figures 3.1 through 3.3 and 3.4 through 3.6 show the changes in the velocity and acceleration respectively, experienced by the tire for each type of terrain.

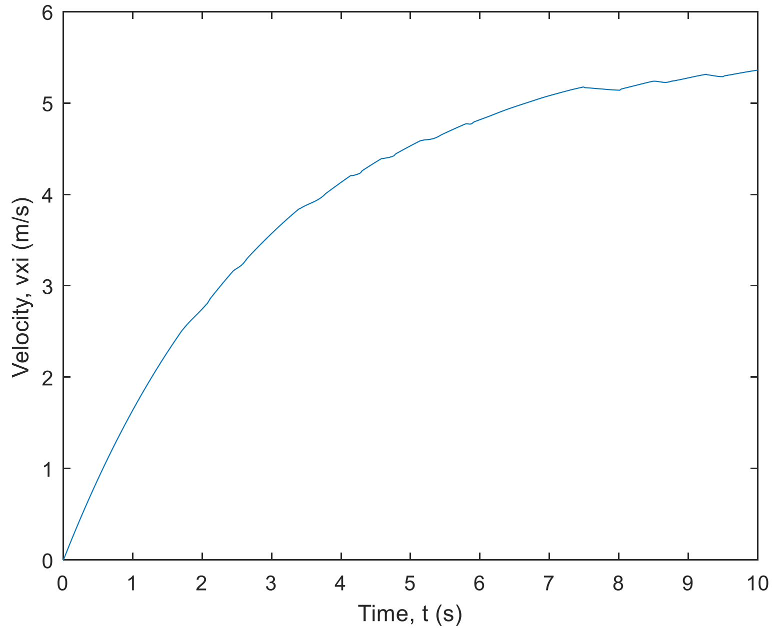

Figure 3.1. Tire Velocity on Asphalt

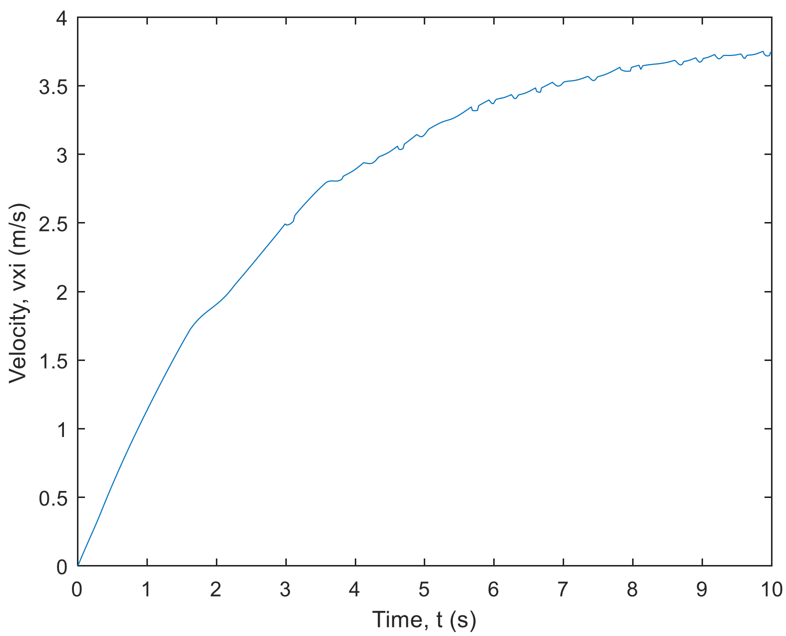

Figure 3.2. Tire Velocity on Meadow

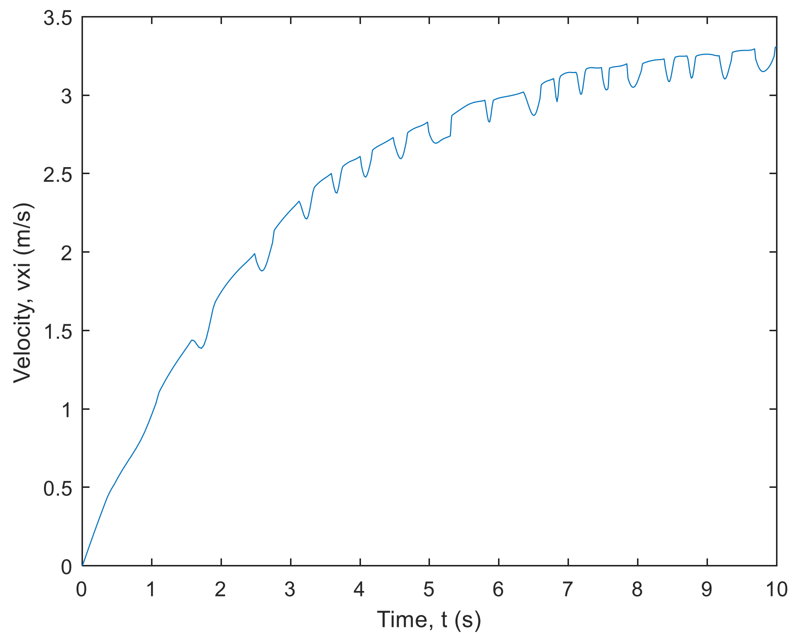

Figure 3.3. Tire Velocity on Soil

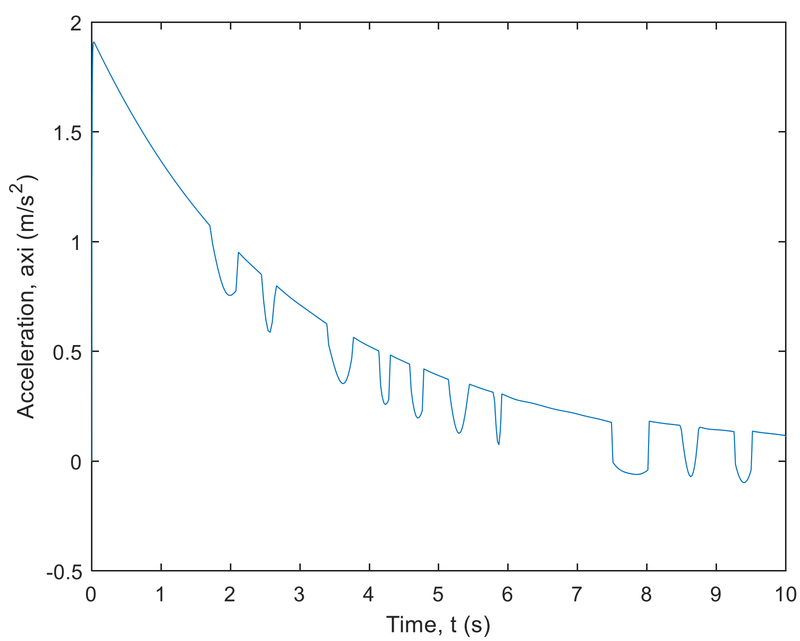

Figure 3.4. Tire Acceleration on Asphalt

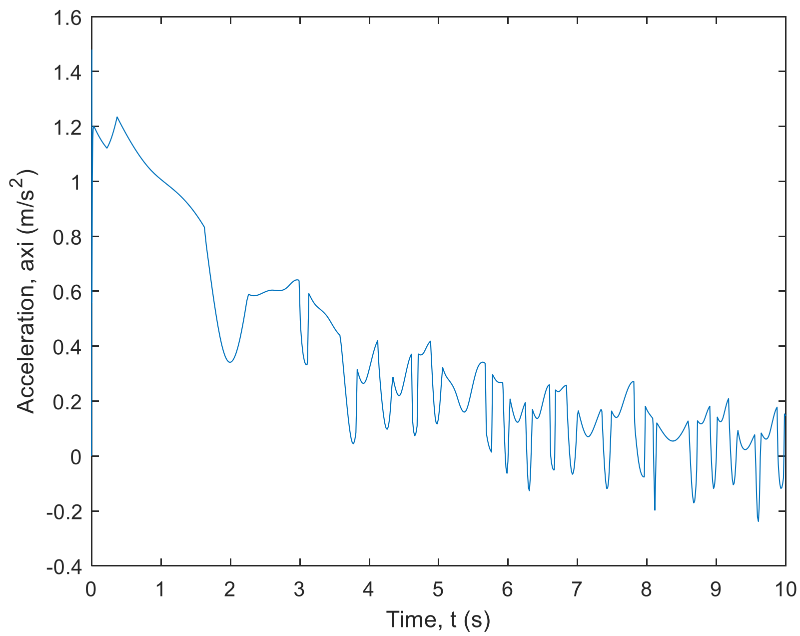

Figure 3.5. Tire Acceleration on Meadow

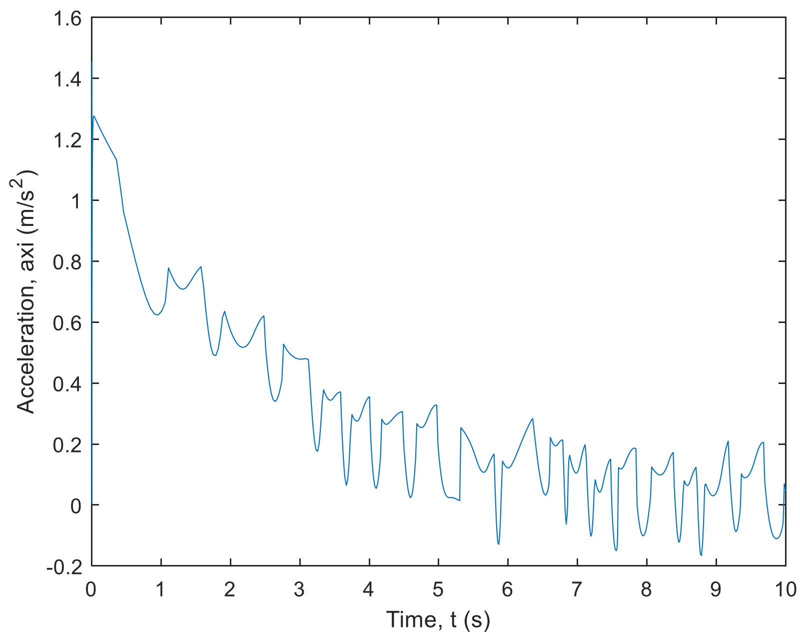

Figure 3.6. Tire Acceleration on Soil

Figures 3.7 through 3.9 and 3.10 through 3.12 show the normal reaction force and the longitudinal reaction force respectively. In Figures 3.7 through 3.9, the static normal reaction is equal to the weight of the quarter model of the vehicle.

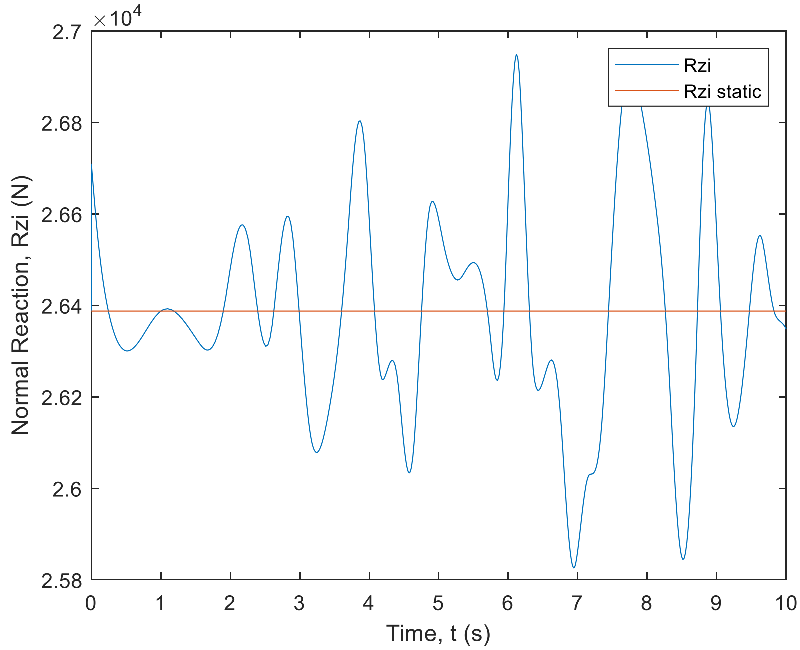

Figure 3.7. Tire Normal Reaction Force on Asphalt

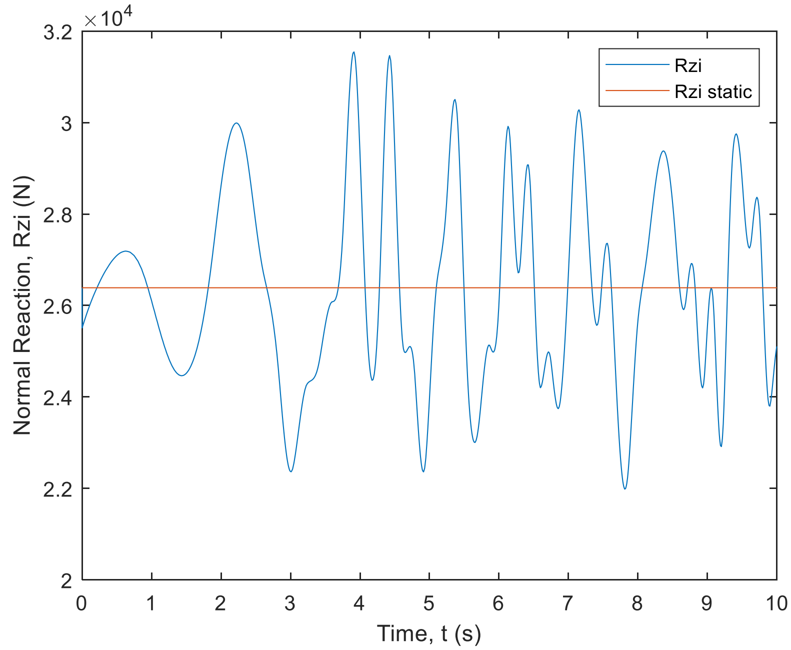

Figure 3.8. Tire Normal Reaction Force on Meadow

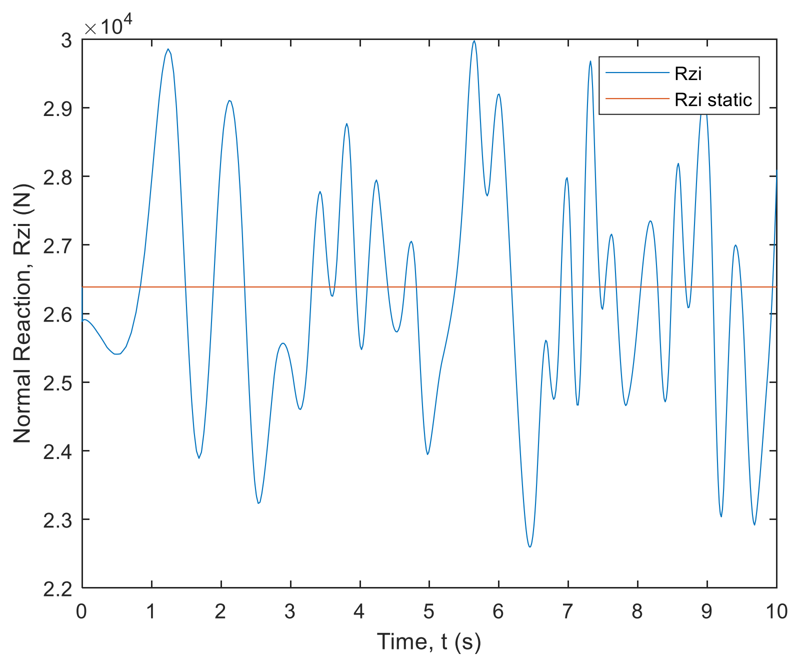

Figure 3.9. Tire Normal Reaction Force on Soil

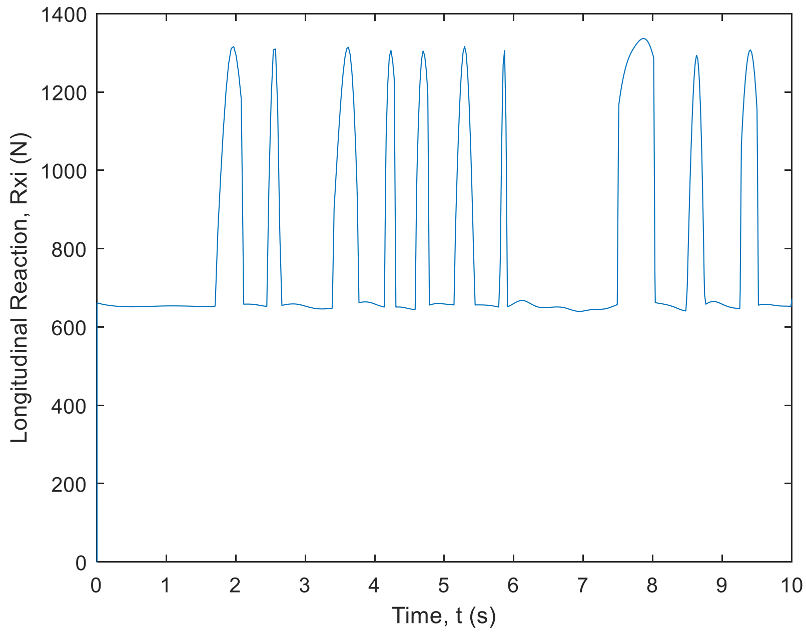

Figure 3.10. Tire Longitudinal Reaction Force on Asphalt

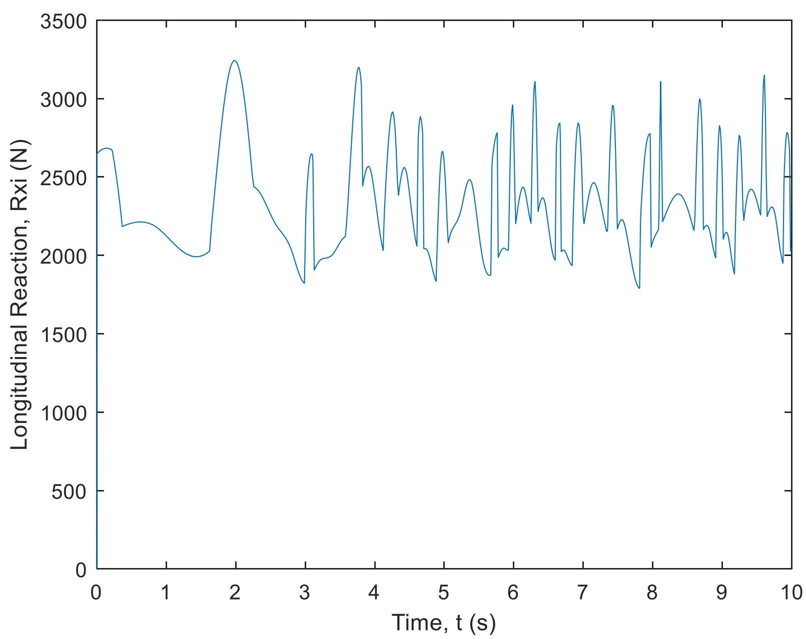

Figure 3.11. Tire Longitudinal Reaction Force on Meadow

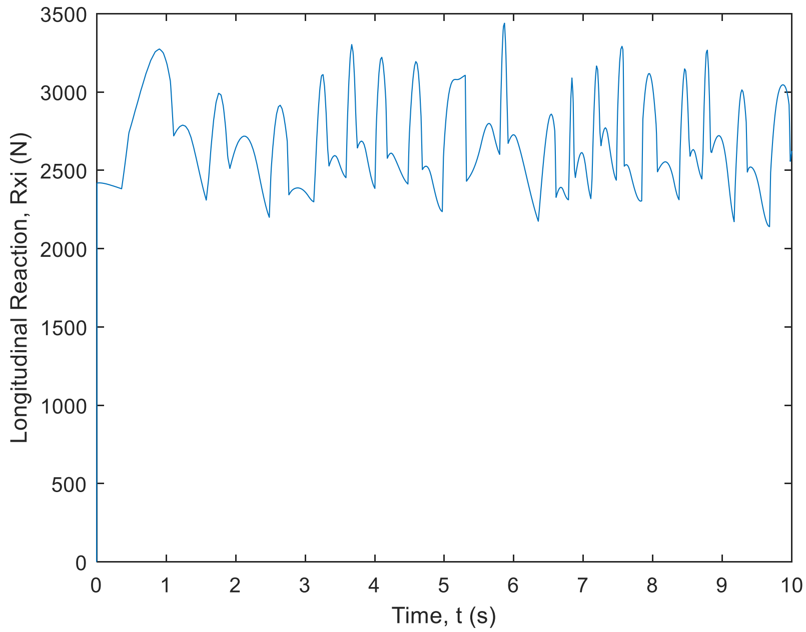

Figure 3.12. Tire Longitudinal Reaction Force on Soil

Figures 3.13 through 3.15 show the circumferential force. Figures 3.40 through 3.43 will illustrate the relationship between the circumferential force, torque, angular velocity, and the angular acceleration. This will explain why the circumferential force does not have fluctuations in its values that correspond to the fluctuations shown in the rolling resistance and peak friction coefficients, which are shown in the Appendix section.

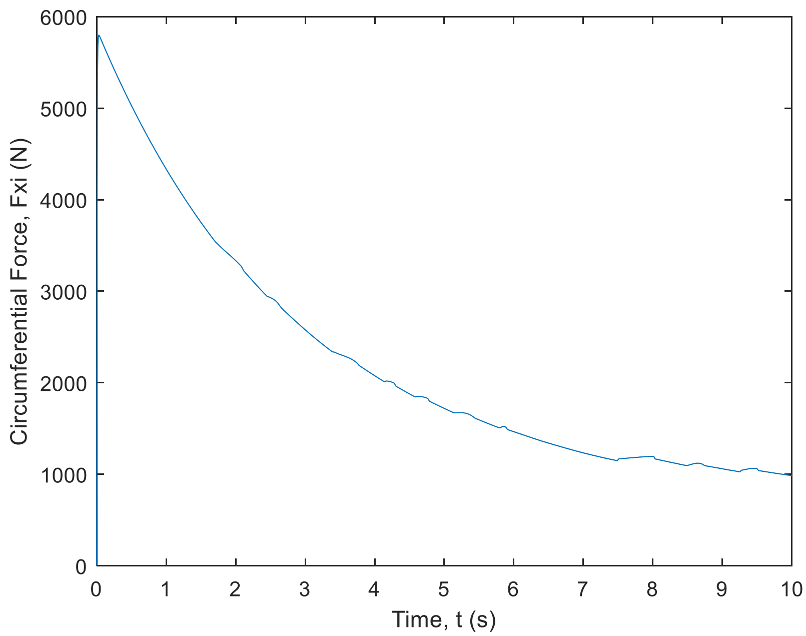

Figure 3.13. Tire Circumferential Force on Asphalt

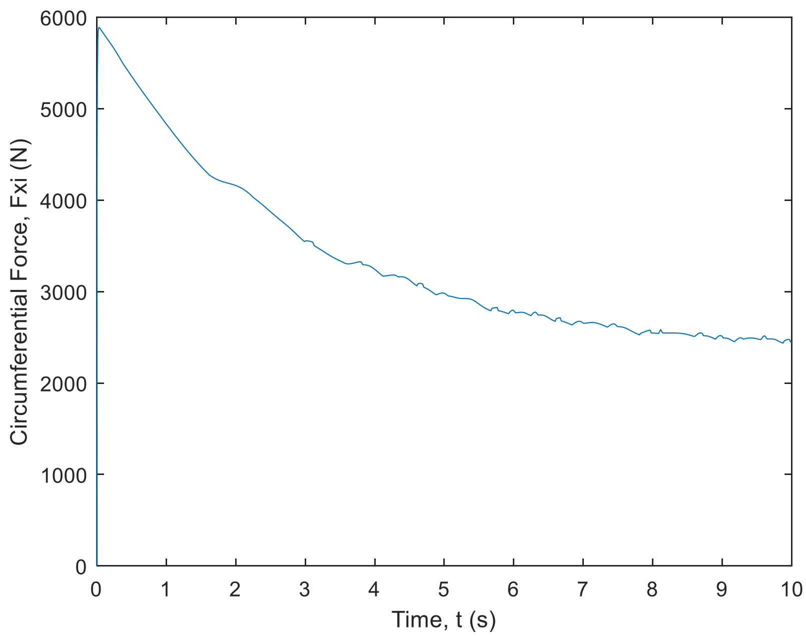

Figure 3.14. Tire Circumferential Force on Meadow

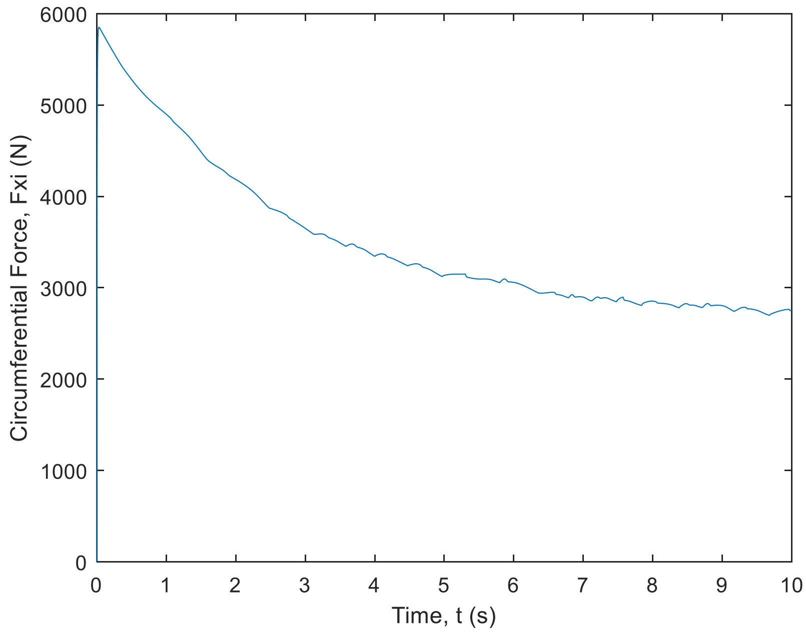

Figure 3.15. Tire Circumferential Force on Soil

In Figures 3.16 through 3.18, the inertia force was calculated using two different methods. Method one finds the inertia force by multiplying the mass and the acceleration together whereas method two is the difference between the circumferential force and the longitudinal reaction force.

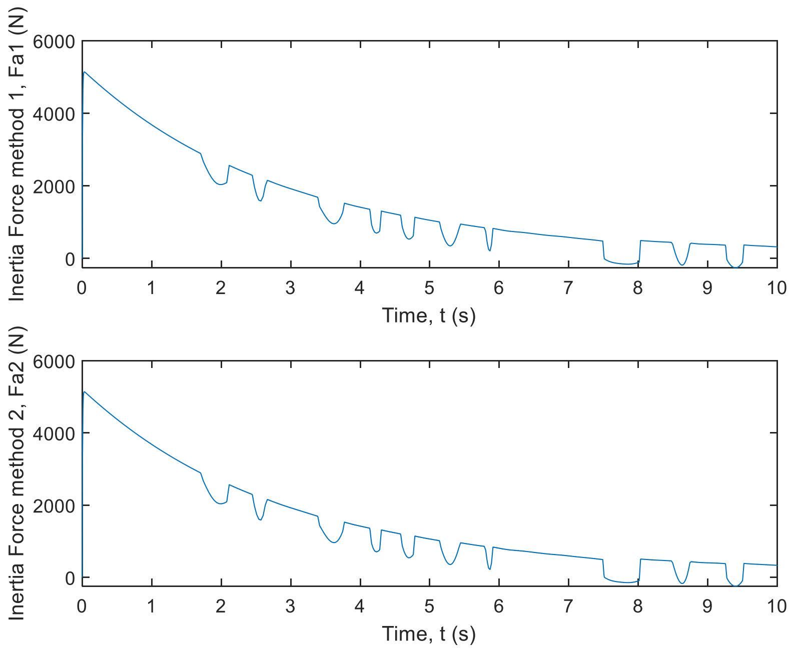

Figure 3.16. Tire Inertia Force: Methods 1 and 2 on Asphalt

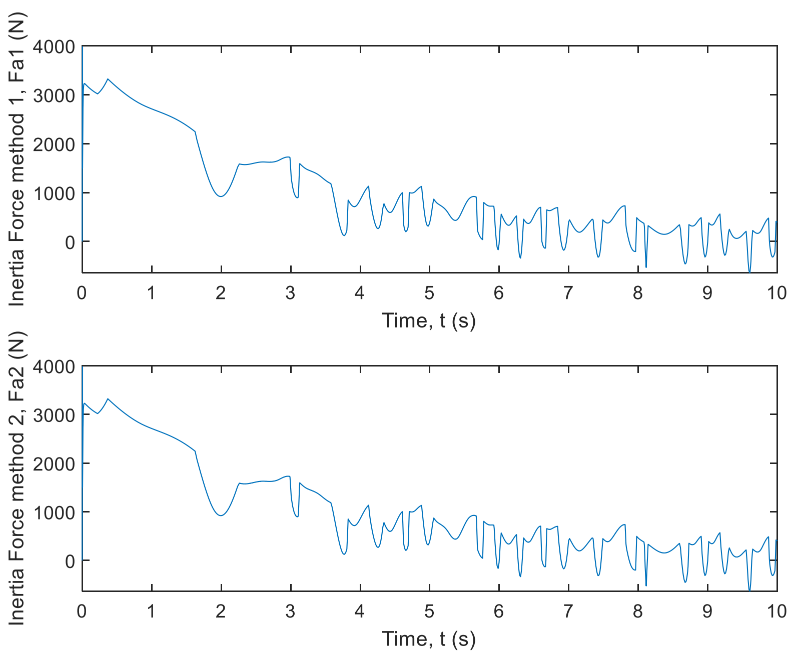

Figure 3.17. Tire Inertia Force: Methods 1 and 2 on Meadow

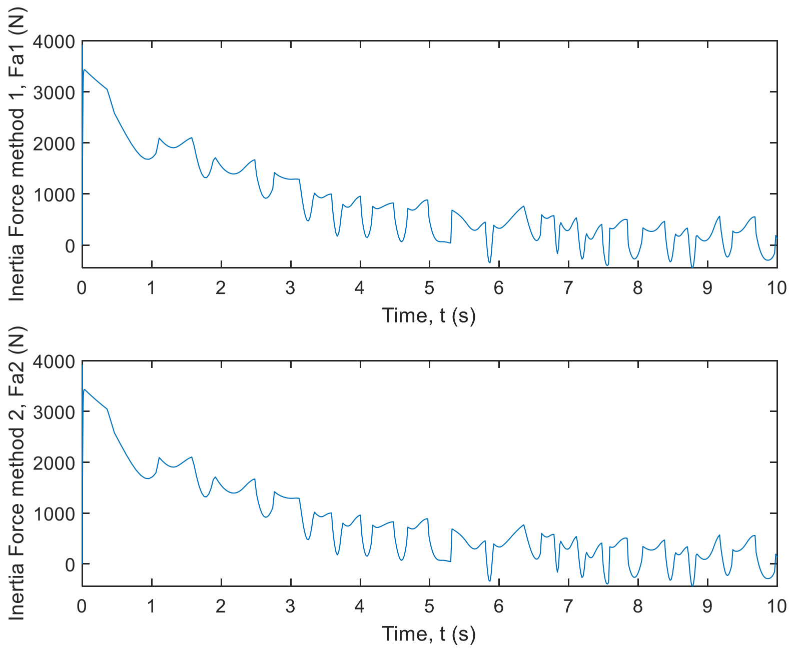

Figure 3.18. Tire Inertia Force: Methods 1 and 2 on Soil

Figures 3.19 through 3.21 are simply comparing the results of the two methods. The two lines are showing the influence that the drag force has on the inertia force. This was done by adding the drag force to the longitudinal reaction force and then subtracting the new value by the circumferential force.

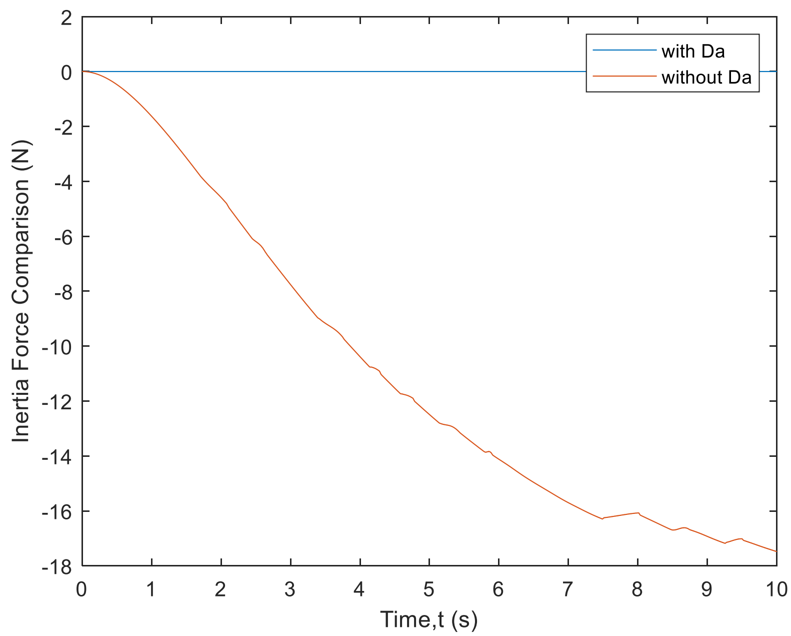

Figure 3.19. Comparison of Methods 1 and 2 for the Tire Inertia Force on Asphalt

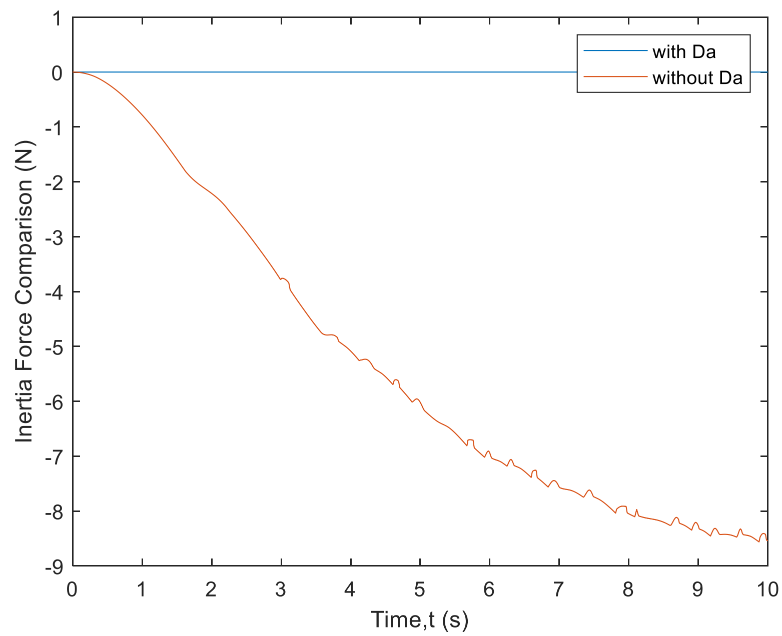

Figure 3.20. Comparison of Methods 1 and 2 for the Tire Inertia Force on Meadow

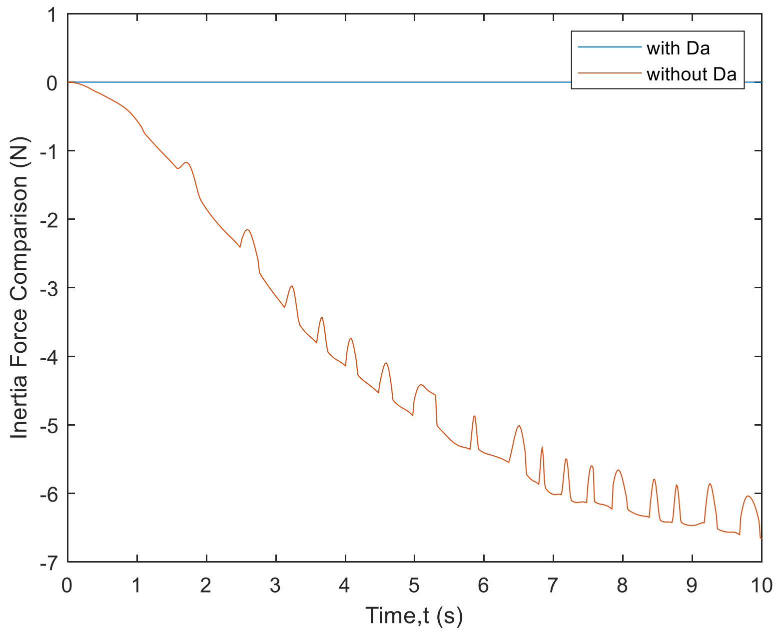

Figure 3.21. Comparison of Methods 1 and 2 for the Tire Inertia Force on Soil

Figures 3.22 through 3.24 show the actual tire slippage. The actual tire slippage was determined by rearranging Equation (2.10) to solve for the tire slippage. Figures 3.25 through 3.27 are showing the relationship between the actual tire slippage and the circumferential force.

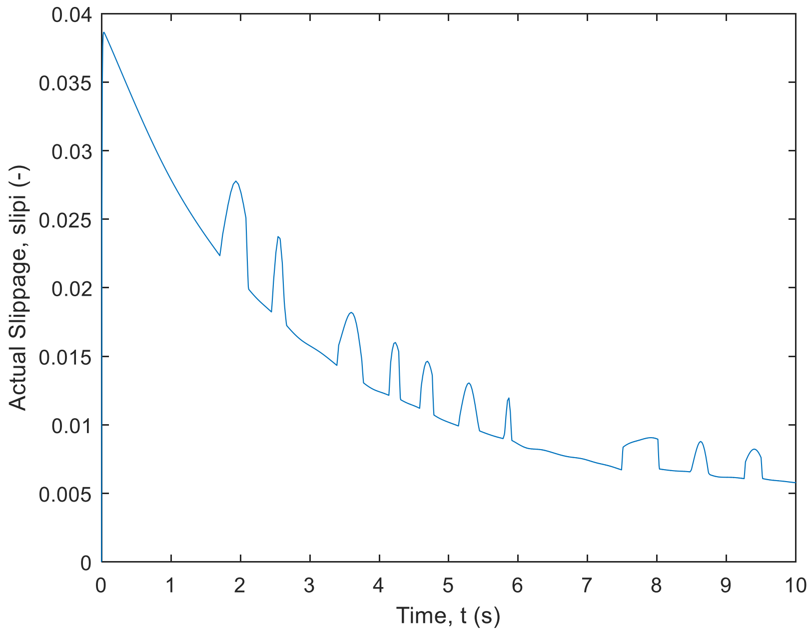

Figure 3.22. Actual Tire Slippage on Asphalt

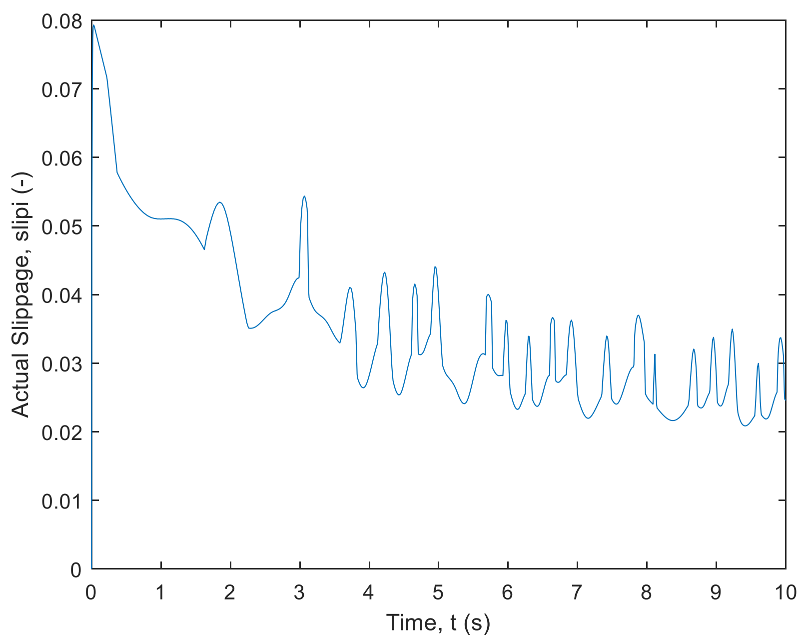

Figure 3.23. Actual Tire Slippage on Meadow

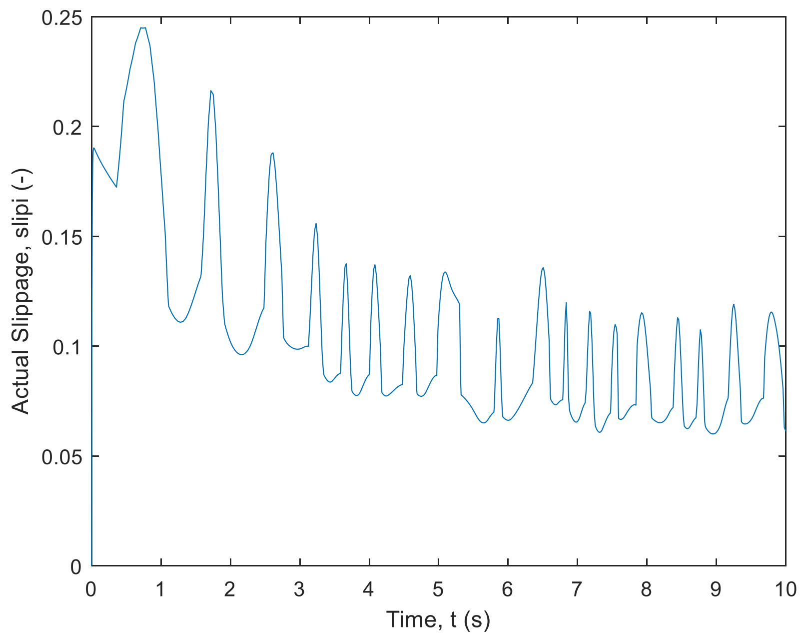

Figure 3.24. Actual Tire Slippage on Soil

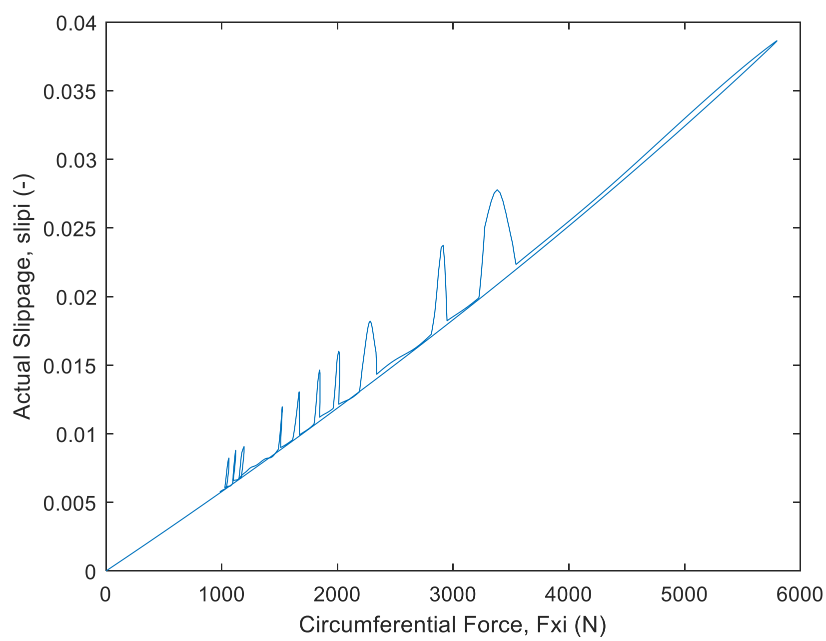

Figure 3.25. Actual Tire Slippage vs. the Tire Circumferential Force on Asphalt

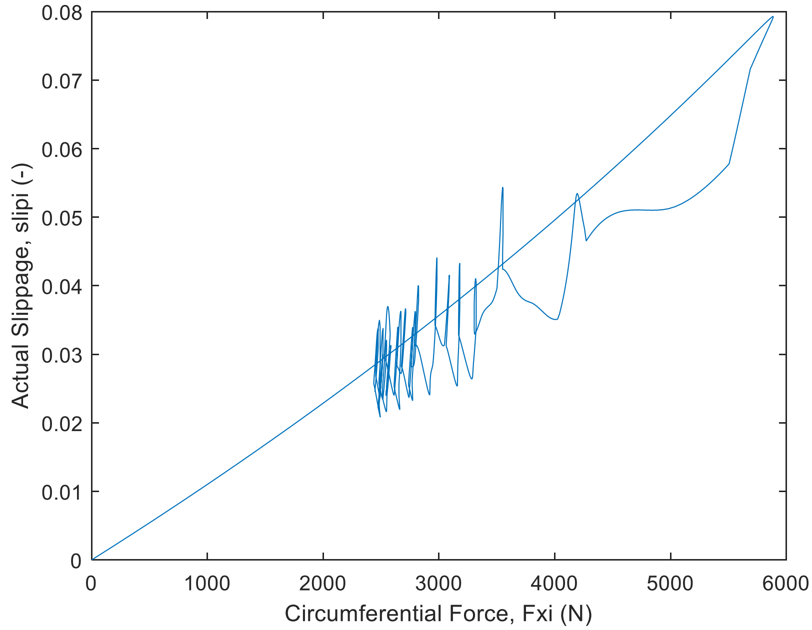

Figure 3.26. Actual Tire Slippage vs. the Tire Circumferential Force on Meadow

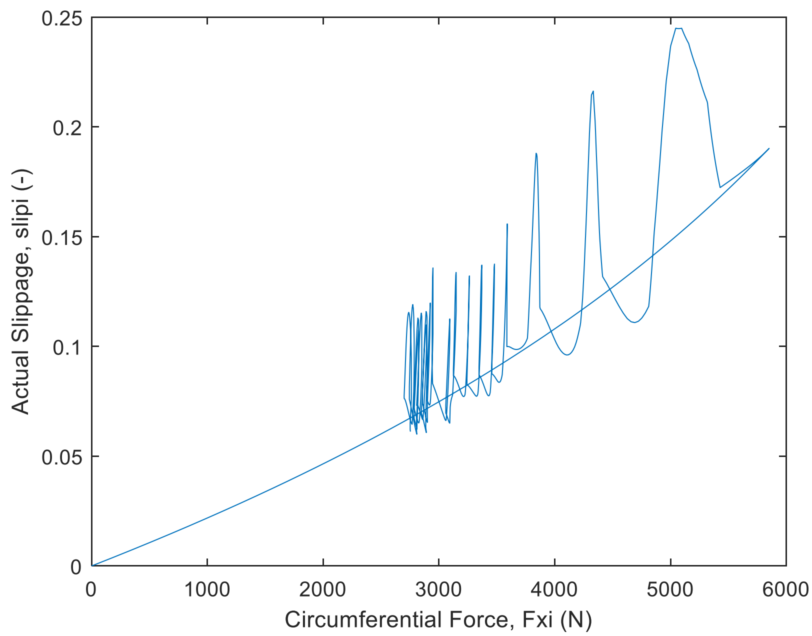

Figure 3.27. Actual Tire Slippage vs. the Tire Circumferential Force on Soil

The tire slippage shown in Figures 3.28 through 3.30 was determined using the conventional method. This method is used to approximate the tire slippage on an actual tire as it is rotating by using the rolling radius in both the driven and driving modes, see Equation (2.5) where the cumulative rolling radii have been plugged in to replace the rolling radii.

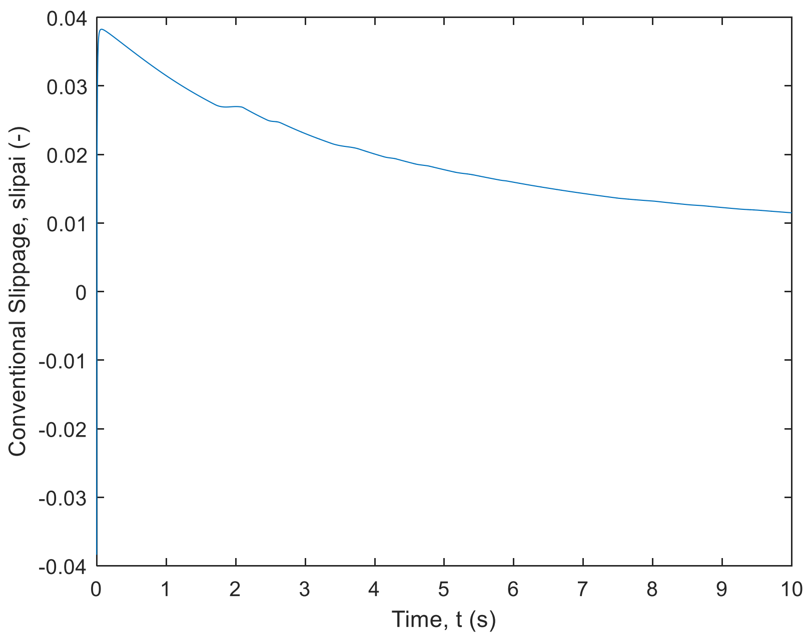

Figure 3.28. Conventional Tire Slippage on Asphalt

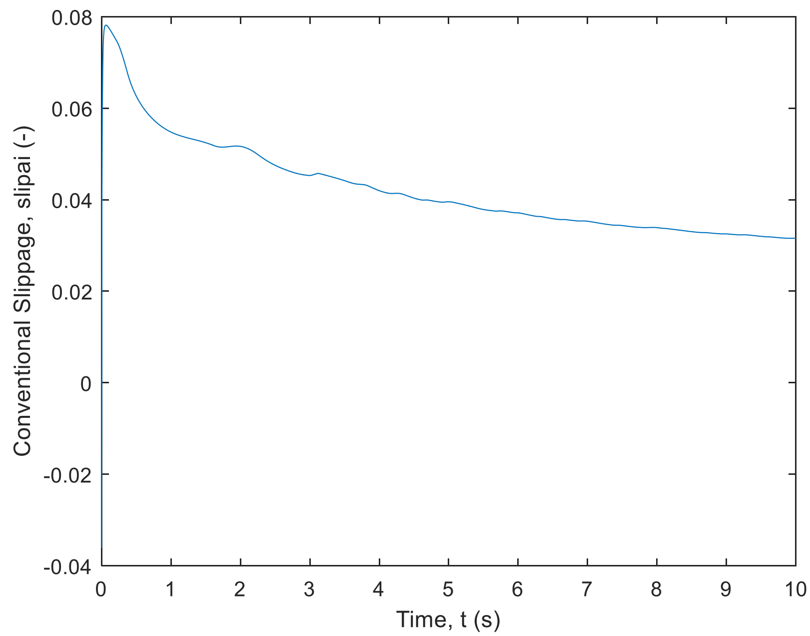

Figure 3.29. Conventional Tire Slippage on Meadow

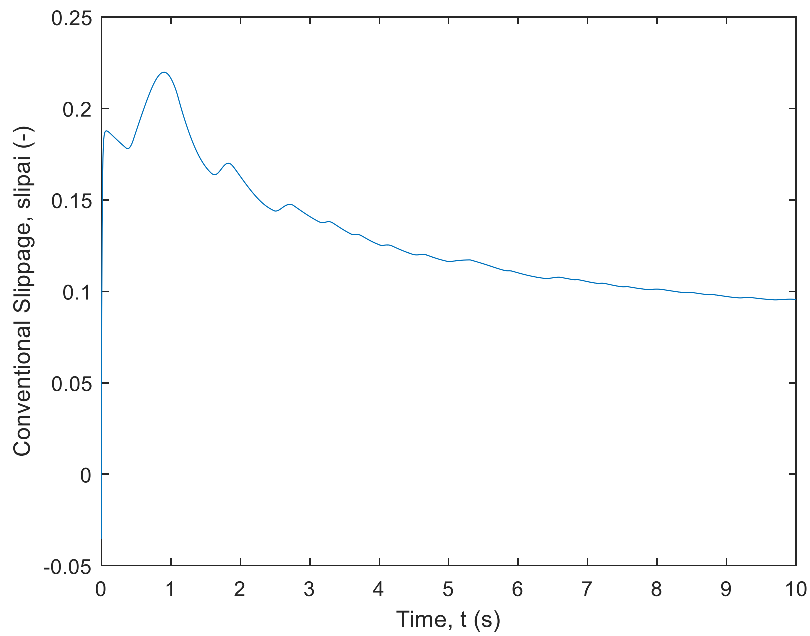

Figure 3.30. Conventional Tire Slippage on Soil

Figures 3.31 through 3.33 show the results for the tire slippage when the agile method is used. This method uses the instantaneous rolling radius in the driven and driving modes in place of the rolling radii in Equation (2.5).

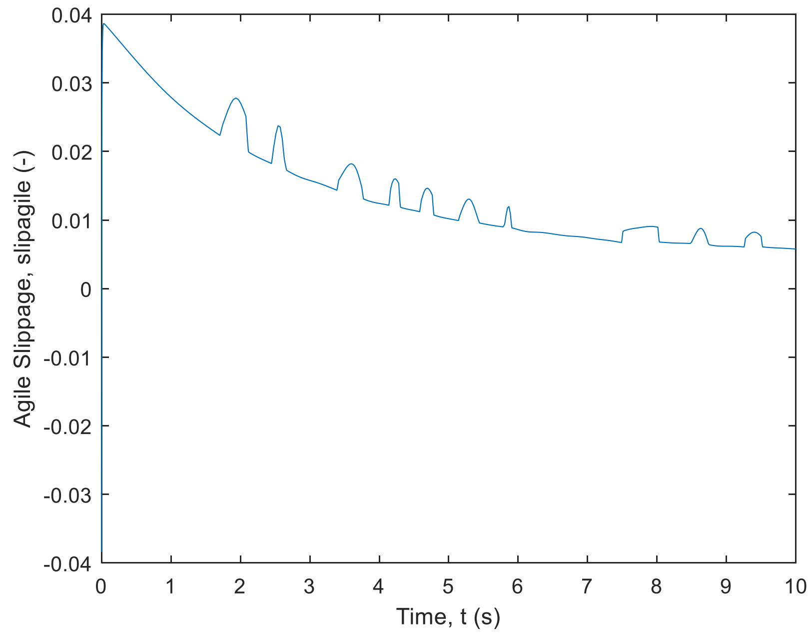

Figure 3.31. Agile Tire Slippage on Asphalt

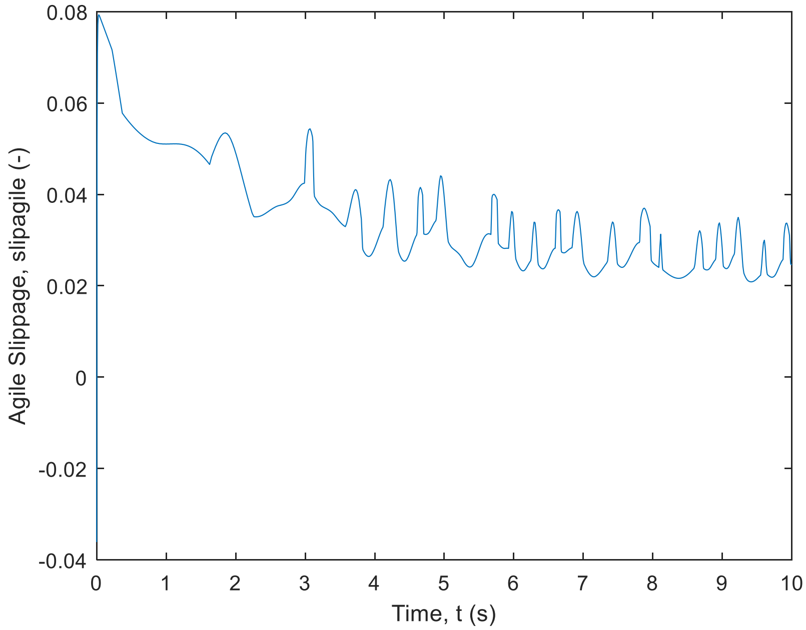

Figure 3.32. Agile Tire Slippage on Meadow

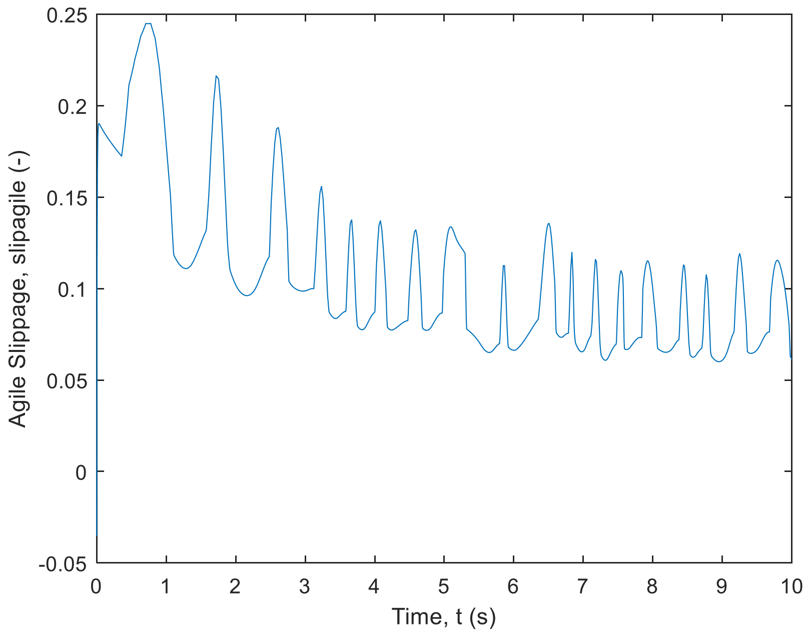

Figure 3.33. Agile Tire Slippage on Soil

Figures 3.34 through 3.36 show the data for the cumulative rolling radii which were used to determine the conventional tire slippage.

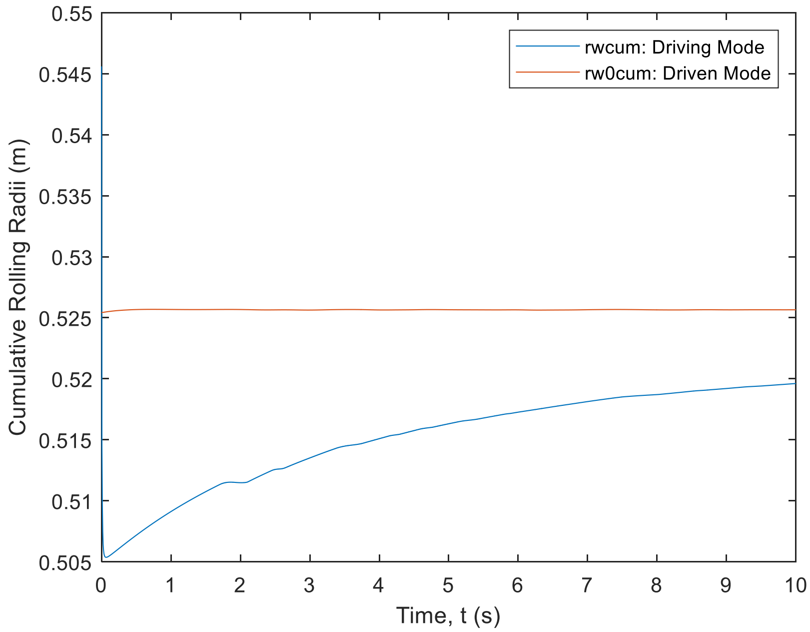

Figure 3.34. Cumulative Rolling Radis in the Driven and Driving Mode on Asphalt

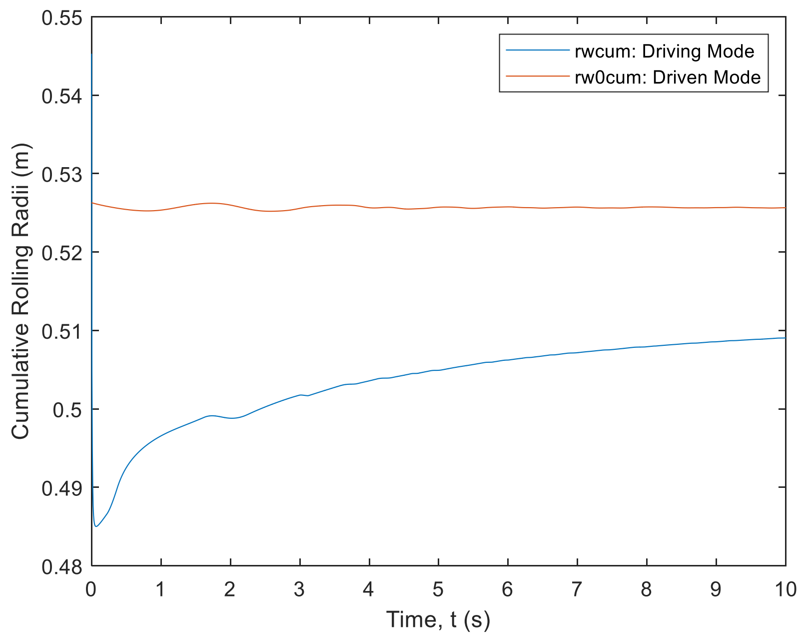

Figure 3.35. Cumulative Rolling Radis in the Driven and Driving Mode on Meadow

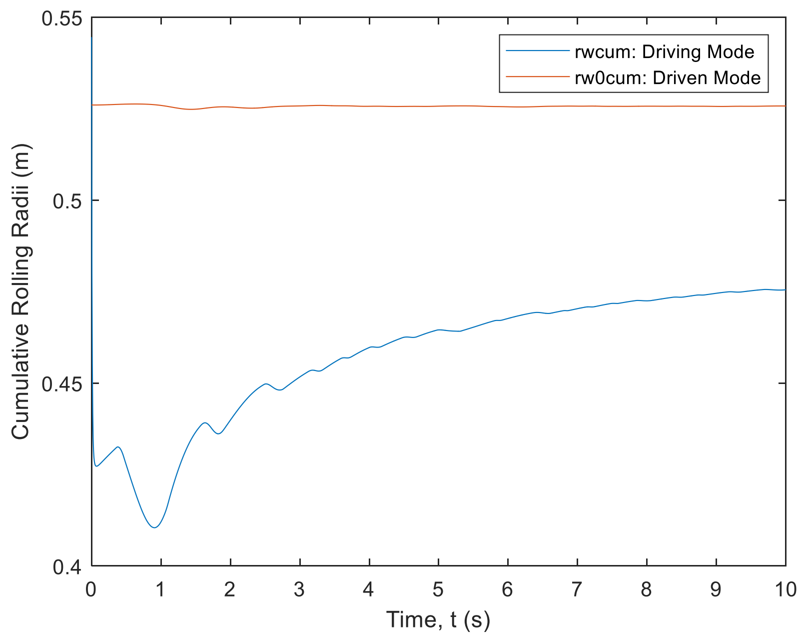

Figure 3.36. Cumulative Rolling Radis in the Driven and Driving Mode on Soil

Figures 3.37 through 3.39 are comparing the actual, conventional, and agile tire slippage. The actual tire slippage is not visible because the agile tire slippage is nearly an exact match. The reason for this is because agile tire dynamics calculates the tire slippage at any given point of time during the computer simulation. While in contrast, the conventional method uses the average of the data from Equation (2.5) for the duration of the simulation when calculating the tire slippage. This average results in a smoother curve for the approximation of the tire slippage.

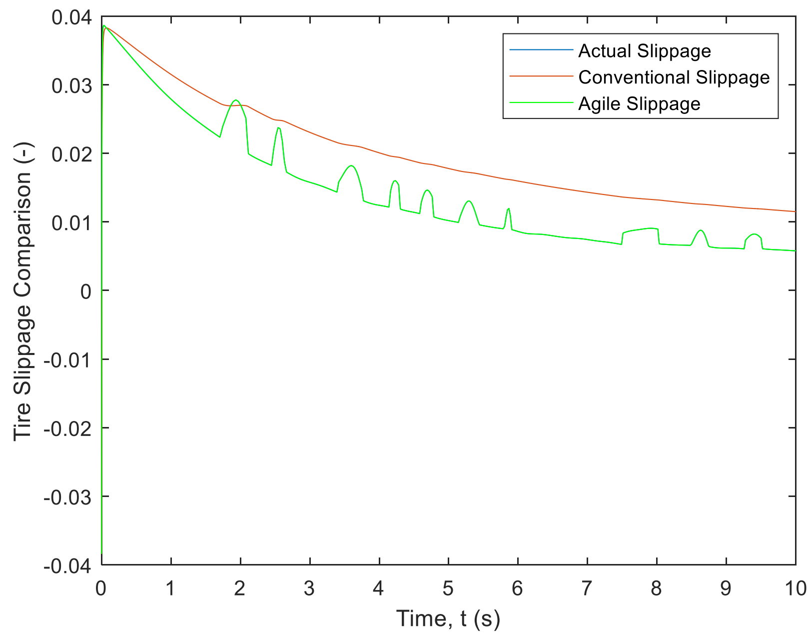

Figure 3.37. Comparison of each Method for Calculating Tire Slippage on Asphalt

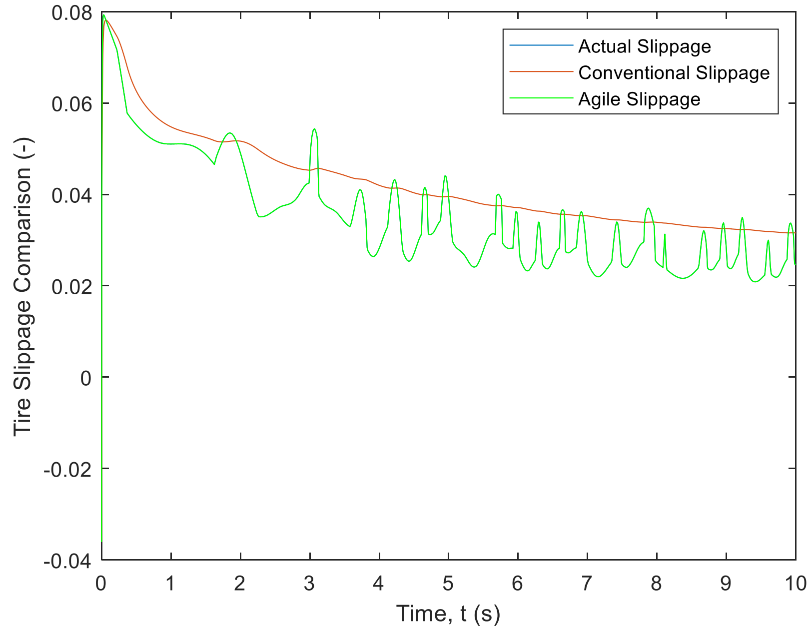

Figure 3.38. Comparison of each Method for Calculating Tire Slippage on Meadow

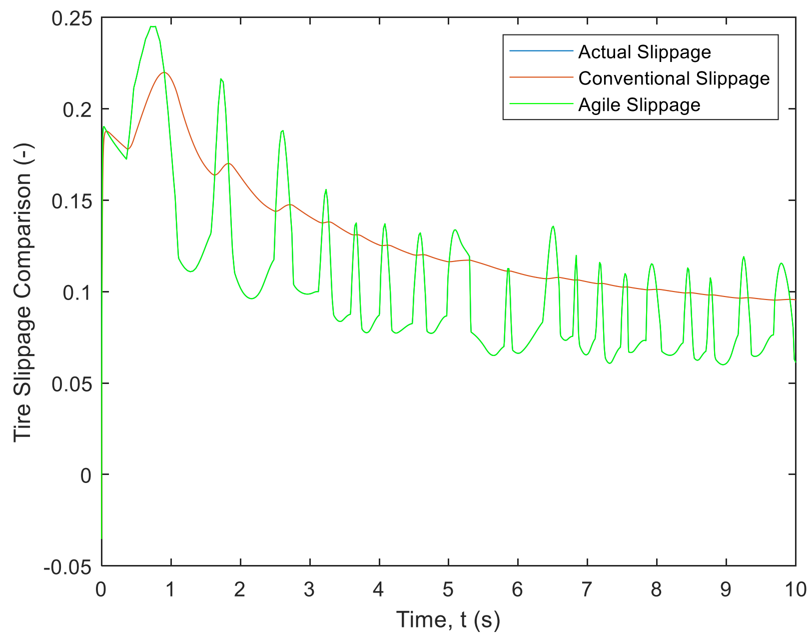

Figure 3.39. Comparison of each Method for Calculating Tire Slippage on Soil

As previously mentioned earlier in this section during the description for the Figures displaying the data for the circumferential force on the tire, Figures 3.40 through 3.43 will illustrate the relationship between the circumferential force, torque, angular velocity, and the angular acceleration. The angular velocity and angular acceleration are shown in Figures 3.40 and 3.41 respectively. Figure 3.42 shows the linear acceleration. For the purposes of this explanation, the simulation was set up to run for 50 seconds on asphalt in order to more clearly illustrate the behavior of the tire characteristics being shown. Looking at Figures A.2 and A.3, it can be observed that both the rolling resistance and peak friction coefficients show several fluctuations in their values; however, as previously shown in Figures 3.13 through 3.15, the circumferential force on the tire does not have corresponding fluctuations in its values. In order for these fluctuations to occur in the circumferential force, the tire would have to exactly follow the speed profile; this would require for there to be changes in the torque. In this case, because the torque and circumferential force show smoother trends, the angular and linear acceleration have noticeable fluctuations to compensate, see Figure 3.43 for the torque. Figure 3.40 shows that the angular velocity came close to being constant after approximately 10 seconds; however, there are still slight fluctuations in the velocity for the remaining 40 seconds. These slight fluctuations cause more significant fluctuations in both the angular and linear acceleration, see Figures 3.41 and 3.42. As a result, the torque and the circumferential force show smoother values without noticeable fluctuations.

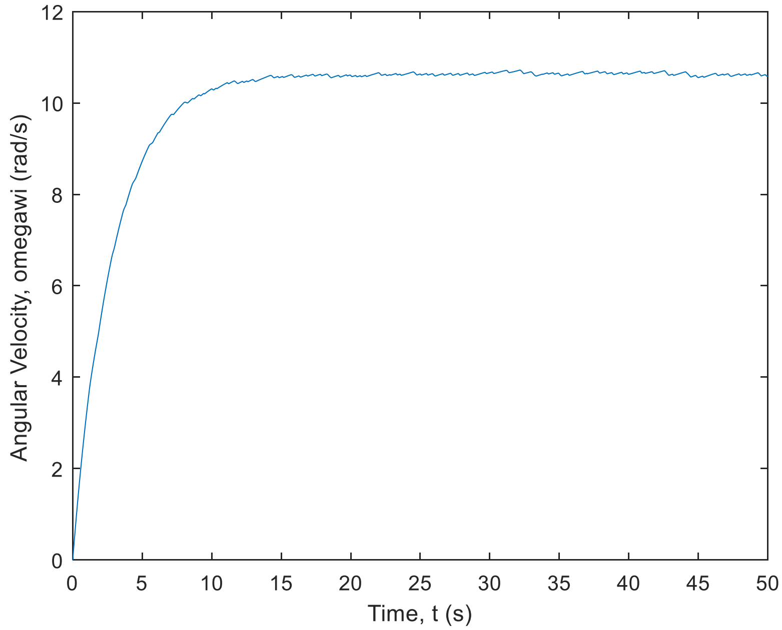

Figure 3.40. Angular Velocity on Asphalt

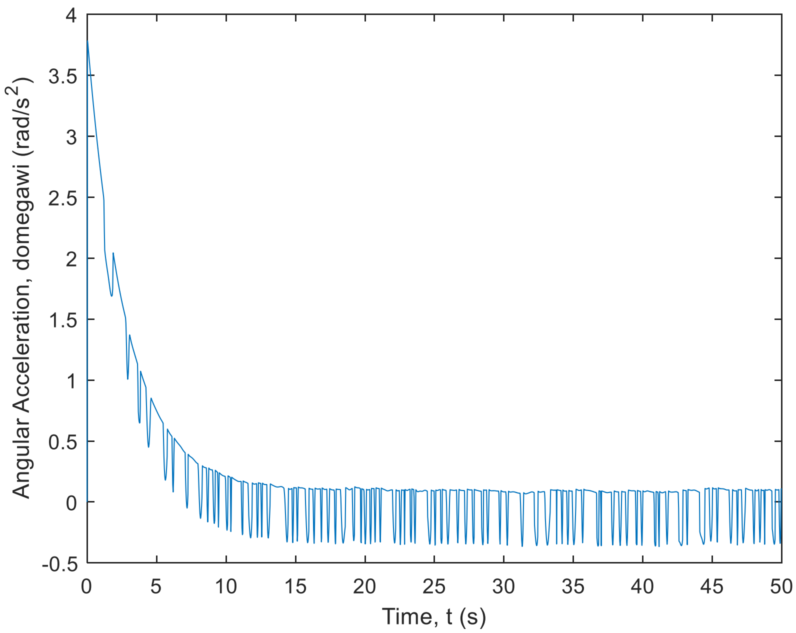

Figure 3.41. Angular Acceleration on Asphalt

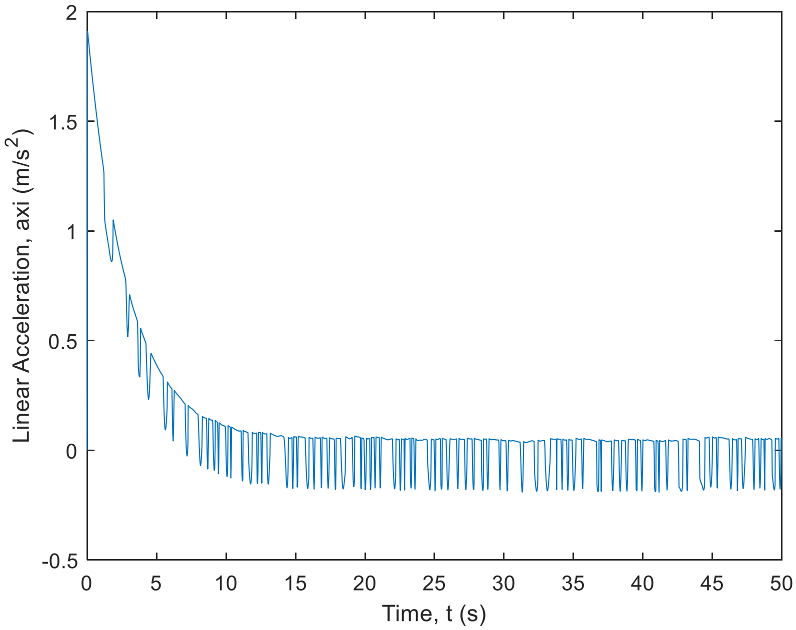

Figure 3.42. Linear Acceleration on Asphalt

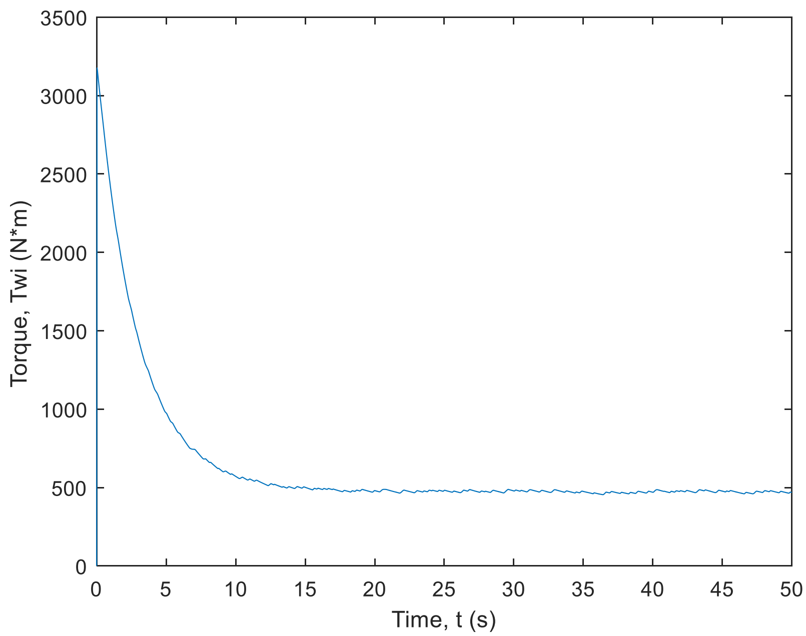

Figure 3.43. Torque on Asphalt

The second objective was to use the computational results to analytically prove that the agile tire slippage dynamics approach provides faster and more precise data in modeling tire slippage as compared to the conventional approach to tire slippage analysis dynamics. The results from the agile tire slippage computer simulation met this objective by confirming that agile tire dynamics are a vast improvement over conventional tire dynamics because the agile method allows for a near perfect approximation to the actual tire slippage making it possible to adjust and control the overall handling of a vehicle much sooner, and with much more precision and accuracy.

3.2 Conclusion

Based on the results from the software analysis in chapter 1, and the decisions made in chapter 2, MATLAB was determined to be the best option for the desired computational and mathematical model of a tire. This model was adapted from a preexisting code by further developing some of the Simulink subsystems and creating a script in MATLAB to generate all of the figures shown in section 3.1 and in the Appendix. The figures shown in section 3.1 especially Figures 3.37 through 3.39 compare the three methods used to determine the tire slippage. Comparing these figures with Figures 3.22 through 3.24 shows how the agile method matched the results from the actual method used. While in contrast, the conventional method showed a general approximation of the tire slippage based on an average of the tire slippage data. This shows that agile tire slippage dynamics or the agile method yields more accurate results than the conventional method making it better suited to be used to further develop vehicle control systems such as traction control. These further developments will ultimately lead to minimizing the negative effects from tire slippage and maximizing the potential safety that can be provided to the vehicle. The computational results help accomplish the main goal of facilitating improvements of the overall vehicle safety by modeling the agile tire slippage dynamics in a computer simulation.

LIST OF REFERENCES

[1] Vantsevich, V. V. (2015). Road and Off-road Vehicle System Dynamics. *Understanding the Future from the Past. Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility* http://dx.doi.org/10.1080/00423114.2014.984726 (Published Online 2015). Retrieved on Spring 2018.

[2] Vantsevich, V. V. (2014). Wheel Dynamics Fundamentals for Agile Tire Slippage Modeling and Control. *ASME DETC2014-34464, August 17-20,* Buffalo, NY. doi: http://dx.doi.org/10.1115/DETC2014-34464 Retrieved on Spring 2018.

[3] Vantsevich, V. V. and Gray, J. P. (2015). Relaxation Length Review and Time Constant Analysis for Agile Tire Dynamics Control. *ASME DETC2015-46798, August 2-5,* Boston, MA. doi: https://doi.org/10.1115/DETC2015-46798 Retrieved on April 2nd, 2020.

[4] MSC. Software. (2017). *Welcome to Adams Tire.* Page 2 and 334. Retrieved from https://simcompanion.mscsoftware.com/infocenter/index?page=content&id=DOC11199& Retrieved on Spring 2018.

[5] Cosin Scientific Software. (n.d.). *FTire.* Retrieved from https://www.cosin.eu/products/ftire/ Retrieved on April 3rd, 2020.

[6] Houcque, D. and McCormick, R. R. (n.d.). *Applications of MATLAB: Ordinary Differential Equations (ODE)*. Retrieved from https://www.mccormick.northwestern.edu/docs/efirst/ode.pdf Retrieved on April 3rd, 2020.

[7] MathWorks. (2017). *Tires and Vehicles*. Retrieved from https://www.mathworks.com/help/physmod/sdl/tires-and-vehicles.html Retrieved on Spring 2018.

[8] CSM Army Tires. (2015). *Continental MPT – 81 – 365/80 R 20 – (14.5R20).* Retrieved from http://www.csmarmytires.com/continental-mpt-81---36580-r-20---145r20.html Retrieved on April 3rd, 2020.

[9] CSM Army Tires. (n.d.). *Continental Multi Purpose Tires (MPT).* Retrieved from http://www.csmarmytires.com/uploads/5/9/4/4/59444003/_mpt_brochure.pdf Retrieved on April 3rd, 2020.

[10] Andreev, A. F., Kabanau, V. I., and Vantsevich, V. V. (2010). *Driveline Systems of Ground Vehicles: Theory and Design.* Pages xxvi, xxx, xxxii, 71 and 74. Boca Raton, FL: Chapman and Hall/CRC and Taylor & Francis Group.

[11] Vantsevich, V. V. (2017). *Vehicle Dynamics: Wheel Kinematic Parameters and Force Factors.* Week 4, Lecture 1: Slide 23. [PowerPoint slides]. Printed.

[12] Vantsevich, V. V. (2017). *Vehicle Dynamics: Wheel Alignment and Stabilization. A Quarter Car Model.* Week 2, Lecture 2: Slide 24. [PowerPoint slides]. Printed.

[13] Gray, J. P., Vantsevich, V. V., and Paldan, J. (2015). Agile Tire Slippage Dynamics for Radical Enhancement of Vehicle Mobility. *Journal of Terramechanics Volume (65,)* Pages 14 – 37. Retrieved from https://www.sciencedirect.com/science/article/abs/pii/S0022489816000045 Retrieved on April 3rd, 2020.

APPENDIX A

ADDITIONAL PLOTS FROM THE MATLAB SIMULATION

Figure A.1. shows the road height, the peak friction coefficient, the rolling resistance coefficient, and the empirical exponential factor over a road distance of 1,000 meters on asphalt.

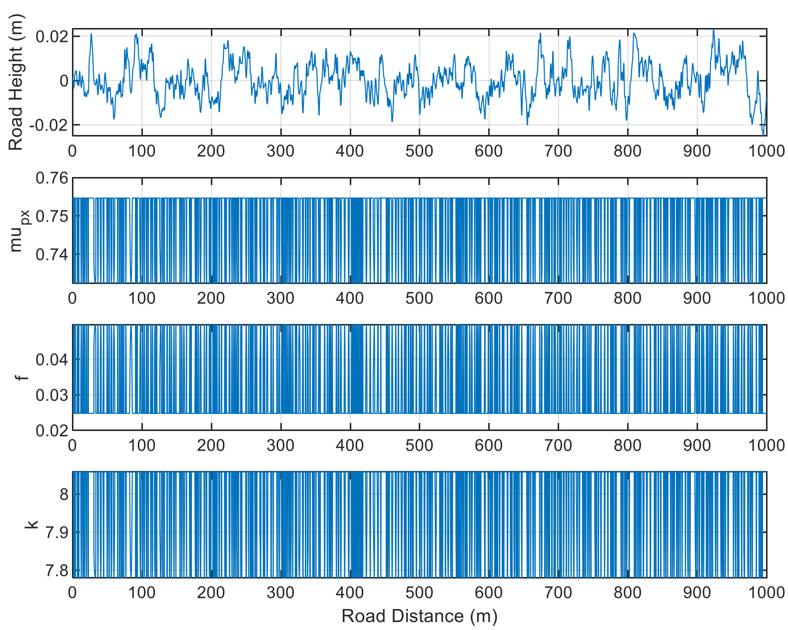

Figure A.1. Road Height, Peak Friction Coefficient, Rolling Resistance Coefficient, and Empirical Factor on Asphalt

Figures A.2 through A.4 are a zoomed in plot of the peak friction coefficient, the rolling resistance coefficient, and the empirical factor respectively that were shown in Figure A.1.

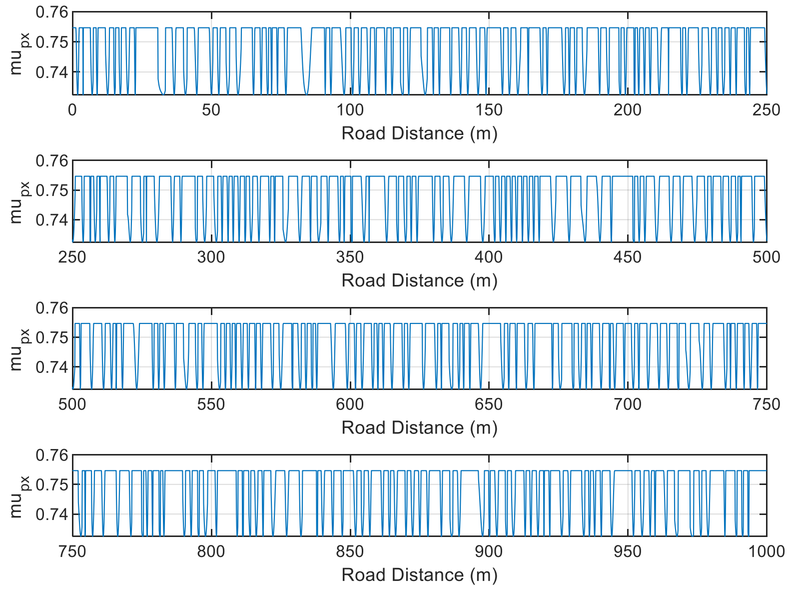

Figure A.2. Peak Friction Coefficient on Asphalt

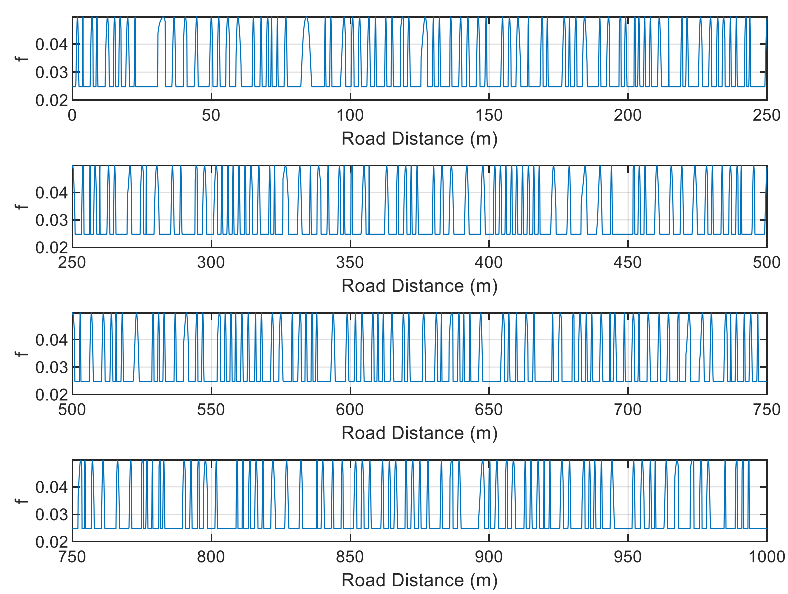

Figure A.3. Rolling Resistance Coefficient on Asphalt

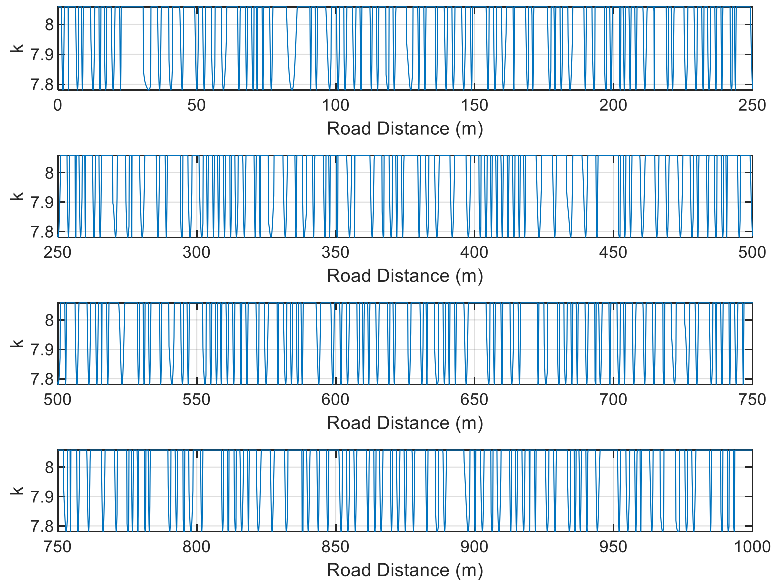

Figure A.4. Empirical Factor on Asphalt

Figure A.5 shows the road height, the peak friction coefficient, the rolling resistance coefficient, and the empirical exponential factor over a road distance of 1,000 meters on meadow.

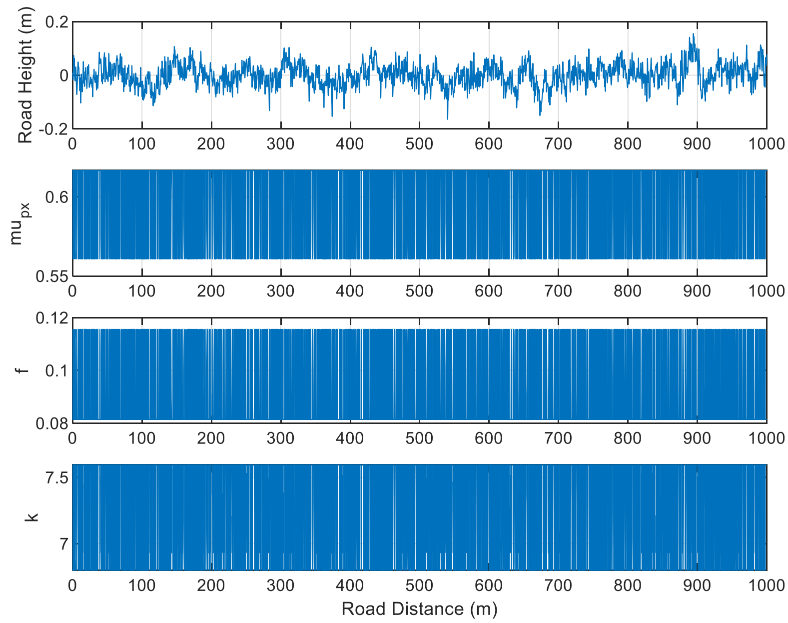

Figure A.5. Road Height, Peak Friction Coefficient, Rolling Resistance Coefficient, and Empirical Factor on Meadow

Figures A.6 through A.8 are a zoomed in plot of the peak friction coefficient, the rolling resistance coefficient, and the empirical factor respectively that were shown in Figure A.5.

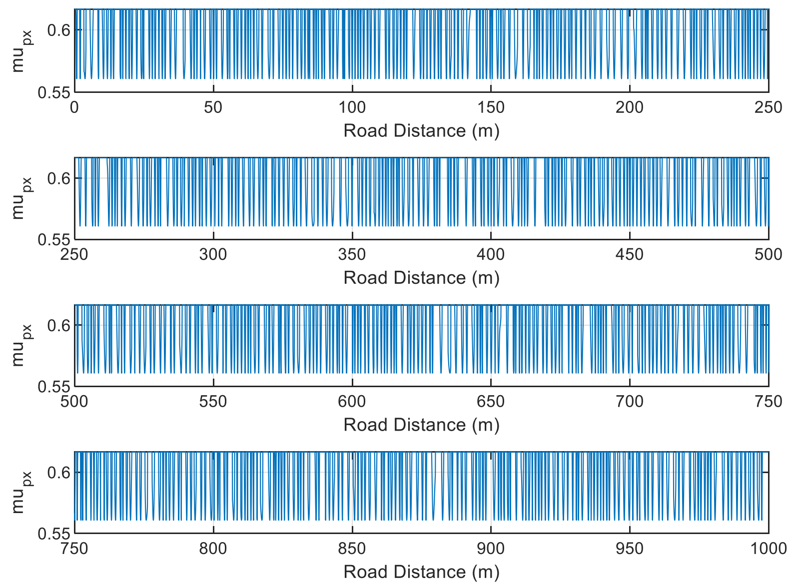

Figure A.6. Peak Friction Coefficient on Meadow

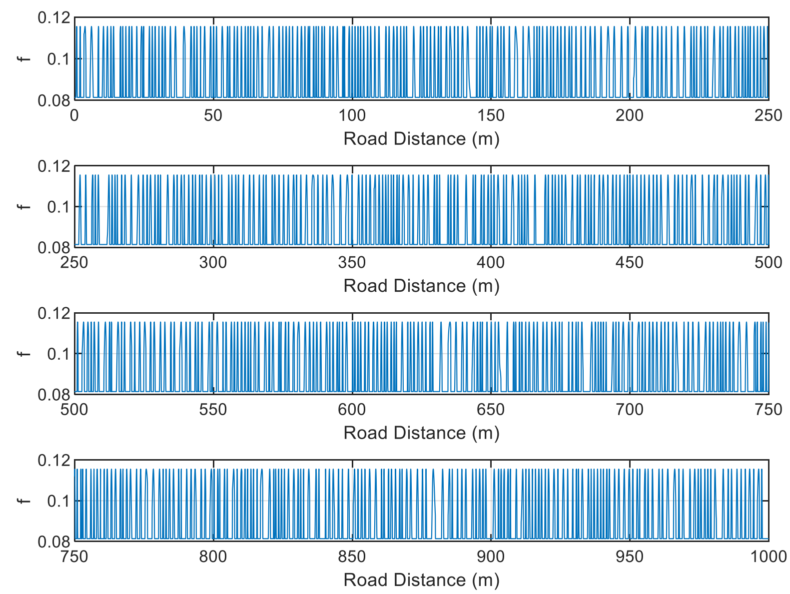

Figure A.7. Rolling Resistance Coefficient on Meadow

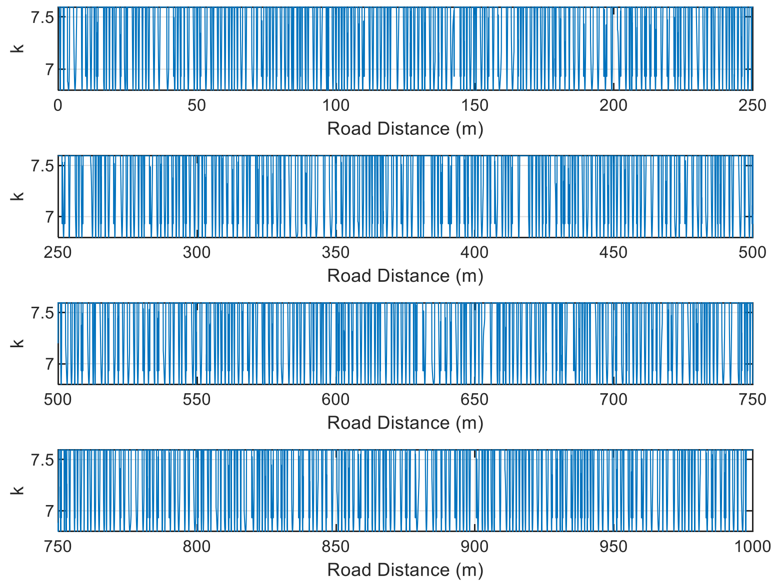

Figure A.8. Empirical Factor on Meadow

Figure A.9 shows the road height, the peak friction coefficient, the rolling resistance coefficient, and the empirical exponential factor over a road distance of 1,000 meters on soil.

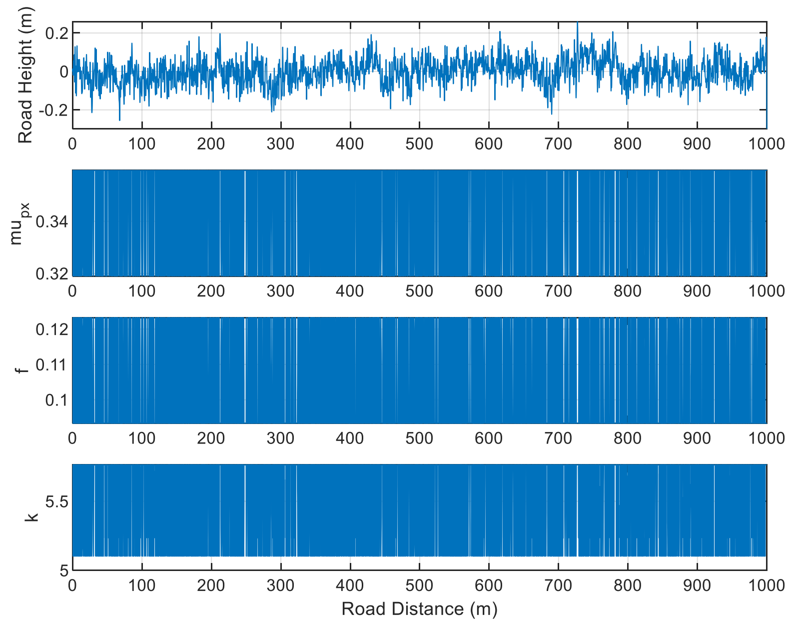

Figure A.9. Road Height, Peak Friction Coefficient, Rolling Resistance Coefficient, and Empirical Factor on Soil

Figures A.10 through A.12 are a zoomed in plot of the peak friction coefficient, the rolling resistance coefficient, and the empirical factor respectively that were shown in Figure A.9.

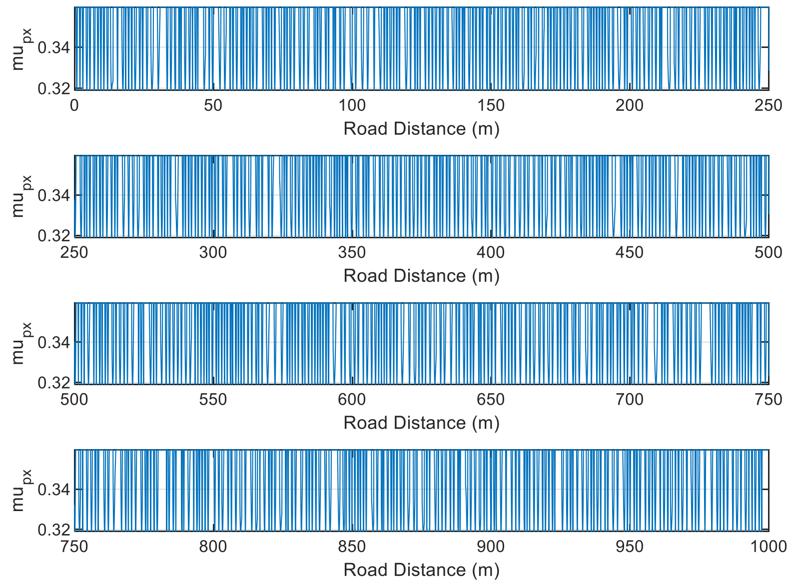

Figure A.10. Peak Friction Coefficient on Soil

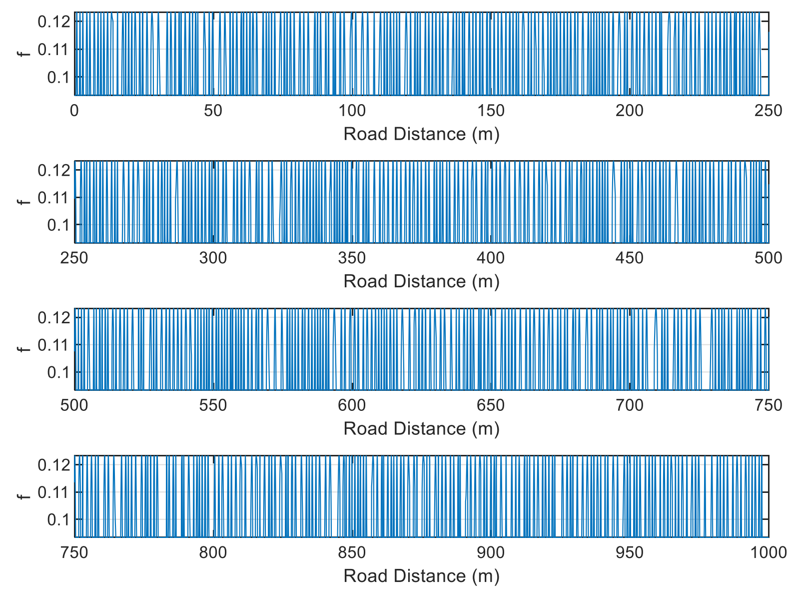

Figure A.11. Rolling Resistance Coefficient on Soil

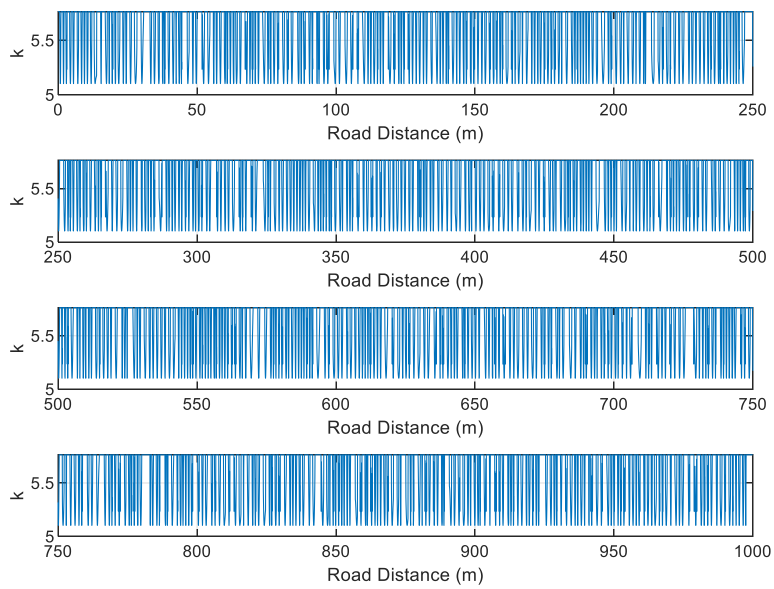

Figure A.12. Empirical Factor on Soil