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Fifth Graders' Understanding of Fractions on the Number Line: A Standard Introduced in the Common Core State Standards for Third Graders

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FIFTH GRADERS' UNDERSTANDING OF FRACTIONS ON THE NUMBER LINE: A STANDARD INTRODUCED IN THE COMMON CORE STATE STANDARDS FOR THIRD GRADERS

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ABSTRACT

This was a case-study design of the various ways students represented fractions on number lines. Students responded to task-based interview questions based on identifying fractions as a number on the number line as well as equivalency and problem solving. The tasks were administered individually to 26 fifth-grade students over a 15-minute time frame in their respective schools. The two groups of 10-year-old students answered most questions in written form on pencil and paper. The students were often asked to explain how they arrived at an answer. Student performance was highest when instructed to plot ½ on a number line of 0 to 1 as well as naming a fraction less than ½. Although no students attempted to solve the word problem with the number line as indicated, they performed lowest when they attempted to plot ½, ¼, and 1 on a number line with a predetermined unit 0 to 1/3. Other low performing concepts consisted of plotting ¼ on a number line from 0-3, identifying ¼ on a non-routine number line, and plotting a unit fraction with an equivalent fraction as well as an improper fraction on a common number line.

Keywords: fractional numbers, understanding fractions, elementary mathematics, number line representation

DEDICATION

I dedicate my dissertation to my two girls - my phenomenal mother, Linda Shuford and my adorable daughter, Alexis Jade Witherspoon. Thank you both for supporting me throughout this journey. Together you served as peer debriefers as you reviewed my work for clarity, provided input on various topics; and Lexi, you even attended several Ed.S. and Doctoral classes with me. I am forever grateful for the long academically-focused weekends and late night phone calls. Because of you, I am here. Thus, we share this moment together, honorary "Dr." Linda you vicariously earned this degree with me and Lexi you are expected to take advantage of these experiences and get your own☺. Although I am grateful for every moment of this journey, I look forward to taking vacations without a computer.

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CHAPTER 1

INTRODUCTION

Elementary students have a difficult time understanding fractions when they are taught with a "one best algorithm" approach. Parker (2006) defined an algorithm as "a set of rules for solving a particular type of problem" (para. 1). According to Aksu (2001), there are some dangers associated with "focusing attention on fraction rules" and that "fraction rules can easily become the focus of rote learning and produce artificial feelings of accomplishment" (p. 375). Adding to the danger of learning rules to solve mathematical problems, Aksu (2001) stated, "Rules do not help students to think about the meanings of operations, and the mastery observed in carrying out those operations is quickly lost" (p. 375).

Several other researchers have expressed the negative effects of introducing standard algorithms with "how-to" steps prior to allowing children to make sense of mathematical problems (Carnine, Jitendra, & Silbert, 1997; Castro, 2008; Hanselman, 1997; Kamii & Clark, 1995; Lappan & Bouck, 1998). Kamii and Livingston (1994) added, "By trying to transmit in ready-made form the results of centuries of reflection by adults, we deprive children of opportunities to do their own thinking" (p. 32). In other words, in order for our children to truly grasp a meaningful understanding of today's algorithms, they will "have to go through a constructive process similar to our ancestors" (Kamii & Livingston, 1994, p. 33). The constructivist approach to teaching mathematics allows children an opportunity to construct knowledge, rather than memorize rules for solving problems.

Despite the growing outcry for children to learn mathematics by constructing knowledge, many educators are still big proponents of enforcing rules as a method for learning. Many teachers apply the “do as I say” practice that has been used in our schools for many years. However, Kamii and Livingston (1994) stated, “Today’s algorithms are the results of centuries of construction by adult mathematicians” (p. 22). Children’s thinking ability is at risk when rote memory is exercised repeatedly for certain tasks. According to Branscombe, Castle, Dorsey, Surbeck, and Taylor (2003), children come from diverse backgrounds and those who are “taught mathematics as if it were social knowledge or algorithms to be memorized, do not learn to think on their own” (p. 82). Branscombe et al. (2003) further attested that individuals learn in a variety of ways, and their learning styles vary. Therefore, when children are taught math concepts, it is important for educators to remember that there is no “one-size-fits-all” strategy and that children should be allowed to construct their own way of coming up with solutions to problems.

In a study conducted by Kamii and Livingston (1994), when fourth grade students who used algorithms were compared to third grade children who had not been exposed to algorithms, it was found that the performance of the fourth grade students was far worse. Not only did these fourth graders have “inadequate knowledge of place value,” but their incorrect verbal response to 837 as “eight, three, seven,” rather than “eight hundred thirty-seven” provided evidence that “algorithms foster mechanical, mindless processing of isolated columns" (Kamii & Livingston, 1994, p. 40). Therefore, it can be surmised that the damage caused by algorithms is not easily overcome in many instances.

Gorsky, McGrath, and Myers (2009) articulated that an excellent mathematical foundation is pertinent in our technologically advanced society for a large number of careers in science, engineering, medicine, technology, and business. These are careers that will definitely require the expertise of individuals who are able to think "out of the box" in order to effectively solve problems that seemingly increase over time in today's society. However, most teachers today were taught math with algorithms and, therefore, "see their role as transmitting information to children to be remembered" (Branscombe et al., 2003, p. 17).

The National Research Council (2001) reported, "Improving teachers' mathematical knowledge and their capacity to use it to do the work of teaching is crucial in developing students' mathematical proficiency" (p. 372). One of the problems with inefficient teaching of algorithms can be traced back to the way teachers were taught math in their primary years of learning. According to the National Research Council (2001), the mathematical education these teachers received both throughout regular school and college failed to provide them with adequate opportunities to effectively learn mathematics. This inefficiency has resulted in the possibility of teachers "knowing the facts and procedures that they teach but often have a relatively weak understanding of the conceptual basis for that knowledge" (The National Research Council, 2001, p. 372). The problem is exacerbated after the foundation has been established and solidified with reinforcement from years of practicing concepts such as algorithms that are more harmful than helpful for developing number sense (Kroll, 2004). This is a problem that is not easily fixed by schools of education that perhaps desire to rectify the problem for future generations by producing individuals who will provide children with guidance that is necessary for constructing knowledge. There is a successive disadvantage associated with the learning and teaching of math solely using an algorithmic approach, and "Students who are studying to become teachers begin their studies with personal preconceived notions about the nature of teaching and learning" (Kroll, 2004, p. 200). Teachers usually teach using the same or similar approach that they were taught.

The Significance and Purpose of the Study

Although numerous studies on the topic of rational numbers focus on the difficulty students experience in locating fractions on a number line (Bright, Behr, Post, & Wachsmuth, 1988; Pearn & Stephens, 2007; Wong & Evans, 2008), minimal literature was found that included fifth graders in these studies (Saxe, Shaughnessy, Shannon, Langer-Osuna, Chinn, Gearlheart, 2007; Wong & Evans, 2008). Even though some studies included fifth graders in various fractional research topics (Charles, Nason, & Cooper, 1999; Lim, 2007; Empson, Junk, Dominguez, & Turner, 2005), to the knowledge of this researcher there is no literature focusing solely on fifth graders' understanding of fractions on the number line. While the importance of qualified teachers was acknowledged, teachers' experiences with fifth graders and their understanding of fractions are often under-studied.

Problem of the Study

The purpose of this study is to explore what fifth graders understand about fractions by examining the reasoning they employ as they solve fractional tasks on a number line, which is a concept that is introduced in third grade. Few studies have examined the perception of educators as they deal with students who struggle to apply and maintain fractional knowledge that is taught on a daily basis. Despite the growing amount of literature relative to mathematics education, past research focused primarily on problems associated with students learning mathematics and various suggested strategies to solve these problems. Additionally, the majority of the studies about teaching and learning of mathematics have been qualitative, which supports the belief of this researcher that qualitative research is the best approach to form a deeper exploration of this issue. Descriptions, understandings, and interpretations are inductively reasoned (Lichtman, 2010). By using the case study qualitative approach, a composite description of lived experiences can be explained with regard to the lived experience of fifth grade students' understanding of fractions (Creswell, 2007).

The 2010 Alabama Course of Study in mathematics and the Common Core State Standards (CCSS) for Mathematics (2010) state that the domain of fractions should begin at third grade and continue through fifth grade. Although the formal knowledge of fractions is a focus in grades three to five, the meaning of fractions is often introduced as early as grades one and two through mathematical vocabulary that focuses on partitioning shapes into equal shares. However, widespread misunderstanding about fraction concepts is well documented (Chan, Leu, & Chen, 2007; Irwin, 2001; Mitchell & Horne, 2008; Vamvakoussi & Vosniadou, 2010), and fractions continue to be an important part of the elementary and middle school mathematics curriculum.

This study was conducted at two elementary schools in central Alabama. There is a lack of research on children's understanding of fractions as numbers that can be placed on a number line. Aksu (2001) stated that conceptual knowledge refers to "understanding relationships that are integrated or connected to other mathematical ideas and concepts," and that procedural knowledge "is the understanding of symbolism that is used to represent mathematics and the rules and procedures used in mathematics tasks" (p. 375). According to Cramer and Wyberg (2009), the number line connects conceptual and procedural knowledge, as well as integrates all number systems i.e. whole, negative, fractions, and decimals (National Research Council, 2001). Wong (2010) exerted that number line models exemplify fractions as quantities. Starting in grade 3, the CCSS for Mathematics 2010 require students to complete the following tasks on the number line:

1. Understand a fraction as a number on the number line.
2. Represent fractions on a number line diagram.
3. Explain equivalence of fractions on the number line.

Additionally, the new CCSS require fifth grade students to solve fraction addition and subtraction word problems using like and unlike denominators with representations such as a linear model. This study would potentially add to the research literature of what understandings and misunderstandings children have about fractions and would hopefully enhance instruction based on the research findings.

This study explored the understanding of fractions on the number line for fifth graders at two elementary schools in central Alabama. Elementary educators from central Alabama will benefit from this research by using it to understand fifth graders' understanding of fractions. Policy makers will find this research valuable because it will be a gateway to students' thinking as well as various strategies that can be used to promote learning provided through this research. At this stage in the research, the fifth graders' understanding of fractions for educators will be generally defined as their process to solve and discuss answers to problems about placing fractions on the number line.

Research Questions

The following question provided a guide for this study: How do fifth grade students in two elementary schools in central Alabama demonstrate an understanding of fractions on a number line? This central research question was further supported by the following sub-questions: What strategies do children use to solve fractional problems on a number line? How do students interpret fractions on a number line?

Limitations of the Study

The length of time to gather the data was the primary limitation of this study. It is believed that a longer period than the four weeks that were used to conduct this research would have yielded more substantial results. The manner in which the data were collected is another limitation of the study. The data collection process consisted of interviews that were conducted with fifth grade students of two different elementary schools that rest on the border of the same major city. Another critical limitation of this study was found in some students' concerns about the interviews serving as an assessment for a grade. Additionally, a limitation of the study was that the choice was given to teachers to select the number of classroom visits for the researcher, which resulted in only one official classroom observation for each school. It is a belief of this researcher that multiple observations would have yielded more data to support the findings of this study. The participant demographics included 88% White and 12% African-American students. This disproportionate representation of the races limited the voices of minority students.

Definition of Key Terms

1. Bracketing - "The analyst must temporarily set aside belief in its reality" (Denzin & Lincoln, 2003, p. 217).
2. Common Core State Standards - define what students should understand and be able to do in their study of mathematics (Common Core State Standards Initiative, 2011). The 2010 Alabama Course of Study adopted the Common Core State Standards in its entirety for grades K-8.
3. Constructivism – described as a "theory of knowing that emphasizes the role each person plays in constructing his or her knowledge rather than absorbing it directly from the environment" (Branscombe et al., 2003, p. 10).
4. Density – the understanding of an infinite number of other fractions between any two fractions.
5. Nonroutine number line – unconventional number line with unequal intervals.
6. Residual Thinking – used interchangeably to describe "gap thinking" where children use the knowledge of a fraction being one away from a whole to solve or reason through problems.
7. Standards for Mathematical Practice – These eight standards described varieties of expertise that mathematics educators at all levels should seek to develop in

their students (Common Core State Standards Initiative, 2011). Mathematically proficient students will:

1. Make sense of problems and persevere in solving them,
2. Reason abstractly and quantitatively,
3. Construct viable arguments and construct the reasoning of others,
4. Model with mathematics,
5. Use appropriate tools strategically,
6. Attend to precision,
7. Look for and make use of structure, and
8. Look for and express regularity in repeated reasoning.

This study focuses on mathematical practice number six.

8. Whole Number Reasoning (WNR) – reasoning strategies and understanding that relate to Whole Numbers (mathreasoninginventory.com). Inappropriate use of WNR occurs when the same strategies are misapplied to all aspects of fractions e.g. a fraction may be seen as two whole numbers rather than a single entity.

CHAPTER 2

REVIEW OF THE LITERATURE

This chapter addresses several areas of research as it relates to students' understanding of mathematics with a special emphasis on fraction concepts. The literature disclosed a wealth of information regarding possible causes of mathematical deficiencies, as well as reform efforts taken to improve mathematical achievement for students within classrooms across the United States. In the literature about fractions, two overarching themes emphasized teachers' lack of mathematical knowledge and barriers to learning fractions.

The three topics that will be discussed throughout this chapter are: (a) teachers' limited knowledge of math concepts, with an emphasis on the impact teacher knowledge has on students' learning of fractions; (b) difficulties associated with learning fractions, including barriers that interfere with acquisition of fractional knowledge; and (c) instructional aides to assist with overcoming barriers**.** This study will also include a brief synopsis of the historical aspect of fractions, as well as information infused throughout the study that was gleaned from national reports.

"One area in which the research evidence is consistent and compelling concerns weaknesses in the mathematical performance of U.S. students" (The National Research Council, 2001, p. 4). Students in the U.S. are lagging far behind students in other leading countries (The National Center for Education Statistics, 2011). The literature attributes much of this gap to the procedural, rather than conceptual, skills that are emphasized more by teachers in the U.S. In a Trends in International Mathematics and Science Study (TIMSS) of classroom performance among teachers in the U.S. with teachers in Japan, it was found that connections among mathematical concepts were the focus of more than 54% of the problems on which Japanese students worked, while only 17% in the United States (Zemelman, Daniels, & Hyde, 2005). Further, it was found in this study that Japanese students were required to use a variety of strategies for problem solving such as discussions, and making connections. Contrariwise, this was not the case with the American teachers. In fact, they reportedly gave the answers in many cases (Zemelman et al., 2005). A remarkable, yet simple observation made by Zemelman et al. (2005), which reflects the difference between American and Japanese teachers, is that "American students were practicing skills while Asian students were thinking" (p. 114).

Mathematical teaching in America is continuously evolving in efforts to effect change that will assist educators in their endeavors to invoke more thinking in the learning process. Despite these evolutionary ideas, so many mathematics educators are content with children being able to provide correct answers to mathematical problems without the ability to satisfactorily rationalize how they solve these problems. However, in light of new initiatives such as the College and Career Readiness Standards (2010) that have surfaced in the mathematical arena of assessing student learning, children's inability to explain their work with understanding is a growing concern.

Historical View of Fractions

The continued difficulty most adults and children experience when learning to solve fraction problems gives the impression that it is a relatively new concept.

However, a brief historical tour on the origin of fractions reveals that fractions were used thousands of years ago “by both the Babylonians and the Egyptians to answer questions involving how much and how many” (Chapin & Johnson, 2006, p. 99). During the era that led to the development of fractions, “counting” numbers such as 28 or 176 were used to express quantities of things. However, some situations that required equal sharing could not be represented with this system.

Although mathematicians of that era were cognizant of the need for a different type of number fractions for expressing amounts that were less than one but greater than zero, “It wasn’t until the Renaissance, that the use of fractions, as we know them today, became commonplace” (Chapin & Johnson, 2006, p. 99). Chapin and Johnson (2006) additionally pointed out that although the terms “fraction” and “rational number” are often used interchangeably; “not all numbers written using fraction notation actually represent rational numbers,” (p. 99). Perhaps this is the type of information that adds to the perceived confusion of learning such a difficult concept. Therefore, it is imperative that teachers take charge and learn, as well as implement, their subject matter with fidelity.

Possible Causes of Teachers’ Inability to Teach Fractions Efficiently

Research consistently shows that although numerous barriers exist as challenges to students’ acquisition of a deeper understanding of conceptual knowledge, perhaps the root cause lies at the level where the most confidence is entrusted: within the classroom. According to Ben-Cahim, Keret, and Ilany (2007), teachers’ lack of content knowledge is one of the most crucial obstacles in students’ knowledge of fractions. This view also gives importance to Son and Senk's (2010) concerns regarding learning institutions that prepare teachers for the arduous task of teaching. These programs need to be designed to equip teachers with the knowledge and skills to teach fractions to elementary school students.

Teacher knowledge of content is crucial for student learning. However, many teachers teach math using the same approach by which they were taught, which, according to Heritage and Niemi (2006), "will most likely have centered on rules and algorithms rather than on concepts and reasoning" (p. 274), thus resulting in an epidemic of non-thinkers in numerous facets at all levels of mathematical instruction. "Young children must be able to think mathematically and access information provided in a variety of ways" (McGee, Kervin, & Chinnappan, 2006, p. 360).

Teachers are in a good position to ensure that children use the various learning strategies to gain experience and that they are able to use each one at the most applicable time. "Students must be encouraged to look for patterns and to use logical reasoning in every problem" (Zemelman et al., 2005, p. 116). Zemelman et al. (2005) recommended that teachers should encourage children to use two important strategies: (a) look for patterns, and (b) apply logic to every problem.

Burns (2002) addressed a concern with the way many educators today are teaching children to memorize rules to help them develop facility with fractions, which does not help them understand the concepts. Burns additionally advised that when students forget rules, they have no way to reason through the process, which is a great risk. Adding rigor to this perception, Keijzer and Terwel (2004) commented on the "landscape of learning" metaphor used by Fosnot and Folk (2002), which provided an explanation for the way children learn. Learning was considered as a journey through the landscape, and that different children, with use of their own experiences, take different routes on their way to the determined destination.

The metaphor further depicts how low-achieving students leave the main route and lack the proper means of ever finding their way back. Another point raised by Keijzer and Terwel (2004) regarding difficulty of fractions for low achievers is that they are not always able to request the right form of assistance due to a compounded difficulty of explaining what they do not understand. The way children learn fractions is something about which teachers need to be knowledgeable in order to provide the assistance needed to correct some of the deficiencies associated with learning.

Another error teachers make that inhibit student learning is their faithful reliance on textbooks. In order for students to detect patterns or cultivate conceptual understanding, teachers will need to go beyond the textbook (Zemelman et al., 2005). A possible cause for this over-reliance on textbooks could be teachers' lack of confidence in their ability to teach math.

Unfortunately, "many teachers who are responsible for teaching math in grades K-8 haven't studied mathematics in-depth" (Chapin & Johnson, 2006, p. xv), which explains their inability to teach in a manner that enables children to gain a deeper understanding of math concepts. Many students in grades pre-K through 8 continue to be taught by teachers who are not prepared to equip students to develop mathematical proficiency (National Research Council, 2001). Chapin and Johnson (2006) further expounded on the misconception that teachers in elementary and middle schools do not need to have the knowledge base of advanced studies to teach, which ultimately leads to exposing children to teachers who have limited mathematical knowledge. Perhaps this is one of the reasons why teachers of mathematics rely heavily on rote procedures with little emphasis on conceptual understanding.

Various Barriers to Learning Fractions

Historically, there has been and still is a growing concern for mathematical achievement of students within the U.S. An even greater alarm is sounded for the need for educational reform as evidenced by the implementation of College and Career Readiness Standards. The operation of fractions is reportedly a concept that is hard to grasp by many students, as well as adults (Son & Senk, 2010). A similar concern was echoed by Pitkethly and Hunting (1996) but more specifically in an accusatory manner toward educators with their comment that the concept of grasping fractions as numbers is something that students, as well as teachers, have a difficult time understanding. This outcry exemplified the importance of teachers having an understanding of fractions to aid the process of teaching and learning of this concept.

Students' struggle with fractions can be attributed to their lack of knowledge of whole numbers. Inappropriate use of whole number reasoning occurs when students see the numerals in a fraction as two separate whole numbers (e.g,. ¾ is identified as 3 and 4) rather than seeing ¾ as a single quantity (Petit, Laird, & Marsden, 2010). This misconception is exacerbated by the overuse and sole exposure of part-whole concepts where they only see a fraction as 3 out of 4 pieces in an area model rather than, for instance, placing ¾ on a number line as a unit of measure. The contrast of whole numbers and fractions could also confuse students.

> Fractions are infinitely divisible. They are not linked by successor relations; no fraction comes immediately before or after another fraction, and between any two fractions are an infinite number of other fractions. This makes it impossible to count fractions directly, which precludes one of the main processes through which people learn about whole numbers. The magnitudes of fractions do not increase in any consistent way with the size of their components; 7 is greater than 4, but a fraction with a numerator of 7 may or may not be larger than a fraction with a numerator of 4. (Schneider & Siegler, 2010, p.1228)

The fact that one quantity could be represented by an infinite number of fractions is another challenge that children face when learning fractions.

Interpretations of Fractions

Much of the difficulty is linked to the various representations and interpretations of rational numbers. Kieren (1976) originally identified these subconstructs of rational numbers. Behr, Lesh, Post, and Silver (1983) further recognized the part-whole concept to accompany the original four subconstructs: measure, ratio, quotient, and operator. The depth of conceptual understanding of rational numbers is grounded in knowledge of each concept individually as well as knowing the interconnections among these five representations (Kieren, 1976).

The *part-whole* construct relies on the ability to equally partition discrete or continuous models. Although this subconstruct is the most common and often overly used, it is essential for students to gain this foundational understanding to comprehend all other interpretations. If students are only exposed to this interpretation, they may gain an understanding of unit fractions (i.e., ¾ is 3 out of 4, but struggle with applying "out of" strategy with improper fractions such as 5/4). The part-whole construct would no longer make sense in the latter context. On the other hand, this subconstruct can be crucial with the role in understanding equivalency and operations of fractions.

A *ratio* is a multiplicative comparison between two quantities or measurements. Clarke, Roche and Mitchell (2007) claimed that this subconstruct is not emphasized enough in the school curriculum. Like the fraction interpretation, ratios can compare a part to a whole, thus identified as the part-whole interpretation. Order is very important with this type of ratio. For example, if you are comparing part-to-whole with 2 girls to 3 boys, it is important to write 2:5 or 2/5 for girls to children. On the other hand, when a ratio compares parts of a set to another part of a set, the order of ratio should only accompany the comparison i.e. 2:3 for girls to boys or 3:2 for boys to girls. This type of representation is known as a part-to-part comparison.

Rational numbers can also play the role of an *operator* to shrink or expand another number. The numerator expands the quantity while the denominator shrinks it. The National Council of Teachers of Mathematics (2010) provides the following information about the difference between the numerator and the denominator:

- If both are the same (i.e., 4/4) then the value is 1; thus, the net effect remains unchanged (4/4 of 12 = 12).
- If the numerator is larger (i.e., 5/4) then the value is greater than 1; thus, it expands the value (5/4 of 12 = 15).
- If the denominator is larger (i.e., ¾) then the value is less than 1; thus, it shrinks the value (3/4 of 12 = 9).

Fraction as a quotient is stated by Clarke (2006) to be a "very useful but often neglected" subconstruct (p. 7). Because fractions are often thought of as parts of a whole, it is likely that students will find this interpretation unusual (Van de Walle, 2007). The idea of a fraction as a quotient defies the misconception that a larger number cannot go into a smaller number, which is often heard when a student solves a division problem with the standard algorithm. This subconstruct is the result of a division problem, such as 3 ÷ 4 = 3/4 or .75, which can be exemplified through partitioning and equal sharing.

Like the operator and quotient constructs, rational numbers as a *measure* is not given adequate attention in schools. Due to the recent adoption of the Common Core State Standards 2010 this is likely to change. The new standards require students to master the concept of understanding a fraction as a number and represent fractions on a number line diagram starting in grade 3. The common thought of fractions being identified solely as part of a set makes it more difficult to also think of fractions as a measure, and thus, obstructs the ability of students to place a fraction on a number line. Fractions can be modeled on a number line to identify the distance from the beginning to the end length (i.e., ¾ of the distance from zero). The understanding of a fraction as a length on the number line created by partitioning units into subunits is the essence of this interpretation (Chapin & Johnson, 2006). A unit of measure is infinitely divisible.

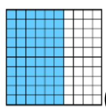
Representations of Fractions

Another difficulty of rational numbers can be attributed to various representations of fractions. Mathematics and, more specifically, rational numbers require representations to help clarify the concept of fractions. Lesh, Post, and Behr (1987) stated that three distinctive types of representations: lexical expressions, numerical notations, and graphical notations where rational numbers can be thought of as words, numbers, a piece, or a point on a number line. For example, 60/100 can be represented as follows:

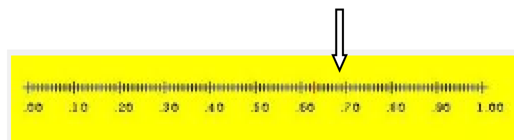
Numerical notations: 60/100 (fraction notation), .60 (decimal notation), 60% (percent notation), or 60:100 (ratio)

Lexical expressions: "sixty out of one-hundred," "sixty-hundredths," "point sixty," "sixty percent," or "60 per 100"

Graphical notations:


(area)


(set)



(linear)

Number line. The number line is a mathematical tool that emphasizes the measure model. It is simply a line that must have at least two points identified to establish a unit of length. This linear model is a representation of numbers that ascend from left to right on a straight line where every unique point correlates with an integer. The progression of a number line starts when the visual numbers on a structured number line with marked segments supports oral. It is suggested that students be introduced to the number line through strings of beads that interchange in color every ten beads (Buys, 2001). The empty number line can be further developed through a dual sided linear model where one side has the decade numbers 0 through 100 while the other side is less defined with only two numbers with zero as the starting point and ending with 100 (Bobis, 2007). Students can position an object along the empty number line to identify designated numbers up to 100 and then flip over to verify if the object is in the correct location.

Although the number line is more abstract than other continuous models, it is considered one of the most important representations for fractions (Cramer & Wyberg, 2009). Keijzer and Terwel (2001) found that when that the number line model was central to support the development of fractional strategies, students outperformed a control group who mainly worked individually with the circle as the major fraction model.

According to Hannula (2003), the ability of locating a number on a number line could be an indication of a confluence of several subconstructs. The National Mathematics Advisory Panel (2008) recommends using the number line during fraction instruction to connect conceptual and procedural knowledge by way of ordering fractions. The number line is a conceptual measurement model that provides a link between arithmetic and geometry (National Research Council, 2001) and is an aid for visualizing the continuous flow of rational numbers (Dienes, 1966). This linear representation reinforces the concept of fraction as a number because it provides a unique point on the line rather than thinking of two whole numbers as in part whole models. Hannula (2003) insinuated that the number line could be indicative of joining together of several subconstructs. The National Research Council (2001) further explained that the number line has the ability to integrate all the number systems from pre-K to 8th grade. In addition to serving as a model for distance, the number line is an effective tool for exploring the concept of equivalency and ordering of fractions. Unlike other models, the symbols play a crucial role with understanding. The number line uses symbols to convey a portion of its meaning where students have to coordinate this symbolic information and visual cues to bring meaning to the model (Cramer & Wyberg, 2009). Contrasting with other models such as hundred charts or arithmetic blocks that rely heavily on visualization, the number line requires more cognitive involvement such as partitioning and labeling (Klein, Beishuizen, & Treffers, 1998).

Difficulty of the Number Line

Although classroom walls are often decorated with number lines, students frequently lack appropriate opportunities to consider the full mathematical meaning of the model (Earnest, 2007). Like Bright et al. (1998), Behr et al. (1983), and Hannula (2003), Yanik, Helding, and Baek (2006) found that students had difficulty working with number lines that were greater than the unit from 0 to 1. When Yanik et al. (2006) used number lines greater than one, the students partitioned the entire number line like a bar as if it was a unit of one. The researchers also found that students were unable to see the number line as a continuous assembly of iterated units. With the linear model children tend to see hash marks as discrete points on a number line instead of as part of a whole unit.

For children to construct a deep conceptual understanding of rational numbers, it is crucial for them to first know that rational numbers can be interpreted a variety of ways and connect the representations to the interpretations, i.e. quotient, ratio, and measurement (National Research Council, 2001).

Teacher Preparation as a Barrier to Effective Teaching and Learning

The National Council on Teacher Quality (2008) finds that very few educational programs cover the mathematical content that teachers need. Although many are unprepared, teachers are usually selected to teach math based on their ability to impressively interview or on their reputation to teach well. The National Council of Teachers of Mathematics (NCTM) (2000) advocated continuing efforts for teachers to learn and improve teaching by “learning about mathematics and pedagogy, benefitting from interactions with students and colleagues, and engaging in ongoing professional development and self-reflection” (p. 19).

In compliance with this suggestion is the perception shared by the National Research Council (2001) that “every school should be organized so that the teachers are just as much learners as the students are” (p. 19). The most common suggestion is to require teachers to study more mathematics with the aid of additional coursework. The goal, however, should not only be to produce teachers who are more proficient in mathematics, but to improve students’ learning by providing teachers with opportunities to learn that will “equip them with the mathematical knowledge and skill that will enable them to teach mathematics effectively” (Ball, 2003, para. 3). “Although the typical method of improving instructional quality has been to develop curriculum and—especially in the last decade—to articulate standards for what students should learn, little improvement is possible without direct attention to the practice of teaching” (Ball, 2003, para. 1). The complex nature of fractions gives importance to a need for improving the instructional quality within the classroom by considering every conceivable barrier, including the language that is needed for a better understanding of this concept.

Language as a Barrier to Mathematical Learning

Another barrier to effective learning of mathematics is the language associated with the problems students are given to solve. Burns (2000) suggested that fractional concepts may be articulated to students as early as preschool age, and that before they start school, they have learned to use fractional language such as ones in the following examples to describe events in their own lives:

- You can have half of my cookie.
- It's a quarter past one.
- I need half a dollar (p. 212).

Due to the complexity of learning about fractions, teachers need to teach and model key vocabulary terms that are necessary for solving these problems. Teacher modeling of appropriate conventional mathematical vocabulary is essential for mathematical development (NCTM, 2000). Chapin and Johnson (2006) asserted that students need to know the multiple meaning of fractions to aid in the development of student learning. More specifically, the use, understanding, and development of fractional language are identified as important factors to grasp the concept of fractions (Clarke & Roche, 2009; Keijzer & Terwel, 2001, 2004; Pearn, 2007). Van de Walle (2007) highly suggests the use of classroom discussions as a catalyst for introducing and teaching vocabulary of fractional parts.

Instructional Aids to Assist with Overcoming Barriers

Although numerous problems dealing with fraction problems were found in the research, a wealth of solutions was also cited. Because of the abstract nature of fractions, "Children should deal with fractions concretely and in the context of real life before they focus on symbolic representations" (Burns, 2000, p. 212). "In order to develop fraction sense, most children need extended periods of time with physical models such as fraction circles, Cuisenaire rods, paper folding, and chips. These models allow students to develop mental images for fractions, and these mental images enable students to understand about fraction size" (Cramer & Henry, 2002, para. 2). Van de Walle (2007) advised educators to make use of various representations such as rectangular, linear, and set models to develop fraction concepts at every grade level. According to NCTM (2000) students should become malleable with creating physical or mental representations that appropriately depict the purpose at hand.

Burns (2002), a leading mathematics educator, stated that educators need to provide multiple strategies for students to learn about fractions including use of concrete materials, geometric perspectives, numerical focus, and problems related to real-life situations. Burns (2002) additionally asserted that children's first encounter with fractions occurs outside of school with examples of the following comments made by adults:

- I'll be back in three-quarters of an hour.
- The recipe says to add two-fifths of a cup of water.
- There's a quarter-moon tonight.
- The dishwasher is less than half full (p. 212).

The Constructivist Approach

The knowledge many children demonstrate about fractions evidences a critical need for math to be taught with more understanding than is currently employed in many schools. The teacher-constructed algorithms most children use to complete fraction problems provide evidence of a widespread pandemic that continues to infiltrate the majority of mathematical classrooms within our schools. Mathematics educators should be concerned with students' ability to construct fraction concepts rather than conform to the norm of providing them with one-size-fits all teacher-learned algorithms for solving problems; however, it is now clearer that using curriculum effectively and working responsibly with standards depends on understanding the subject matter (Ball, 2003). Contrary to the belief that many teachers portray in their instructional practices, leading experts report that simply telling students what a concept is does not develop conceptual understanding, and that "Concepts are built by each person; understanding is created" (Zemelman et al., 2005, p.114).

A report from National Assessment of Educational Progress (NAEP) voiced the frustration they hold regarding students' difficulty in learning fractions. Students' inability to solve fraction problems with proficiency is chronicled in the following examples cited by Perie and Moran (2004) in the NAEP report. The results of this assessment were based on research that was conducted with students in grades 8 through 12 and which emphasizes the great weakness American students portray in understanding fractions.

- 50% of 8th graders demonstrated an inability to order 2/7, 1/12, and 5/9 from least to greatest.
- Only 29% of 17 year olds could translate 0.029 as 29/1000 correctly.

Although students should be held accountable for their learning, the bulk of what they learn is dependent on teacher knowledge. Therefore, if teachers have greater understanding of fractions and know how to teach their students for understanding, students should gain a deeper understanding of fractions.

The Use of Background Knowledge as an Instructional Aide to Teaching Fractions

Children's intuitive knowledge is another catalyst that can be used to aid instructional practices. According to Mack (1988), children come with a wealth of prior knowledge that can serve as a foundation for instructional knowledge. The National Center for Education Statistics (NCES) reported that children as early as preschool age have a rich store of prior knowledge that reflects fractional understanding (2011). This is clearly evident when children successfully engage in equal sharing activities. Based on this understanding of early literacy in fractional knowledge, NCES (2011) encourages educators to build on students' intuitive knowledge to facilitate connections between prior knowledge and formal fractional concepts. Contrary to this belief, Siemon (2003) shared that children do not need to be taught how to half because this notion is intuitive to most children.

Another component involved in the intuitive process is students' informal notions of partitioning and measuring, which provide a point of origin for developing the concept of rational numbers. Although the teaching of fractions fills many teachers with trepidation, Hunting (1983) reported that children have ideas about fractions prior to formal instruction, and teachers must take children's prior experiences into account when teaching fractions.

A Comparison of the Circular and Linear Models for Instructional Purposes

According to research, number lines can be a valuable instructional tool within the classroom. The National Research Council (2001) pointed out how they allow users to integrate all the number systems, i.e. integers, decimal fractions, common fractions, equivalent fractions, and mixed numbers. Cramer and Wyberg (2009) reported how they can be used to link conceptual and procedural knowledge. According to the National Research Council (2001), procedural fluency and conceptual understanding are often seen as processes competing for attention in mathematics. However, focusing instruction on understanding helps students to learn more easily, be less susceptible to common errors, and be less likely to forget.

Teaching children in a manner that enhances understanding would put to rest the myriad of complaints teachers make regarding how students have to be taught the same concepts year after year. Keijzer and Terwel's (2001) comparison of the linear and circular model reported the effectiveness of the number line over the use of the circular model. A similar comparison of the circular and linear models by other leading researchers found that children were able to promote effective strategies for partitioning a linear model when they had difficulty partitioning a circular model (Charles et al., 1999).

According to Bright et al. (1988), successful use of the number line with fractions requires integration of two forms of information, visual and symbolic. This idea is taken a step further with Wong and Evans (2008) who added the referent units to this simultaneous process. Despite the numerous advantages of the number line, the simultaneous process that Bright et al., as well as Wong and Evans (2008), described is partially to blame for the difficulty associated with the number line. For example, Pearn and Stephens (2007) reported that 60% of students sampled from "Years 5 and 6" were able to name a fraction between 0 and ½, while only 38% could accurately place the fraction they selected on the number line. According to Ni and Zhou (2005) this level of difficulty could be attributed to the manner in which they perceive the unit as a problem.

Other problems associated with the number line are reported by Baturo and Cooper (1999) who stated that students have difficulty unitizing linear models due to their perception that the marks are discrete points on a line instead of parts of a whole unit. Bright et al.'s (1988) recognize, "The use of symbols to label points on a number line may focus a student's attention on those symbols rather than the pictorial embodiment of the fractions" (p. 2), which identifies the points as a distractor.

Another key point emphasized by the National Research Council (2001) is the importance of teachers stressing to students the necessity of counting lengths rather than the end-points where the numbers are. Wong and Evans (2007) found it to be a challenge for students to successfully place unit fractions on a number line labeled from 0 to 1. Bright et al. (1988) reported on a greater success rate of helping students with representing fractions on number lines from 0 to 1 than on number lines from 0 to 2.

Summary

A review of the literature unveiled several possibilities for students' lack of conceptual understanding, many of which can be connected to the teachers' lack of understanding as the root cause for so many of these hindrances. The academic success of children is no longer measured by their ability to compete with fellow classmates or with students in neighboring schools, but rather on a global plane. Therefore, it is imperative that students are taught mathematics at a level of understanding that will allow them to master skills that will propel them to the next level with greater ease. The literature provides numerous suggestions and ideas that can be used as instructional tools that allow children to manipulate learning, which will have a lasting effect on their learning ability.

Although much blame has been placed on teachers for students' inability to perform at their highest potential, notice is also given to the shallow and undemanding curriculum implemented in many school systems (National Research Council, 2001). We must not lose sight of hope in our learning institutions, as efforts of the past decade have shown that good instruction can make a difference, and that teachers can learn from their work, as well as work with curriculum materials.

Since no literature was found that focused solely on fifth graders' understanding of fractions on the number line, this study addressed a major gap in the literature. The purpose of this case study is to explore the understanding of fractions on the number line for fifth graders at two elementary schools in central Alabama.

CHAPTER 3

RESEARCH METHODOLOGY

In this case study, the researcher investigated fifth grade students' understanding of fractions on a number line. Collective case studies are an exploration of "bounded systems over time through detailed, in-depth data collection involving multiple sources of information," such as student interviews, classroom observations, and artifacts (Creswell, 2007). The data collection process that was employed was more conducive for qualitative methodology.

According to Merriam (2009), qualitative research begins with assumptions or a world view with which to study research problems by probing into the meaning individuals or groups attribute to a particular problem or concern**.** Qualitative research allows the researcher to gain a deeper understanding of phenomena by collecting data through interviews, observations, and written artifacts within the natural settings that are unique to concerns of the people under investigation. Merriam (2009) pointed out how any data collection method, including testing and interviewing, is appropriate for a case study. Adding to this thought, Lincoln and Guba (1985) highlighted the importance of the environment for which the phenomenon under scrutiny is to the investigation with their comment that "inquiry must be carried out in a 'natural' setting because phenomena of study, whatever they may be…. take their meaning as much from their contexts as they do from themselves" (p. 189). Qualitative researchers study things in natural settings in efforts to rebuild the interpretations participants use to understand their world (Hatch, 2002), which is another characteristic of qualitative research.

The manner in which direct application of qualitative research to educational concerns lends itself to social research has an interesting historical background. Hatch (2002) asserted that the first professional qualitative researchers were most likely anthropologists who, through their ethnographic narratives, described primate cultures in remote places. One of the first social scientists known to spend time in natural settings and to use an inductive approach to understand culture was Boaz. "Boaz was a cultural relativist who believed that the object of anthropological study is to describe the knowledge that members use to make sense within their own culture" (Hatch, 2002, p.3). Fieldwork practices such as participant observation, interviewing, and artifact-gathering, all of which are important characteristics of qualitative research, reportedly were developed by anthropologists such as Bronislaw Malinowski, Margaret Mead, and Alfred Radcliffe-Bown (Hatch, 2002).

According to Hatch (2002), qualitative researchers seek to understand a phenomenon from the perspectives of those actually living in it. Merriam's (2009) partial description of qualitative research stated how it is instrumental in knowing more about or improving one's practice. The qualitative research method was used for this study because it is subjective in nature and leans heavily toward classroom observations, student interviews, and physical artifacts as methods for collecting data that would help to understand student strategies and how students understand and solve problems on the number line.

Although this approach is somewhat time consuming, it was appropriate for this study because it relies heavily on what people do and say, which played a major role in the manner the data was collected. The qualitative approach was also appropriate for this study due to the number of participants, specifically 26 fifth grade students and 2 classroom teachers, that was necessary to complete the study. The meaning for the phenomenon experienced by these study participants will be described in this research (Creswell, 2007). Several research questions were addressed in this study, which is another reason for using the qualitative research approach.

Qualitative researchers strive to understand phenomenon from the perspectives of those who experience it (Hatch, 2002). One method of qualitative research that lends itself well to the study of a single phenomenon that is bounded by a unique system is a case study. The researcher studies a case which was described by Stake (1995) as "a specific complex functioning thing" which could include a child, a classroom, a school, or all the schools in a city (p. 2). Yin (2009) described a case study as "an empirical inquiry that investigates a contemporary phenomenon in depth in its real life context" (p. 18).

According to Merriam (2009), the stance that researchers bring with them to a study gives rise to the theoretical framework of the research. If researchers are not careful, these preconceived ideas about a concern to be understood could possibly skew the data. Hatch (2002) advised that epoche "requires that we work to become aware of our own assumptions, feelings, and preconceptions, and then, that we strive to put them aside" in order to become more amenable to the phenomenon of interest (p. 86). With the data collected from this study, this researcher was able to analyze the situation by minimizing it to significant statements, which were combined into themes. The passion that the researcher has for teaching math in a manner that is understood by children stimulates preexisting ideas which forces the concept of epoche in order to allow the experience of students' use of the number line to illuminate an understanding of the phenomenon being studied.

Fifth grade students have a difficult time constructing mathematical knowledge when they are taught fractions with algorithms. This was a study of fifth grade students' descriptions of interpretations of the concept of fractions within two elementary schools within central Alabama. The purpose of this qualitative study was to explore educators' and students' perception of fifth grade students' understanding of fractions on a number line in two elementary schools in central Alabama. Denzin and Lincoln (2005) advised that in order to make sense of or interpret phenomena in light of the meanings people bring to them, qualitative researchers will study them in the setting for which they naturally occur. Several questions were used for collecting data for this study through interviews and observations, which are characteristics of qualitative research.

A collective case study perspective provided the philosophical foundation for this study. This approach was used to give voice to students to an extent that would illuminate not only their understanding but also to serve as a foundation for additional research on this topic. Case study is an approach that can be used to recognize and restore philosophical beliefs that have confined internal ideas that are caused by philosophical confusions.

Theoretical Framework

Piaget is noted for his theory of cognitive development and is commonly referred to as a child psychologist. A noteworthy observation was his view regarding the construction of knowledge. According to Piaget, knowledge is a continuous process of development that requires mental and/or physical action on behalf of the child and not simply a collection of stored information. Piaget's theory on the evolution of knowledge is provided with his explanation of how such terms as schema, assimilation, accommodation, and equilibrium work together in the environmental adaptive process.

According to Piaget, the manner in which children satisfactorily adapt to the environment is known as schemas. The new information children take in that has to be matched with existing schemas is referred to as assimilation. Accommodation occurs when children have to alter schemas they already have in order to aid comprehension of events through assimilation that otherwise would not take place. Equilibration is the balance between the two—assimilation and accommodation—that children maintain as they progress through the stages of cognitive development.

Piaget is also well known for his views on four major stages of development in a child's cognitive growth. Although Piaget's theory was not specifically applied to education, many educational programs are established on his views that children should be taught at levels for which they are developmentally prepared. According to Branscombe et al. (2003), most children are able to reason abstractly around age 11 throughout adulthood and thinkers no longer need to consider concrete objects in order to problem solve. Additionally, Cramer and Wyberg (2009) asserted that "children's conceptual development evolves from concrete experiences to abstract ones" (p. 228).

Philosophical Assumptions

The philosophical assumptions chosen for this qualitative research consist of a constructivist position that is taken toward things that have been acknowledged as psychological, which is an ontological, or nature of reality, assumption (Sokolowski, 2000). "Knowledge produced within the constructivist paradigm is often presented in the form of case studies…" (p. 15). The axioms available to researchers include generalization and causal linkage possibilities (Lincoln & Guba, 1985). The epistemological assumption held during this research emerged as interaction with teachers and students were made during the study. It was through the works of these individuals as well as their numerous conversations that the resulting pedagogy was continuously developed. An intentional reliance on problem-solving strategies these fifth graders used to understand fractions after extensive instructions were provided underscored the constructivist paradigm. Strong passionate beliefs held by the researcher concerning teaching and learning mathematics necessitated a constant reminder throughout the study to suspend personal and academic presuppositions while analyzing information obtained through observations and interviews in order to inductively allowed findings to evolve as data was gathered.

Participants

Several sampling methods were used in this study. One was purposeful sampling described by Creswell as a method of selecting a small number of participants in which data collection took place to understand a central occurrence (Creswell, 2007). A variation of purposeful sampling was also used, which is referred to as maximal variation sampling. Two educators and their students who have had the experience of teaching and learning fractions in a fifth grade environment were purposefully sampled as participants in this study. This sampling method provided a wealth of information from a few participants who shared the phenomenon of interest.

Participants and Settings

The research was carried out in two fifth grade classes (one at each of two elementary schools) in suburban cities in central Alabama. A total of 28 participated in this study, 14 girls and 12 boys as well as one male and one female teacher. All participants within the two classrooms were recruited in the same manner, in which every student with a signed consent form from a parent was allowed to participate. No participants were included or excluded due to gender, race, academic history, or any other personal factor. One of the schools selected for this research had a diverse demographic population while the other was more closely aligned in population and socio-economic status.

School A was one of six schools within a relatively small school district that started in 1959. In the beginning, there were only three schools with minimal enrollment. At the time of this study, the school system served approximately 4,515 students. The elementary site of School A that was selected for this project was located in the central part of the suburbs of a major city. According to the 2010 Census Reviewer, the city of residence had approximately 20,000 people with city's makeup of 47.73% male and 52.27% female. The ethnicity make up was 97.23% White, .93% Asian, 1.03% Black, .06% American Indian and Alaskan Native, and .19% other races. The state's demographic report indicated a median income of $100,483 with a median property value at $336,300. This was a non-Title I public school in a metropolitan environment with a 98.39% White, 1.24% Asian, and 0.37% Black population. This school had a girl to boy ratio of 413 males and 395 females. Four elementary schools were included in the district, and three of these schools had a similar student body demographic representation as the school in this research study.

School B was one of several city schools within a district that started as an independent system after it was removed from the county system in 1988. At that time, there were only four schools within the district that consisted of a combined total enrollment of 5,243. By 2007, the school system had grown to a total of 16 schools with 12,406 students enrolled. During the 2004 school year, the school system was set to open a large middle school that resulted in a need for rezoning. The rezoning plan was met with strong opposition from major stakeholders. Despite strong oppositional forces, the plan ultimately increased parental support and resulted in an unusually effective Parent Teacher Organization. According to the school principal, it was a transient school where approximately 9% of the students are constantly enrolling or transferring from the school.

The elementary site of School B that was selected for this project was located in the suburbs of a major city. According to the 2010 Census Reviewer, the city of residence had approximately 81,000 people with the makeup of 48.05% male and 51.95% female residents. The ethnicity if the population was composed of 75.11% White, 14.84% Black, 5.08% Asian, .25% American Indian and Alaskan Native, and 3.18% other races. The city's chamber of commerce reported a median income of $71,964 with the median property value of $203, 327. This was a Title I public school in a metropolitan environment with a composition of 53.11% White, 30.13% Black, 10.73% Hispanic, and 6.03% Asian residents. This school had a girl to boy ratio of 279 males and 252 females. Of the 531 students enrolled, 23% received Free Lunch and 7% received Reduced Lunch. Ten elementary schools were included in the district, and three of these schools had a similar student body that was similar to the school in this research study. The other seven schools had areas zoned to them that included fewer apartments and a substantial difference of free and reduced lunch.

Important common patterns within this research were identified, which again constitutes the use of maximal variation sampling, a form of purposeful sampling. Given the fact that all participants of this study were educators or fifth grade students, it was safe to conclude that homogeneous sampling was also used. During the selection process for participants, consideration was given to teachers who worked in schools where the districts encouraged progressive education for mathematics. The stark contrast of the student body in both schools with regards to demographics was another determining factor for participants. Two educators were utilized in this project: one white male and one white female.

General Settings

Campus A was in the heart of downtown within a quaint community. The school was one block from the shopping district, various restaurants, and a public library. These were all places where students frequently went after school. In contrast, Campus B was nestled on a secluded plot of land at the end of a busy intersection. Although there were a few stores and restaurants in close proximity of the school, the location was not conducive for independent student travel.

Classroom Settings

There were several commonalities noted in the fifth grade classrooms during the scheduled observations. A rich mathematical environment was evidenced with mathematical posters that contained math vocabulary and strategies for every operation. The classroom arrangements fostered a collaborative environment where the students sat at desks or tables in groups of four. Both teachers used the Investigations curriculum as the primary source of instruction. Both classrooms had document cameras that projected on the white dry-erase boards. Classroom A was arranged with student desks in four groups of five while Classroom B arranged student in groups at rectangular tables of four.

Study Participants

Teacher A (TA) was a Caucasian female who appeared to be in her mid-50s. TA spoke with an eloquent voice, yet in a very authoritative manner. She started her educational career 34 years ago as kindergarten aide. She then taught in self-contained fourth and fifth grade classrooms for 15 years. TA held a master's degree in Elementary Education. For the past 18 years she had been employed as a fifth grade teacher of mathematics. She taught with much patience and respect.

Teacher B (TB) was a Caucasian male who appeared to be in his early 50s. He started his career as a sales representative in corporate America. Prior to working at this school, he taught in a rural town about 50 miles from the school of research. His hard work and dedication has earned him "Teacher of the Year" for his school. He held a master's degree in Education. Although he had been in education for a total of 13 years, he had been teaching fifth grade at this school for the past 8 years.

Students one through fifteen were in Teacher A's class (7 boys and 8 girls). This class had an all-white student body. Students 16 through 26 were in Teacher B's class (7 boys and 4 girls) with 8 whites and 3 blacks.

Table 1

Participant Codes

School A	SA
School B	SB
Teacher A	TA
Teacher B	TB
Students 1-15	S1 – S15 (SA, TA)
Students 16 – 26	S16 - S26 (SB, TB)

Role of the Researcher

In a case study approach, researchers do not assume that they know in advance what they are going to find or how people react to different or even similar situations. This approach required flexibility as discoveries and findings emerged that might suggest tweaking the approach. In other words, each discovery could influence the ongoing process of this study.

A middle-aged black female who is employed as a math coach in a predominantly white school district conducted the study. She received a Bachelor of Science in Finance from Alabama State University and a Master's and an Educational Specialist degree from the University of Alabama at Birmingham. Her prior employment history was in the area of finance, where she was employed in administrative positions in corporate America for several years. She taught a self-contained fourth grade inclusion classes for five years. She is currently employed as a Math Coach where her primary role is to provide continuous professional development for teachers in two schools. She is also pursuing a doctoral degree in education.

Data Collection

The data set for this study included multiple sources of data with classroom observations, interview notes, completed tasks, and transcribed interviews. The qualitative approach to inquiry was used to gather data for this research and included several methods. First, the researcher observed both classrooms for an entire lesson on fractions. Then the researcher interviewed fifth grade students to explore their understanding of fractions. The interviews and observations took place in the schools during school hours.

To ensure the data was reproduced exactly as it occurred in the field (Lincoln & Guba, 1985; Merriam, 2009), the interviews were video recorded with the use of an iPad and a Flip Camera. This proved to be useful during the inter-rater coding process. During the interviews, the recorders were turned off when the initial discussions had nothing to do with the purpose of the interviews. During the first meeting, participants were informed in greater detail about the methods that would be used to generate data for this study. They were advised that, with their permission, all interviews would be video-recorded and would last approximately 15 minutes. Hatch (2002) argued, “Researchers should take time to go over what the study will involve and what will be expected of the participants before the study begins" (p. 51). Participants were also informed that no harm or risk would come to them during the research and that all information would be held in strict confidence except as reported in the aggregate. Participants were further advised that extra precautions would be taken to conceal identities in every conceivable manner, even to the extent of not labeling their classrooms, nor would they be identified in the study. They were advised that they would be inconspicuously referred to as "one student at an elementary middle school" or "one teacher at an elementary school," etc. The procedures for collecting data were also communicated to the participants in the interview protocol (Appendix A), as well as in the recruitment letter. The participants' expressions, mannerisms, and reactions to questions were also observed which necessitated note taking. As the observations were completed, taking notes was found to be invaluable. Bryman (2004), an expert in the field of research, stated that jotted notes are very brief notes written down on pieces of paper or kept in a small notebook to jog one's memory about events that should be written at a later time. Notes were used to assist with the memory of facial expressions, other body language, and tone of voice observed during those observations, which provided hints of secure and insecure responses. Most importantly, a better understanding for their lived experiences was gathered. Merriam (2009) suggested that researchers should write detailed notes that can be used as sources of data.

The assessment was comprised of eight tasks, which measured understanding of the concept of fractions on the number line. The task directions and questions are as follows:

1. How do you see or think of a number line?
    a. Can you draw one? Put the numbers1, 2, and 3 on your number line.
2. This number line shows 0 to 1. Put an X where the ½ would be on the number line below.
3. This number line shows 0 to 3. Put an X where the ½ would be on the number line below.
4. Name a fraction that is less than ½.
    a. Place your fraction as accurately as possible on the number line below.
5. Figure out what this point is called on the number line. Circle the correct answer.
    a. How did you figure out this answer? 2/6, 2/7, 1/4, 2, 2/4
6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.
7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location. 8/12, 8/3, 2/3
8. Use the number line below to solve the following problem.
    a. Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?

The questions addressed the focus of the research as well as gave the participants latitude to respond to the tasks in their own ways. During the interviews, every effort possible was made to help the participants complete the tasks in a non-threatening environment.

Another type of data collected for this study involved informal meetings and observations (see Appendix D) within the educators' classrooms to document teaching and learning of fraction concepts. Yin (2009) identified various sources of evidence that may be used for qualitative research, including interviews, observations, and artifacts, which were used during this study. According to Hatch (2002), documents are classified as unobtrusive data, which "provide insight into the social phenomenon under investigation without interfering with the enactment of that social phenomenon" (p. 116).

The qualitative research method was used for this study because it was subjective in nature and leans heavily toward interviews and observations of individuals as methods for collecting data. Although this approach was somewhat time consuming, it was appropriate for this study because it relies heavily on what people do and say, which was the focus of this research. Often, what participants say and what they do were different. The qualitative approach was also appropriate for this study due to the number of participants, specifically 26 students and 2 teachers, that were necessary to complete the study. The following research questions were addressed in this study:

- How do fifth grade students in two elementary schools in central Alabama demonstrate an understanding of fractions on a number line?
- What strategies do children use to solve fractional problems on a number line?
- How do students interpret fractions on a number line?

Data Analysis

Data analysis in qualitative research is a systematic search for meaning, which consists of organizing and interrogating data to see patterns, identify themes, and make interpretations (Hatch, 2002; Merriam, 2009). As suggested by Hatch (2002) and Merriam (2009), organizing and transcribing the data are initial steps during the analyzing process. The researcher scanned the students' written work. This scanned document was further developed to include what they said and did from the video recording, thus creating a comprehensive transcription. All task-based interviews were transcribed over a period of two weeks. The researcher read the transcripts as well as reviewed the videos and observation notes multiple times to identify patterns. After the first read and viewing, the transcriptions were coded into commonly used strategies, words, phrases, and misconceptions. Students' verbal and written responses were entered into a spreadsheet with detailed notes and explanations for each task. The tasks were also analyzed for accuracy by percentages and number of repeat occurrences for statements, strategies, and errors.

The essential elements under investigation were isolated which underscores the idea of "bracketing" (Denzin & Lincoln, 2003). Firstly, data were bracketed through several readings of transcripts in order to develop a comprehensive coding system. Codes were noted and marked. The most popular codes were density, partition, unit, precision, tool, strategy, and question. Codes were compiled into several categories, e.g. number line as a tool, question levels, student reasoning, procedural understanding, and successful strategies. Four themes and nine subthemes were identified as emergent from the analysis of student interviews, classroom observations, and research questions of this study. The researcher consistently analyzed the data to ensure the themes were coherent with the participants' data.

Establishing Credibility

The case study approach allowed for use of several validation methods, including triangulation, to make the findings more authentic and trustworthy. This research method involved using more than one data collection source in order to increase internal validity, as well as minimize any existing biases. As the study develops, “steps should be taken to validate each against at least one other source, e.g. an observation in addition to an interview” (Lincoln & Guba, 1985, p. 283). The process of triangulation enabled the researcher to compare data that were acquired through interviews and observations. According to Lincoln and Guba (1985), “No single item of information should ever be given serious consideration unless it can be triangulated” (p. 283). The data that was gathered consisted of the observation of the various strategies children employ while solving fraction problems. To further promote trustworthiness, the researcher recruited a peer-debriefer in order to ensure validity of interpretations (Lincoln and Guba, 1985). This member was a fifth grade teacher and a doctoral student who was not connected to the study and provided authentic feedback as the study progressed.

Another verification strategy used for this study was rich, thick descriptions using precise details and descriptive words to hopefully transport our readers to the setting (Geertz, 1983). “The purpose of a thick description is that it creates verisimilitude, statements that produce for the readers the feeling that they have experienced, or could experience the events being described in the study” (Creswell & Miller, 2000, p.129). The ability to transport readers to the situation with rich descriptions will establish credibility through the lens they use to read the narrative. This strategy provides “enough description so that readers will be able to determine how closely their situations match the research situation," thus strengthening transferability (Merriam, 2009).

Ethical Considerations

Throughout this research, the researcher kept in mind that this study must not bring any harm to the participants nor allow any preconceived biases to skew the findings. Therefore, caution was given to ensure that all interviews were conducted in a private setting to protect the participants. One ethical issue of concern was informed consent. During the initial visit with teachers and students, the purpose of this study was described as well as possible benefits. The teachers and parents of the participants were given a recruitment letter that provided a delineation of their role and the role of the students, as well as that of the research member. Upon receipt of signed Informed Consent Forms (See Appendices A and B) from teachers and parents, the each student was provided with an Assent form (See Appendix D). Each participant was advised that this study was voluntary and that they could withdraw at any time.

A second ethical issue was privacy. All participants were made aware that their identity would remain anonymous. Also, all information was maintained on the researcher's personal computer, which requires a password. Transcriptions and notes were coded with pseudonyms (e.g., Teacher A, Student 1). The Consent and Assent forms were locked in a personal file cabinet and will be destroyed in approximately one year.

A third ethical issue was deception. The participants were advised of the actual purpose of the study, and the researcher did not venture from the purposes of this study.

Participants were also made aware that this study would provide valuable information regarding how fifth grade students understand fractions to help with the teaching and learning of fractions.

Trusting relationships with the teachers and parents of students were established. They were provided with telephone numbers as well as email addresses of the researcher. The intent was to make the participants realize that they could contact the researcher at any time with questions or concerns. The research process was conducted with the greatest amount of respect for each participant. After each interview, these participants were given expressions of gratitude for their time. Also, the teachers were given a $20 gift certificate as an expression of appreciation for their participation in the study.

CHAPTER 4

RESULTS

This chapter describes the themes identified from the data analysis. The discussion of the findings of this qualitative study is structured around the research questions. Data analysis consisted of organizing data and identifying common themes from students' responses and classroom observations. Participants were required to complete a variety of tasks using fractions as numbers represented on the number line. The oral and written responses were examined for comprehension based on the manner students approached and worked with each task.

The eight task directions were administered as follows:

1. How do you see or think of a number line?
    a. Can you draw one? Put the numbers 1, 2, and 3 on your number line.
2. This number line shows 0 to 1. Put an X where the ½ would be on the number line below.
3. This number line shows 0 to 3. Put an X where the ½ would be on the number line below.
4. Name a fraction that is less than ½.
    a. Place your fraction as accurately as possible on the number line below.
5. Figure out what this point is called on the number line. Circle the correct answer.
    a. How did you figure out this answer? 2/6, 2/7, 1/4, 2, 2/4

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.
7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location. 8/12, 8/3, 2/3
8. Use the number line below to solve the following problem.
    a. Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?

On the initial task, students were asked to elaborate on their perception of a number line followed by directions to draw a number line including numbers 1, 2, and 3. The majority of the students described the number line as a tool, system of order, and in terms of density. Density is the understanding of an infinite number of other fractions between any two fractions. Some students simply described it as a line with numbers on it while others considered it as a unit of measure from 0 to 1. Table 2 shows examples of students' perceptions of a number line.

Table 2

Students' Perceptions of a Number Line

Students' Perception	**Student Responses**	**N=26**
Number line as a tool	S2: start at 0….count up 1 that helps you figure out S3: a line that's counting to a hundred S16: a line with numbers on it…It helps you with your strategies S17: a helping strategy when you have a really big…fractions S19: a way to get back and forth to numbers S26: a line with numbers on it and you can use it to subtract back to find an answer	23%
Order including ascending S6 & 14 concept of infinity	S6: a line with arrows pointing at each end with numbers on it…. With numbers in order S8: a line that you put things in order from smallest to greatest S14: a line..would have certain little arrows… a big one that could describe 2 or maybe 1, 2, 3, 4, and then there's a bigger one for 5. Biggest one for 1 S23: the smaller number is on the left hand side and the bigger numbers are on the right hand side S24: It places the numbers in order	19%
Density	S15: two numbers.. and lots of smaller numbers in between them S20: two whole numbers and in between it there will be decimals and fractions S22: it has numbers, kind of like a ruler….some in the middle like one and a half… S25: a line with numbers between 0 and 1..	19%
Line including time line	S1: I kind of think of it as a timeline S7: I think of a line S10: A line with numbers on it	12%
Unit of measure, mostly 0 to 1	S4: 0 at the first part and a 1 at the other part S11: I think of 0 to 1 S13: a line that says 0 and 1 at the end S21: I think of a scale between 0 and 1 S18: a line with numbers ranging from a certain distance	12%
No attempt	S5, S9, S12	8%

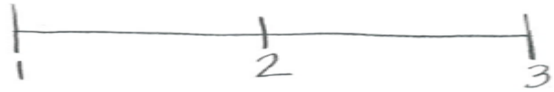
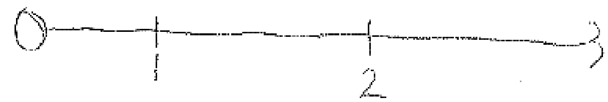
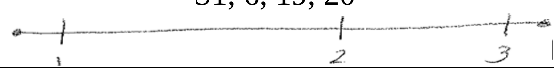
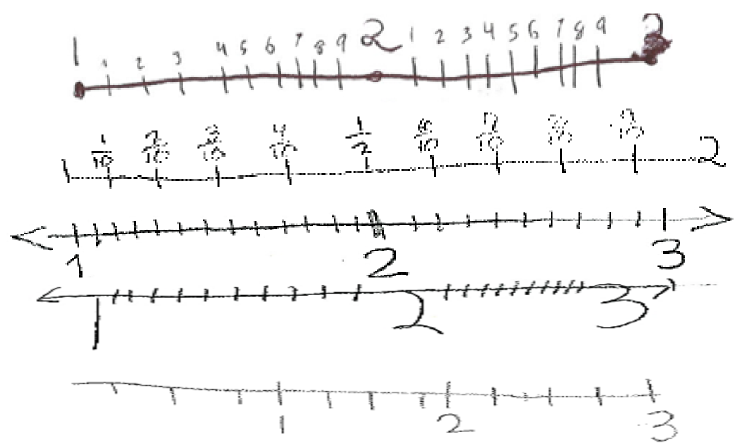
Number Line Illustrations

Only seven students demonstrated a concept of infinity by way of arrows at both ends of the number line, while another student only represented an infinite arrow on the positive side of the illustration. Although six students included 0, no students extended their number lines to include negative numbers.

Seventeen students (65%) demonstrated the concept of equality while 13 of these 17 students partitioned models with precision on the number lines they drew. When students were asked to draw a number line, half of 26 students drew equal distances, equal partitions, and units of measure. Some drew lines with unequal partitions, while others drew equal partitions with equal distances with all units except the 0-1 unit. Some students demonstrated an awareness of the concept of density evidenced in the lines they drew, e.g. the units between the equally partitioned whole numbers were segmented with unequal partitions. Table 3 shows examples of students' illustrations of a number line.

Table 3

Students' Illustrations of a Number Line

Number line Illustrations	**Exemplary Examples**	**N=26**
Equal distances, partitions, and units of measure	S3, 4, 8, 10, 11, 13, 14, 17, 21, 23, 24, 25, 26 	50%
Equal distances, partitions, and units of measure- *Except for 0 to 1*	S2, 5, 16, 18 	15%
Unequal partitions	S1, 6, 19, 20 	15%
Concept of density	S7, 9, 12, 15, 22 	19%

Themes

From the remaining tasks, four themes were identified as emergent from the participant data (student interviews, narrative descriptions, and classroom observations): (1) prevalence of lower level teacher guided questions and greater success with students' completion of lower level tasks, (2) students' reasoning and procedural knowledge exceeds conceptual understanding, (3) the number line revealed more clearly student misconceptions with fractions, and (4) successful strategies students use when solving problems on the number line. Each of the four themes will be further explored in the following paragraphs.

Theme 1: Prevalence of Lower Level Teacher Guided Questions and Greater Success with Students' Completion of Lower Level Tasks

Classroom observations and interview tasks

- Lower level questions were more prevalent in classroom
- Greater success with lower level tasks

In many cases teachers ask questions to simply find out what facts students have committed to memory (Walsh & Sattes, 2011). This was evidenced within this research from the level of questions that were observed in the classrooms. Using Sanders (1966) version of Bloom's Taxonomy the questions teachers asked of students were classified into five categories in Table 4.

The majority of the classroom questions fell into the least rigorous categories of Memory, Translation, and Interpretation at a combined total of 74%. The higher order thinking questions for Application, Analysis and Synthesis were noted at 26%. These findings parallel students' performance with the levels of questions they were asked.

Table 4

Bloom's Taxonomy Questions Observed in Classrooms

Bloom's	Classroom 1	Classroom 2
Memory / Recall or lower 9/27= 33%	• That's great math, but did you read the problem? • Did someone read the question incorrectly? • Why did you add 5/12+1/3=9/12? • Which is it ¾ or ¼? • Which one will you change?	• What other percentages are in there? *(teacher pointed to 75%)* Is 50%? • What plus 50% equals 75%?
Translation 4/27 = 15%	• Can anyone see it another way? • Can you think about this (1½) in twelfths? • Can you add the fractions first?	• What other percentages are in there? *(teacher pointed to 75%)*
Interpretation 7/27 = 26%	• What questions do you have about number 2? • Do you know why 2/3 is closer to 1? • Where did you get 12/12 from? • Look at your denominator, what do you notice about them?	• Where is the relationship in the problem 8x4=32? • What relationships have to be there with the denominator? • What do you know about 75%?
Application 4/27 = 15%	• Where would you put fraction 0/12? • How can you change 2/4?	• How can you prove that? • Why did the student want to get to the denominator of 32?
Analysis 2/27 = 7%	• Why did I say 1 ½ is closer to 1 than 5/6? • How do you know 1/12 is smaller than 1/6?	
Synthesis 1/27 = 4%		• Is there a simpler strategy?
Evaluation		

During the task-based interviews, students were able to demonstrate greater success with lower level tasks. For example, 96% of the students could successfully operate with the least rigorous tasks such as naming a fraction less than one-half and appropriately plotting ½ on the number line between 0 and 1. When the students were directed to put an (X) where the ½ would be on a number line from 0 to 3, the accuracy rate dropped to 31%. Although plotting self-selected fractions on the number line dropped in accuracy, it was the only other category that more than half of the participants (58%) were able to complete. In direct contrast, the students performed at their lowest (0%) when no student made an attempt to use the number line to compute 2/3 + 3/4. Table 5 shows examples of accuracy rates for each task.

Table 5

Percentage of Items Answered Correctly

Task	Task Description	N=26
2	Put "X" to locate ½ on 0-1 number line	96%
3	Put "X" to locate ½ on 0-3 number line	31%
4a	Name a fraction <1/2	96%
4b	Plot fraction accurately	58%
5	Identify correct point (1/4) on non-routine number line	27%
6	Put ½, ¼, and 1 on number line with 0 and 1/3	4%
7	Plotted all three (8/12, 8/3, 2/3) in reasonable locations	4%
7a	• Place 2/3 on number line	27%
7b	• Place 8/12 on number line	19%
7c	• Place 8/3 on number line	23%
8	Used number line to compute 2/3 + ¾	0%
8a	• Correct sum (2/3 + ¾)	42%
8b	• Plotted correct sum in a correct location	23%

Theme 2: Students' Reasoning and Procedural Knowledge Exceeds Conceptual Understanding

The CCSS for Mathematics (2010) are promoting a balance of procedural and conceptual understanding. Thus, educators are encouraged to shift from solely relying on step-by-step procedures to focusing first on a thorough understanding of concepts in mathematics. Students' responses exceeding their conceptual understanding was noted in the way they answered questions about ¼. All of the students, with the exception of one, could name a fraction less than ½; 14 of them selected ¼. Although 10 of those students successfully selected and plotted ¼ on the number line from 0 to 1 in Question 4, only one student (S14) validated that understanding in task number five when he correctly identified the point (1/4) on the unconventional number line. Another student (S16) was the only student to correctly place ¼ and ½ in sync when directed to plot as accurately as possible on a number line that shows 0 and 1/3 on task number six. Even though four students (S1, S5, S6, and S7) explicitly stated that "half of a half is a fourth" on task number five, none of them plotted ¼ and ½ in sync on task number 6. Student 13 even plotted them out of order as follows (Figure 1):

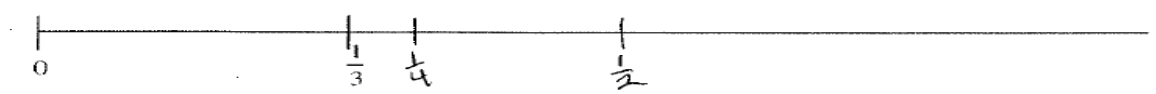


Figure 1. Student Who Plotted the Fractions Out of Order

These results seem to reveal a progression with understanding the concept of one-fourth. Many of the children in this study successfully selected and represented ¼ as a fraction that is less than one-half and some articulated the relationship as being half of half. Full conceptual understanding seems to be achieved when children can apply the knowledge by plotting ¼ in a location that marks half of a half when a unit of measure is already established. Table 6 shows a potential progression that emerged from the 14 out of 26 students who selected ¼ as a fraction less than ½.

Table 6

Potential Progression of ¼

Number of Students	Accuracy Correct	Description
14	14/14=100%	Named ¼ as a fraction less than ½
10	10/14=72%	Successfully plotted the self-selected fraction (1/4)
4	4/14=29%	Reasoned "half of ½ is a fourth"
1	1/14=7%	Plotted ¼ in the location half of ½
1	1/14=7%	Correctly identified ¼ on a nonroutine number line

Although students 14 and 16 performed most of the tasks relating to ¼ correctly, they both missed one of the three tasks. Thus, the combined results for the three tasks indicate that none of the students in this study had a solid understanding of ¼ as a number on the number line. Of all the participants, 96% could name a fraction less than ½; only 58% could plot their self-selected fractions in reasonable locations on the number line. The accuracy rate declined to 27% when students were required to identify ¼ on a non-routine number line. Only one student (S16) considered the existing unit of measure 0 to 1/3 and accurately plotted ¼, ½, and 1 on a number line in reasonable locations (See Figure 2).

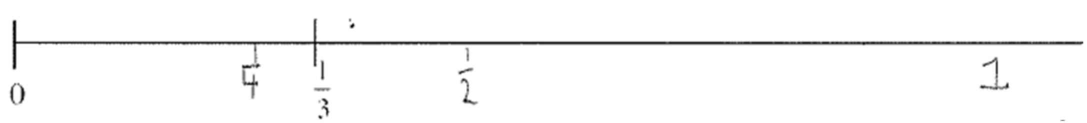
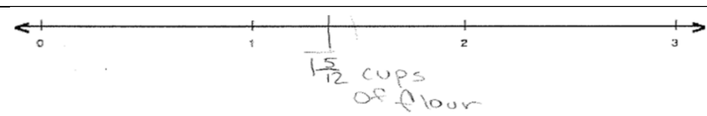
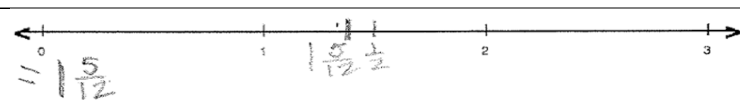
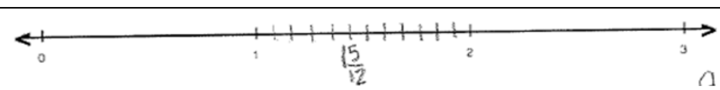


Figure 2. Student Who Accurately Plotted ¼, ½, and 1 with Existing Unit 0 to 1/3

When participants were directed to solve the final task, 2/3 + ¾, on a number line in context through a word problem, students were more successful with the procedural tasks than with actual use of the number line. None of the 26 students used the number line to determine the sum of the two fractions. Although 11 of them (42%) successfully calculated the sum, 5 of them (19%) used the number line solely to plot their answer, $1\frac{5}{12}$, on the number line in a reasonable location. Students' use of benchmark and physical partitions were strategies noted to help them plot their answers on the number line (Table 7). Of the 11 students, 8 solved with use of paper and 3 solved mentally. Inconsistency between students' ability to plot a number on the number line and their stated reasoning about a number suggest that students' conceptual understanding of fractions may still be fragile.

Table 7

Physical Partitions and Benchmarks to Plot Fractions

Student	**Plotting with Benchmark ½**
8	
12	
Student	**Physical Partition**
17	

Theme 3: The Number Line Revealed More Clearly Student Misconceptions of Fractions

Participants expressed difficulty with understanding fractions on a number line in three areas:

- Whole Number Reasoning
- Lack of Precision
- Students' Responses Compared to Actual Plots
- Common Errors

Whole number reasoning. Oftentimes children are exposed to the part-whole construct in which a fraction such as 3/4 can be seen as three out of four. A child's interpretation of the part-whole construct is highly dependent on the ability to equally partition quantities or discrete objects. In order for children to fully understand fractions they need to be exposed to other sub-constructs. Clarke and Roche's (n.d.) position on the part-whole construct is that it is "the most common interpretation of fractions and is likely to be the first interpretation that students meet at school." Educators tend to expose children to the part-whole construct without exposing them to other constructs, which adversely affects fractional learning. This becomes evident when adding and subtracting as well as understanding magnitude and density of fractions. If a child is bound to part-whole knowledge, they may be likely to add three fourths plus two fifths by adding the numerators and denominators separately, thus resulting in five ninths. This type of part-whole perception understandably fuels the inappropriate use of whole numbers.

Of the 26 participants in this study, 23 (88%) demonstrated examples of whole number reasoning, which resulted in 24 instances that evolved from four different tasks.

When students were asked to figure out a point on a nonroutine number line, 19 students selected options 2/6, 2/7, 2 and 2/4 rather than the correct option ¼. Of these, 16 counted hash marks rather than the units of measure. Similar to the findings of et al. (2010), one student placed unit fractions in order based solely on the magnitude of denominators (see Figure 3). Three students placed two-thirds on the number line between the unit of two and three (See Figure 4). Two students simply added the numerators and denominators straight across to result in an incorrect sum (see Figure 5). One student selected 2 on task number 5, because it was the second hash mark and without any regards to the unit of zero to one. In a study conducted by Vermont Mathematics Partnership Ongoing Assessment in 2005, approximately 44% of the incorrectly analyzed responses were also attributed to inappropriate reasoning of whole numbers (Petit et al., 2010). The following illustrations provide examples of the various ways in which students misapplied whole number reasoning in this research.

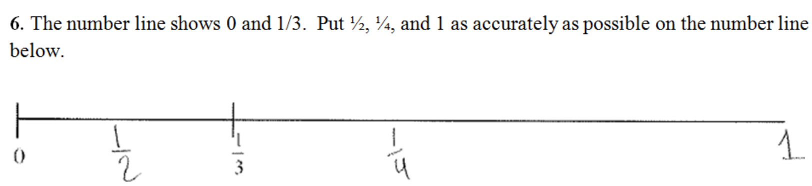


Figure 3. Student Who Plotted Fractions Based on the Magnitude of Denominators

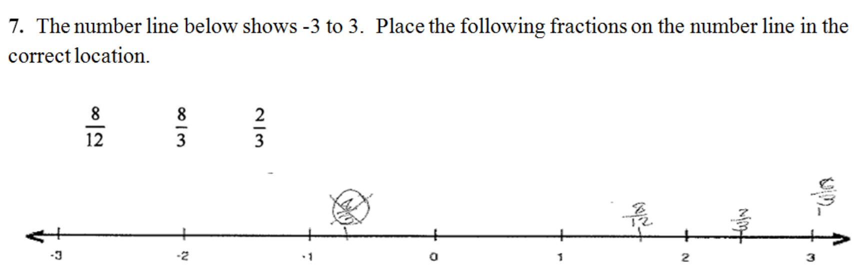


Figure 4. Student Who Plotted 2/3 Between 2 and 3

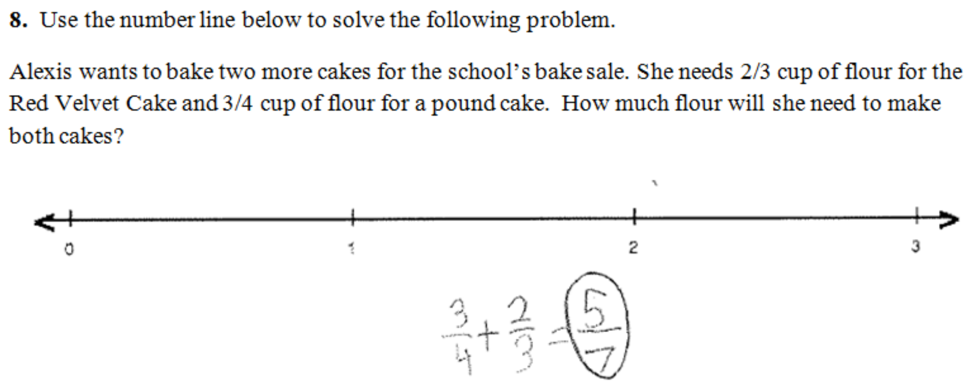


Figure 5. Student Who Added Numerators and Denominators Straight Across

Lack of precision. The Common Core State Standards for Mathematical Practice encourage educators to set up their classroom environment so students will be using these practices. One of these practices calls for students to attend to precision. Although students in this research study could often correctly answer questions about fractions within a unit of 0 to 1, they seemed to lack understanding that a fraction is a number with a specific location on the number line. In this study, 21 students plotted ¼ and ½ in the correct order on the number line with a preexisting unit of 0 to 1/3. Of these 21 students, 20 of them (77%) demonstrated a lack in precision when plotting ¼ and ½ (see Figure 6). In all these cases, 1/4 was not represented as half of ½. Six students demonstrated this same lack of precision when they placed ½ and 1 on the number line out of sync (see Figure 7). On task number seven, 15 students demonstrated a need for improved proficiency when plotting one or more of the three fractions 8/12, 8/3, and 2/3. Lack of precision in plotting fractions occurred in more instances with plotting 8/3 (10 occurrences), 8/12 (6 occurrences), and 2/3 (4 occurrences). See Figures 8-10. Of the 15 students, 7 of them seemed to understand the unit in which the fraction belonged but did not plot the fraction with enough precision. Some students' lack of precision occurred when they used the strategy of physically partitioning 8/12 or 2/3 with imaginary lines to plot. As shown in the data less success was noted with partitioning 8/12.

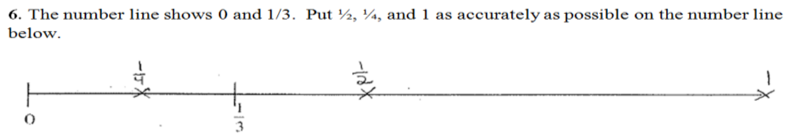


Figure 6. ¼ and ½ Out of Sync

Figure 7. ½ and 1 Out of Sync

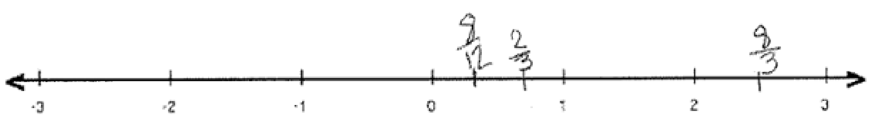


Figure 8. Imprecise Plot for 8/3

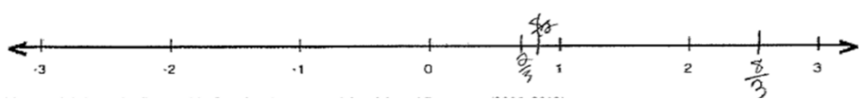


Figure 9. Imprecise Plot for 8/12.

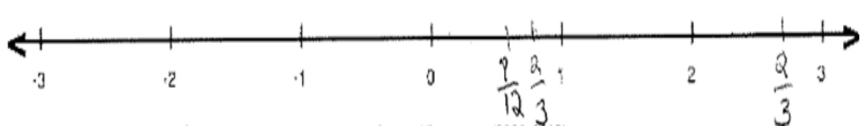
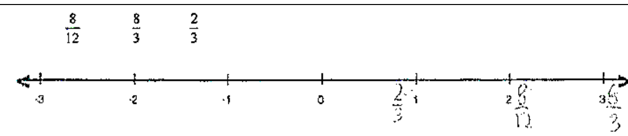
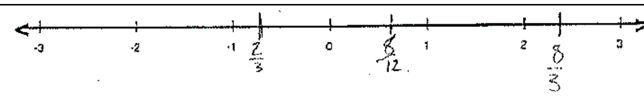
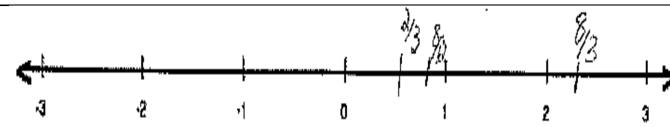
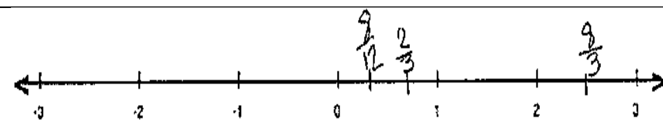
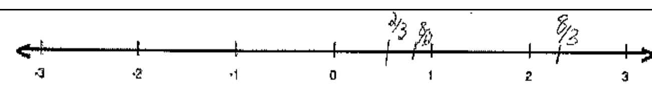
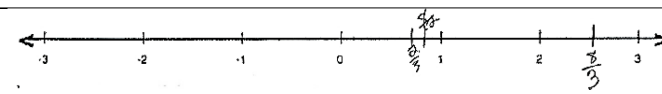
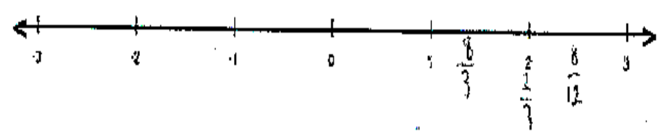
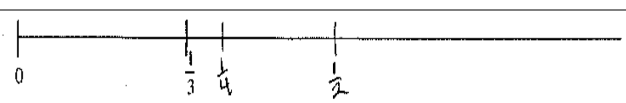
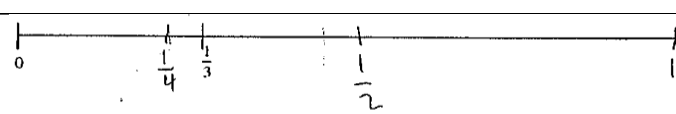


Figure 10. Imprecise Plot for 2/3.

Students' responses compared to actual plots. Students' misconceptions were more clearly revealed when their reasoning was compared to their actual plots on the number line. Although six students (23%) made a connection between 8/12 and 2/3, Student 15 was the only one to plot the two fractions accurately together on the number line. Student 11 made an initial connection but stated, "I know 8/12 could be converted to 2/3, but it couldn't fit in the same spot." The responses to this question suggested that only one student demonstrated a solid understanding of equivalency. Table 8 highlights students' misunderstanding by comparing what they said with what they did. In most cases the students' responses did not match their actual understanding of fractions as numbers on a number line.

Table 8

Students' Responses Compared to Actual Plots

Student	Students' Responses	Number line plots
2	*S: Umm 'cause it's more than 100%. Cause you could do 1 and. Oh wait it would be 2 and 1/3…..*	
10	*S: Oh I knew it was more than 2 and I knew it would be 2 and 2/3.*	
11	*S: So it was 8/3. So I knew 3 (pointed at 0-1 segment) and 6 (pointed at 1 to 2 segment) and then I just did two more.*	
18	*S: Cause 8/3 is a mixed number so I did umm make it into a … into an improper fraction …So I got 2 and 2/3.*	
11	*S: I know 8/12 is equal to 2/3, but I couldn't fit it in the same spot.*	
13	*I know that 2/3 is a little over 1/2 and I knew that 8/12 is a sixth over uh ½. So it's like 2/3.*	
16	*S: Uh, I knew that 8/12 could be converted to 4/6, and so I knew that could be converted to uh 2/3. And so I put 2/3 right there.*	
13, 24	***Exemplary** Reasoning "half of a half is a fourth"*	
1, 5, 14	***Exemplary** Reasoning "half of a half is a fourth"*	

Common errors. Even though none of the students in this study utilized the number line as indicated to solve 2/3 + ¾, 11 students calculated the sum correctly. The errors of the remaining 15 students' made primarily fell into four categories:

1. Fragile procedural knowledge (8 students, 31%);
2. Whole number reasoning (2 students, 8%);
3. Guessed without calculation (3 students, 12%); and
4. Mental miscalculation (2 students, 8%).

Table 9 shows examples of fragile procedural knowledge and whole number reasoning.

Table 9

Fragile Procedural Knowledge and Whole Number Reasoning

Students	Fragile Procedural Knowledge		Students	Fragile Procedural Knowledge
9	$\frac{2}{3} = \frac{4}{12}$ $+\frac{3}{4} = \frac{3}{12}$ $\frac{7}{12}$		19	$\frac{3}{4} \times 3$ $\frac{2}{3} \times 4 = 7$ 12 $\boxed{\frac{7}{12}}$
13	$\frac{2}{3} = \frac{7}{7}$ $+\frac{3}{4} = \frac{5}{7}$ $\frac{9}{7}$		21	$\frac{2}{3}\ \frac{4}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$
14	$\frac{3}{4} = \frac{9}{12} + \frac{2}{3} = \frac{8}{12}$ $\frac{9}{12} + \frac{8}{12} = \frac{17}{12}$ $1\frac{1}{5}$		23	$\frac{3}{4} + \frac{2}{4} = \frac{5}{4}$ $4\overline{)5}$
18	$\frac{2}{3} \underset{\times}{+} \frac{3}{4} = \frac{5}{12}$		24	$\frac{2}{3} + \frac{3}{4} = \frac{18}{12} = 1\frac{6}{12}$
Student	**Whole Number Reasoning**		**Student**	**Whole Number Reasoning**
22	$\frac{3}{4} + \frac{2}{3} = \frac{5}{7}$		26	$\frac{2}{3} + \frac{3}{4}$ $\frac{5}{7}$

Three students appeared to guess an answer without any indication of calculations with the following responses:

- Student 2: “4 pounds of flour;”

- Student 3: “1 ¼,”
- Student 5: “I think it will be more than one.”

Other students calculated mentally in their heads resulting in incorrect sums such as $1\frac{7}{12}$ (S1) and 11/12 (S6).

Theme 4: Successful Strategies to Solve Problems on the Number Line

- Common Denominator
- Mathematical Reasoning and Mental Calculations
- Residual Thinking

Common denominator. Several successful strategies evolved during this study when students solved various problems on the number line. Unlike the results of a study conducted by Clarke and Roche (2009) where 22% of the students who attempted to use a common denominator strategy were unsuccessful in finding the correct answer, 10 students (38%) in this study successfully used the common denominator strategy to calculate 2/3 + ¾ in task number 8. Of the 10 students who used the common denominator strategy, 8 of them used the traditional algorithmic approach to find like denominators as seen below in Figure 11; whereas, the other 2 students solved mentally and verbally explained their thinking.

Figure 11. Student's Use of Finding a Common Denominator Strategy

Mathematical reasoning and mental calculations. Mathematical reasoning seemed to play a role for nine students while solving two tasks. To solve task number 4, four students reasoned, "I knew that was one-half and I know that half of a half is one-fourth." The remaining five students used mathematical reasoning to accurately determine the locations of 8/3, 2/3, and 8/12 on task number 7 with statements such as, "Well if you did 3/3 that would be 1; 6/3 would be 2; since it's over 6/3, it would be 8/3 so there would be two more thirds," (Student 9). Three students successfully calculated sums for task number 8 without using paper or pencil.

Residual thinking. Clarke and Roche (2009) contend that residual thinking is likely to be a successful strategy for solving fractional problems that are conducive for building up to the whole, e.g. fractions that are one away from a whole such as 1/6 more to make a whole for 5/6, etc. Residual thinking became evident when students were deciding where to locate 2/3 on the number line in task number 7. Table 10 consists of responses that evolved while completing this task.

Table 10

Examples of Residual Thinking

Student	**Residual Thinking**
12	Ok 2/3 is 1/3 away from 1.
18	2/3 is close one and it only takes a third to get there.
19	2/3 is one away from one whole.

Students' ability to plot successfully seemed to help solve problems with fractions on the number line more efficiently. Students 4, 6, 8, 17 and 25 created imaginary or physical plots to help meet the requirement of representing the self-selected fraction that was less than one-half on the number line in task number 4. Some of these same students (4, 6, 8, and 17) along with students 12 and 13 demonstrated a greater success rate with placing 8/12 and 2/3 in close proximity when they drew imaginary hash marks with twelfths and thirds.

Summary

In this chapter, results from eight tasks that 26 students who participated in this study revealed that locating a fraction between 0 and 1, as well as simply naming a fraction less than ½, were their greatest strengths. The critical area for growth became evident when students were asked to apply the knowledge by physically plotting fractions on the number line that went beyond 0-1. The results of the study also indicate a strong connection between the levels of questions teachers ask and the performance level of students.

CHAPTER 5

DISCUSSION

In this chapter, the reader will find a brief summary of the entire study, including the significance of the study, theoretical framework, study methodology, and major findings. A reiteration of the research questions and discussion of the results, as well as the recommendations for practice and future research are also presented.

Summary of the Study

This dissertation is a case study of fifth graders' understanding of fractions on the number line. The participants consisted of 26 students and 2 teachers from two elementary schools in suburban cities within central Alabama. The interview tasks were extrapolated from previous scientifically-based research questions (e.g., Pearn & Stephens, 2007; Saxe et al., 2007; Vermont Mathematics Partnership, 2012; Wong & Evans, 2008).

The method's assessment consisted of seven open-ended instructions and one multiple-choice question. The tasks allowed students to describe their perceptions of number lines and demonstrate their understandings of fractions equal to or less than ½. Participants also exhibited their knowledge of a unit fraction, a mixed number, and an equivalent fraction on the number line. Students had another opportunity to use the number line for adding fractions with unlike denominators.

Thematic coding (Denzin, 2001) was initially used with the research questions in mind. The researcher analyzed the data for specific themes (Creswell, 2007; Lincoln & Guba, 1985) and reported the percentage of accurate responses for both.

Research Questions

One overarching question was used to guide this study.

Q1: How do fifth grade students in two elementary schools in central Alabama demonstrate an understanding of fractions on a number line?

Two additional research questions were framed to further support the central research question.

Q2: What strategies do children use to solve fractional problems on a number line?

Q3: How do students interpret fractions on a number line?

Discussion of Results

The purpose of this section is to discuss fifth graders' understanding and reasoning as they solved fractional tasks on a number line that is connected with the current mathematics canon of literature. The researcher explored five categories pertaining to number lines: students' perceptions and illustrations of a number line, understanding ½ as a number, understanding fractions less than ½, understanding unit fractions, mixed numbers, and equivalent fractions, and understanding adding fractions with unlike denominators. The first category was the students' perceptions of a number line. Most of the students perceived the number line as a tool, as well as a model for order and density. The majority of the participants (65%) illustrated the number line with a concept of equivalence. More specifically, ½ of the population partitioned the units into equal measures of distance including the unit 0 to 1.

With the exception of one student, Student 26, students' understanding of ½ as a number started strongly for the remaining students. Most students placed ½ in a reasonable location on the number line between 0 and 1, which is an expectation for third graders. During the subsequent task, over ½ of the participants overgeneralized their knowledge of placing ½ on the number line with the notion of placing "X" at half of the entire number line with 3 units, thus suggesting a confusion between knowing the difference between the number ½ and half of a quantity.

Understanding fractions less than and equal to ½, the third category for this research directed students to "name a fraction that is less than ½" and then place the fraction as accurately as possible on the number line. Every student (96%), with the exception of Student 23, successfully named a fraction less than ½. Over half of them (58%) plotted the self-selected fraction with precision. In a comparable study, 44% of the fifth grade students successfully chose and marked a number between 0 and ½ (Pearn & Stephens, 2007). More than ½ of the students in this study (58%) selected ¼ when requested to name a fraction that is less than ½.

A little less than 30% of the participants were able to correctly identify ¼ on a "non-routine" number line when provided with 2/6, 2/7, ¼, 2, and 2/4 to choose from. Most of the students that selected the correct response of ¼ used mathematical reasoning to solve the problem, i.e. "one-fourth is half of ½." There was a remarkable difference in the findings of the study conducted by other leading researchers where 60% accuracy was noted for 6th graders (Saxe, Shaughnessy, Shannon, Langer-Osuna, Chinn, & Gearhart, 2007). Saxe et al. (2007) reported that children had more difficulty identifying points on the number line when the segments did not match the denominator. For example, children demonstrated difficulty identifying 3/8 when the number line was partitioned into fourths, halves, or sixteenths. This thought was further emphasized in the findings of this research study as 19 (73%) students failed to correctly identify ¼ on a non-routine number line. This entire number line was partitioned into fourths, but with half of that line partitioned into eighths. The most popular choice, 2/6, coupled with another common incorrect choice, 2/7, indicated that students in this study counted hash marks rather than equal units of measure. Rather than counting hash marks, it was important for students to understand that the number line represents distances from zero (Mills College Lesson Study Group, 2010). The Mills College Lesson Study Group (2010) also concluded that initial confusion between fractional and whole units on a number line was due to the fact that both were indicated with line segments. This brought clarity to the understanding of the thought process of Student 26 when he selected two wholes to represent one-fourth.

Only one student in this research study considered the existing unit of measure and plotted ½, ¼, and 1 correctly on the number line. The majority of the students (81%) automatically placed 1 at the end of the number line, thus suggesting a disregard for the existing unit of measure 0 to 1/3. In a similar study conducted by Pearn and Stephens (2007), students were required to mark 1 on the number line when provided with the unit of 1/3. They reported in their findings that 46% of the fifth graders completed this task with success

The fourth category of the researcher's study focused on the understanding of unit fractions, mixed numbers, and equivalent fractions on a number line. When asked to place 8/12, 8/3, and 2/3 on the number line, the three fractions yielded varied results for correct answers. The students performed at 27% accuracy for 2/3, which suggested that 27 out of 100 students would plot correctly. Even when verbal connections were made between 8/12 and 2/3, most students were unable to apply the knowledge via a number line. When students placed the equivalent fractions in close proximity on the number line, they drew imaginary lines to partition, primarily for 8/12, and reasoned mostly for 1/3, e.g. "one away from a whole." A small percentage (15%) of participants were successful in placing two of the three given fractions in a reasonable location on the number line. Petit, Laird, and Marsden (2010) reported that it was fundamental that understand the concept of equivalency, density, and magnitude of fractions in order to demonstrate proficiency with this task. According to Lamon (1999), partitioning was an essential component of understanding fractional concepts such as representing fractions on a number line and understanding the density and equivalency of fractions. Using a similar statement to address this concern, Pothier and Sawada (1983) stated that the concept of understanding fractions was grounded in partitioning as understanding the concept of whole numbers was grounded in counting. It is imperative that students understand that fractional knowledge were grounded in the concept of unit fractions. For example, in order for students to model 8/3 on the number line, they "must first be aware that '1/3' is the appropriate subunit" and that '8/3' is found by traveling from the zero point, a distance of '8' iterates (Behr, 1992, p. 43).

The last category of this researcher's study was "understanding adding fractions with unlike denominators on the number line." This resulted in 11 of the 26 students successfully finding the sum of 2/3 + ¾ in context through a word problem. For three separate classifications associated with this task, interestingly, 42% of students were assessed for each category. Of the students, 42% correctly calculated the sum of the two fractions and 42% made no attempt to use the number line. Although 69% attempted to use the common denominator strategy, only 42% used it with success.

Not one student in this study utilized the number line to solve the problem as indicated in the directions; thus, this connected with Bright et al.'s (1988) findings that some students who were provided a symbolic solution were unable to do so with the number line. Bright et al. (1988) further stated that it is essential for students to grasp the skill with equivalent fractions prior to using the number line to model addition. This theory coincides with the findings of this researcher's study. Only two students within this study demonstrated a deep conceptual understanding of equivalency by correctly plotting 8/12 and 2/3 together on the number line in task number 7. Wantanabe (2002) further noted that number lines do not help students develop fractional number sense, but rather number lines only made sense to those students "who already understand fractions as numbers" (p. 463). Currently, the CCSS expectations have students in grade 3 learning fractions as a number on the number line.

The CCSS (2010) stated that third graders should "understand a fraction as a number on the number line and represent fractions on a number line diagram." Ni and Zhou (2005) suggested that rational numbers should be introduced to children at an earlier age simultaneously with whole numbers. Sophian (1997) conducted a research study and emphasized how five year olds have the ability to successfully complete equal sharing tasks. Empson (1995) gave another example where equal sharing tasks were beneficial for introducing fractions to five-year-old children using constructivist approaches. Other researchers, Nunes and Bryant (1996), recounted that children as young as six and seven years old have the ability to develop an understanding of several relationships among measurements, units, and numbers. The CCSS (2010) requires that second graders "represent whole numbers as lengths from 0 on a number line diagram with equally spaced point corresponding to the numbers 0, 1, 2, …" Petitto (1990) stated that first through third graders transition from sequential to proportional thinking to place numbers on a number line. "This suggests that teachers should be sure that students are thinking proportionally when using a number line with whole numbers, not just sequentially, before asking students to locate fractions on a number line" (Petit et al., 2010, p. 103). It occurred to this researcher from engaging in this study and relating it to other studies that the emphasis placed on fractions on the number line in earlier grade levels with younger children may be the solution to the researchers' challenges.

Discussion of Research Questions

Research questions from this study were aimed to gain an understanding of how fifth graders understand fractions on a number line. Results of the analysis performed are used to address each question below.

Overarching Question

How do fifth grade students in two elementary schools in central Alabama demonstrate an understanding of fractions on a number line? All the data from the interviews were considered as the researcher endeavored to describe students' understanding of fractions on the number line. Students were noted to use various strategies to demonstrate their understanding of fractions on a number line. Explaining their thinking through oral conversations was one of the most powerful strategies noted that children employed to reasonably respond to various tasks. Whitin and Whitin (2000) posited, “Writing and talking are ways that learners can make their mathematical thinking visible.” Although more than half of the students used a common denominator as a strategy for solving problems on the number line, the use of a common denominator was also noted as one of the most incorrectly applied strategies. The majority of the students (81%) in this study demonstrated an inappropriate use of whole number reasoning on at least one of the eight tasks. This over-arching question was supported by findings from the supporting questions that were used to guide this research.

Research Question One

What strategies do children use to solve fractional problems on a number line? All the data from the interviews were considered as the researcher sought to describe students’ understanding of fractions on the number line. The students used similar strategies to solve the fraction problems on the number line. Several students used mathematical reasoning such as “half-of-a-half” or “one away from a whole.” Students additionally utilized equivalency relationships, which are also a form of mathematical reasoning, to plot the fractions 8/12 and 2/3. When required to place 8/3 on the number line, some students reasoned, “3/3 equals to 1, 6/3 equals 2, so 8/3 equals 2 2/3,” which is also employing the use of equivalency. Students’ use of imaginary lines to partition units for plotting fractions on a number line was another strategy that was frequently used.

Landmark numbers such as ½ was another strategy students used to identify fractions on the number line in order to solve problems. A large number of students (69%) used common denominator to find the sum of a fractional word problem. However, only 42% used a common denominator and solved the problem correctly. A small number of students from both cases used skip counting and procedural strategies to place 8/3 or 8/12 on the number line. Some students made use of whole number reasoning when required to find the sum of two fractions with different denominators. Two of the students added the numerator and multiplied the denominator, while another two added the numerators and multiplied the denominator.

Research Question Two

How do students interpret fractions on a number line? All the data from the interviews were considered as the researcher sought to describe students' understanding of fractions on the number line. Four students were able to reason and procedurally convert 8/3 to 2 2/3. Another three students articulated a connection between 2/3 and 8/12. Although the students were able to reason or solve procedurally, none of them could represent this information accurately on the number line, which suggests that students do not see fractions as an extension of the number system. This finding coincided with Petit, Laird, and Marsden's (2010) position that the use of number lines helps students to think about a fraction as a number.

The students' overgeneralization of finding one-half of the entire number line zero to three at 58% bears a striking resemblance to a comparable study by Wong and Evans (2008), where 57% of the participants placed ½ half-way between the numbers 1 and 2. This observation was confirmed in another research finding by Yanik et al. (2006), which found that “many students seemed to use a number line as if it was a fraction bar, partitioning it as if the entire visual number line was a unit of one” (p. 324). Stated in a similar fashion, Larson (1979) reported that some students ignore the scaling and treat the number line as a unit regardless of length when the number line is of greater length than one. Shaughnessy (2011) referred to this overgeneralization as redefining the unit where the students treat “the entire distance shown as the unit distance rather than the distance between zero and one” (p. 432), which indicates that students do not perceive fractions as a number on the number line. According to Mitchell and Horne (2008), it is not uncommon for students to find a fraction of a whole line rather than locating the fraction when students first encounter number lines. This suggested that students need more exposure and experience with the number line. Larson (1979) additionally provided a valid rationale for this misconception in his explanation that when a zero to one model is used, the measure construct is not fully utilized, thus being simply another part-whole model. As previously stated, the part-whole subconstruct is overused in many classrooms.

Table 5 allows the reader to review the accuracy rates of participants for each task. The students performed well on naming a fraction less than ½ (96%) and identifying ½ on a number line ranging from 0 to 1 (96%), which suggests a stronger understanding with these concepts. In contrast, both cases performed at their lowest on tasks 8c and 4a which requires using the number line to add fractions as well as plotting fractions ½, ¼, and 1 on a number line with a preexisting measure from 0 to 1/3. There was a 4% difference among students who placed 2/3 (27%), 8/3 (23%), and 8/12 (19%) on the number line.

Recommendations for Target Audiences

Policymakers should involve educators in making curriculum decisions related to students' understanding of fractions on the number line. They should recognize educators as the experts to get a full understanding of students' knowledge of fractions and more specifically, fractions on the number line. This would give a better understanding as to which procedures are most effective in building on current understanding and addressing the misunderstandings or misconceptions regarding students' knowledge of fractions with the number line. Elementary educators and early childhood educators will benefit from this study because it will give first-hand accounts of students' understanding as they plan for teaching the concept of fractions. It also emphasized the implications for training math educators using constructivist rather than algorithm approaches.

Implications for Teaching the Concept of Fractions on a Number Line

Implications of this study suggest that students tend to respond at a level that correlates with the questions teachers ask. Therefore, the goal of teachers should be to expose students to questions and activities that will allow them to perform at a level that focuses on higher order thinking skills in accordance with Blooms taxonomy.

This study also revealed a profound and progressive concept about students' knowledge of the fraction "1/4." Although 14 students could name 1/4 as a fraction less than ½, only 10 could successfully represent this fraction on a number line. Also, four students stated one-fourth was one-half of a half; however, only one student could physically plot one-fourth as half of a half.

The results of these findings correlate with suggestions recommended by leading researchers on the topic of fractions. Petit et al. (2010) advised that students should be provided with a variety of number lines with varying units that contain different types of numbers, such as negative and positive, as well as partitioning or repartitioning. For example, on task number 4, students were able to demonstrate the concept of ¼ with accuracy when asked to name and plot the fraction. However, when asked to identify and plot ¼ in subsequent tasks on numbers five and six, which were presented in a different format, students failed to apply knowledge of this concept, thus stressing the importance of providing students with various and multiple tasks to demonstrate understanding.

Of the participants, 73% used a "counting parts" strategy where they simply counted the number of segments without regard for the size of the partitions. Because the students focused on the amount of segments rather than equal units of measure, they failed to identify ¼ on a nonroutine number line. This suggests that students have been primarily or exclusively exposed to equally partitioned models. In order for students to gain a deeper understanding of ¼ as a quantity, students must be exposed to various models with unequal regions in area models and unequal partitions on number lines (McNamara, 2006).

The Common Core State Standards for Mathematics (2010) require third graders to "understand a fraction as a number on the number line," and more specifically, "represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts." Because students at this age might use the entire number line to determine half rather than locating ½ on the number line precisely between 0 and 1, it is suggested that during the early stages of the Number and Operations-Fractions Progression they use other representations like the number line such as area models, tape diagrams, and strips of paper to develop understanding (Common Core Standards Writing Team, 2011). Since the CCSS were implemented in the state of Alabama during the 2012-2013 year, the participants of this study may not have been exposed to the number line with fractions starting in third grade. Thus, it is unclear when the students were able to construct understandings about the meaning of ½ between a number line of 0 to 1. It is sensible to believe that if the students accommodated this knowledge in grade three, they would have mastered the concept of ½ of a number line of any length on any number line e.g. 0 to 3 or -2 or 2, etc. by grade five.

Implications for Future Research

Three opportunities for further investigation emerged from this dissertation. First, the theoretical development for understanding the concept of one-fourth merits further examination. There appeared to be a progression starting with simply stating a fraction to plotting a fraction on a number line, and finally identifying a fraction in a nonstandard format such as with a nonroutine number line. Secondly, in addition to the assessing comprehension of fractions with the measure model, future research could be extended to include other models such as area and set. Finally, foundational concepts of fractions are taught within grades three through five. Therefore, a longitudinal study of students with the number line should occur first in third grade with a follow-up study with the same students in grade five.

Summary

Over the past 35 years, numerous researchers have explored students' understanding of fractions on the number line (Baturo & Cooper, 1999; Bright et al., 1988; Hannula, 2003; Larson, 1979; Merenluoto, 2003; Mitchell & Horne, 2008; Ni & Zhou, 2005; Pearn & Stephens, 2007; Pititto, 1990; Wong & Evans, 2008; Yanik et al., 2006). However, no research was found that focused solely on third or fifth graders' understanding of fractions on a number line. The findings of this researcher's study were related to the existing body of research on fifth graders' limited exposure to fractions with use of the number line. A recurring finding in children's difficulty with locating and identifying fractions on a number line beyond 0 to 1 was established in this study, which was also exemplified in several other studies (Bright et al., 1988; Larson, 1979; Yanik, 2006). Bright et al. (1988) reported that it was easier for children to identify fractions on number lines from 0 to 1 than from 0 to 2. In a similar study, Yanik et al. (2006) explained that children had more difficulty locating ¾ on a number line from 0 to 5 than from 0 to 1. In a study conducted by Larson (1979), students had difficulty finding 1/5 on a number line with 2 units.

Analysis of the data and review of the literature disclosed other implications for mathematics educators. First of all, children need to be exposed to number lines that extend beyond the 0 to 1 unit. Also, students should be introduced to fractions at an earlier age as required in the Common Core State Standards. These standards formally introduced the number line to second graders with whole numbers and this knowledge is extended with fractions on the number line for third graders. Another implication of this study for early and elementary educators is that students should be provided with a variety of number lines in their daily mathematical practices. The majority of the students in this study used a counting parts strategy regardless of the units of measure when they identified fractions on the number line. This study's analysis suggested that students have been exposed primarily to equal or pre-partitioned models.

Although this researcher identified the importance of students' using number lines to solve fractions, additional research is needed for third through fifth grade students regarding how and when to teach this topic. If we want to educate children to become proficient in mathematics, or more specifically with fractions on the number line, we must develop teachers' mathematical knowledge with this concept. Furthermore, children must start using number lines earlier.

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APPENDIX A

INFORMED CONSENT DOCUMENT TEACHERS

Appendix A: Informed Consent - Teachers

Informed Consent Document

TITLE OF RESEARCH:	Fifth Graders' Understanding of Fractions on the Number Line: A Standard Introduced in the Common Core State Standards for Third Graders
IRB PROTOCOL:	**X120807006**
INVESTIGATOR:	Taajah Witherspoon
SPONSOR:	Department of Education at the University of Alabama Birmingham

Explanation of Procedures

You are being asked to take part in a research study, which will give insight as to how children understand the concept of fractions. The researcher is asking to observe while you teach one or two fraction lessons and to allow your students, with parental permission, to be individually interviewed as they solve 8 short fraction problems. You were selected because you are a 5th grade teacher who can help provide insight as to how students understand fractions. The timeframe for this study is October 2012 to November 2012.

If you enter the study, there will be observation(s) of your instruction during one or two math lessons focused on fractions. Following the observation(s), each student with parental consent will be interviewed for approximately 15 minutes. The interview will be video recorded. The recording will only be used for data collection and analysis. All recordings will be securely filed and marked confidential. Initial interview questions will be asked and other follow-up questions may be necessary to clarify responses. There will be strict anonymity of your name, your school's name, and all students' names throughout the research and you may choose to withdraw from this study at any time. The participants in this study will include two teachers and approximately 40 students in the Birmingham area.

Risks and Discomforts

Although there are no foreseeable risks or discomforts to the participants, there is always a potential for loss of confidentiality.

Benefits

You may not benefit directly from taking part in this study. However, this study may help the researcher find out something that will help fifth grade students to gain a better understanding of fractions.

Alternatives

The alternative is always possible; you may choose not to participate in this study.

Confidentiality

Information obtained about you for this study will be kept confidential to the extent allowed by law. However, research information that identifies you may be shared with the UAB Institutional Review Board (IRB) and others who are responsible for ensuring compliance with laws and regulations related to research, and the Office for Human Research Protections (OHRP). All written work and questionnaires will be kept in a locked drawer in the researcher's possession. The results of this research may be published for scientific purposes; however, your identity will not be given out.

Refusal or Withdrawal without Penalty

Whether or not you take part in this study is your choice. There will be no penalty if you decide not to be in the study. You are free to withdraw from this research study at any time. Your choice to leave the study will not affect your relationship with the University of Alabama at Birmingham.

Cost of Participation

There will be no cost to you for taking part in this study.

Payment for Participation in Research

You will receive a $20 gift card for taking part in this research study.

Questions

If you have any questions, concerns, or complaints about the research, please contact the researcher, Taajah Witherspoon. Ms. Witherspoon's phone number is 205-470-1455 or you can reach her at this email address: witherspoont@mtnbrook.k12.al.us or taajah@uab.edu.

If you have questions about your rights as a research participant, or concerns or complaints about the research, you may contact Dr. Ann Dominick. Dr. Dominick is the supervising professor at the University of Alabama at Birmingham in which this research is a requirement of my graduate studies. Her email address is adominic@uab.edu.

If you have questions about your rights as a research participant, or concerns or complaints about the research, you may contact the Office of the IRB (OIRB) at (205) 934-3789 or 1-800-822-8816. If calling the toll-free number, press the option for "all other calls" or for an operator/attendant and ask for extension 4-3789. Regular hours for the OIRB are 8:00 a.m. to 5:00 p.m. CT, Monday through Friday. You may also call this number in the event the research staff cannot be reached or you wish to talk to someone else.

Legal Rights

You are not waiving any of your legal rights by signing this informed consent document.

Signatures

Your signature below indicates that you agree to participate in this study. You will receive a copy of this signed informed consent document.

Signature of Participant Date

Signature of Principal Investigator Date

APPENDIX B

INFORMED CONSENT DOCUMENT PARENTS

Appendix B: Informed Consent - Parents

Informed Consent Document
Parents

TITLE OF RESEARCH: Fifth Graders' Understanding of Fractions on the Number Line: A Standard Introduced in the Common Core State Standards for Third Graders

IRB PROTOCOL: **X120807006**
INVESTIGATOR: Taajah Witherspoon
SPONSOR: Department of Education at the University of Alabama Birmingham

For Children/Minors (persons under 19 years of age) participating in this study, the term *You* addresses both the participant ("you") and the parent or legally authorized representative ("your child").

Explanation of Procedures

Your child is being asked to take part in a research study, which will give insight as to how children understand the concept of fractions. Fifth grade students are selected for this research because of the instruction they have already had up until this point, which can provide insight as to how students understand fractions. If you agree to allow your child to participate, he or she will be individually interviewed as he or she solves 8 short fraction questions that will reflect knowledge that is expected of a fifth grader. The student tasks will also require the students to identify and place various fractions on a number line diagram. The timeframe for this study is November 2012 to December 2012.

If you enter this study, there will be one interview, approximately 15 minutes in length needed for this process. The interview will take place in the closest conference room to your child’s classroom and will be video recorded. The recording will only be used for data collection and analysis. All recordings will be securely filed and marked confidential. Initial interview questions will be asked and other follow-up questions may occur within two weeks of the interview to clarify responses. There will be strict confidentiality of your child's name and school's name throughout the research and your child has the choice to withdraw from this study at any time. With the permission of the classroom teacher, the teacher's instruction on fractions will be observed. The participants in this study will include two teachers and approximately 40 students in the Birmingham area.

Risks and Discomforts

Although there are no foreseeable risks to the participants, there is always a potential for loss of confidentiality. Your child may also be uncomfortable being pulled out of the classroom or being videotaped. Should your child display any sign of discomfort, the interview will be stopped immediately and the child will be escorted back to the classroom with no future obligations.

Benefits

You may not benefit directly from taking part in this study. However, this study may provide new information that will help fifth grade students to gain a better understanding of fractions.

Alternatives

The alternative is not to participate in this study.

Confidentiality

Information obtained about you for this study will be kept confidential to the extent allowed by law. However, research information that identifies your child may be shared with the UAB Institutional Review Board (IRB), the UAB Department of Education, and others who are responsible for ensuring compliance with laws and regulations related to research, including the Office for Human Research Protections (OHRP). All written work and questionnaires will be kept in a locked drawer in the researcher's possession. The results of this research may be published for scientific purposes; however, your child's identity will not be given out.

Refusal or Withdrawal without Penalty

Whether or not you allow your child to take part in this study is your choice. There will be no penalty if you decide not to allow your child to be in the study. Your child is free to withdraw from this research study at any time. Your choice to leave the study will not affect your relationship with the University of Alabama at Birmingham. Participation will not affect your child's class standing or grade.

Cost of Participation

There will be no cost for taking part in this study.

Payment for Participation in Research

There will be no payment for taking part in this research study.

Questions

If you have any questions, concerns, or complaints about the research, please contact the researcher, Taajah Witherspoon or Dr. Ann Dominick. Ms. Witherspoon's phone number is 205-470-1455 or you can reach her at this email address: witherspoont@mtnbrook.k12.al.us or taajah@uab.edu.

Dr. Dominick is the supervising professor at the University of Alabama at Birmingham in which this research is a requirement of my graduate studies. Her email address is adominic@uab.edu.

If you have questions about your rights as a research participant, or concerns or complaints about the research, you may contact the Office of the IRB (OIRB) at (205) 934-3789 or 1-800-822-8816. If calling the toll-free number, press the option for "all other calls" or for an operator/attendant and ask for extension 4-3789. Regular hours for the OIRB are 8:00 a.m. to 5:00 p.m. CT, Monday through Friday. You may also call this number in the event the research staff cannot be reached or you wish to talk to someone else.

Legal Rights

You are not waiving any of your legal rights by signing this informed consent document.

Signatures

You are making a decision whether or not to have your child participate in this study. Your signature indicates that you have read the information provided above and decided to allow your child to participate. Please sign this document and return to your child's classroom teacher. A copy with all signatures will be returned to you via your child for your records.

Signature of Parent or Guardian Date

Signature of Principal Investigator Date

APPENDIX C

CLASSROOM OBSERVATION PROTOCOL

Appendix C: Classroom Observation Protocol

Classroom Observation Protocol

Teacher ______________________________ Date ___________________
School _______________________________ Grade/Level 5th
Observer Taajah Witherspoon ____________ Time of class: ______________

What is the physical set up of the classroom?

What classroom activities does the teacher use?

What materials does the teacher use for instruction?

What questions does the teacher ask?

What feedback does the teacher provide during the lesson?

What assessment strategies does the teacher use to determine learning?

APPENDIX D

ASSENT FORM - STUDENTS

Appendix D: Assent Form

Assent Form

Title:	Fifth Graders' Understanding of Fractions on the Number Line: A Standard Introduced in the Common Core State Standards for Third Graders
IRB Protocol No.:	X120807006
Sponsor:	UAB Education Department
Investigator:	Taajah Witherspoon

The investigator named above is doing a research study.

These are some things we want you to know about research studies:

We are asking you to be in a research study. Research is a way to test new ideas. Research helps us learn new things.

Whether or not to be in this research is your choice. You can say Yes or No. Whatever you decide is OK.

Why am I being asked to be in this research study?

You are being asked to be in this study because you are a 5th grade student and can provide valuable information regarding how students understand fractions.

What is the study about?

The researcher needs to learn more about the way fifth graders understand fractions to help with the teaching and learning of fractions.

What will happen during this study?

If you agree to be in this study, you will have to participate in one interview for about 15 minutes to show what you know about fractions and number lines by solving 8 short problems. You will not be graded on any of the tasks.

Will the study hurt?

You will not experience any pain or discomfort during this study.

What are the good things that might happen?

The researcher might find out something that will help fifth grade students to gain a better understanding of fractions.

What if I don't want to be in this study?

You do not have to be in the study if you do not want to.

Who should I ask if I have any questions?

If you have any questions about this study, you can stop and ask me anytime.

Do I have to be in the study?

No, you do not have to be in the study. Even if you say yes now, you can change your mind later. It is up to you. No one will be mad at you if you don't want to do this.

Now that I have asked my questions and think I know about the study and what it means, here is what I decided:

_________OK, I'll be in the study. ________ No, I do not want to be in the study.

The researcher has told me about the research. I had a chance to ask questions. I know I can ask questions at any time. I want to be in the research.

If you sign your name below, it means that you agree to take part in this research study.

______________________________	____	________________
Your Name (Printed)	Age	Date
______________________________		________________
Your Signature		Date
____		________________
Signature of Person Obtaining Consent		Date

APPENDIX E

INTERVIEW PROTOCOL

Appendix E: Interview Protocol

Fifth Graders' Understanding of Fractions on the Number Line: A Standard Introduced in the Common Core State Standards for Third Graders

Interview Protocol

Name___________________________ **Date___________________________**

School__________________________

Title___________________________

Introduction

I want to thank you for taking the time to talk to me today. I will be recording and writing what we say today. It is important that I reflect in my writing what you mean. Therefore, I may ask you to elaborate during the interview. The information that we discuss today will be confidential and only used within the scope of this research project. Your name and your school will not be made public.

I am interested in finding out in this study how fifth grader's understand the concept of fractions on the number line. Please give these questions and tasks some thought as I really want to know your perspective. Please feel free to discuss and elaborate on your views. Are you ready to start?

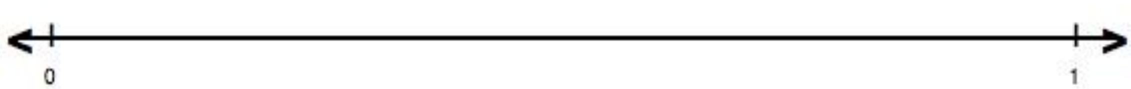
Opening Statement:

I would like to start with the number line.

1. a. "How do you 'see' or think of a number line?" Can you draw one?

   b. Put the numbers 1, 2, and 3 on your number line. *Merenluoto, K. (2003).*

2. This number line shows 0 to 1. Put an (X) where the ½ would be on the number line below.

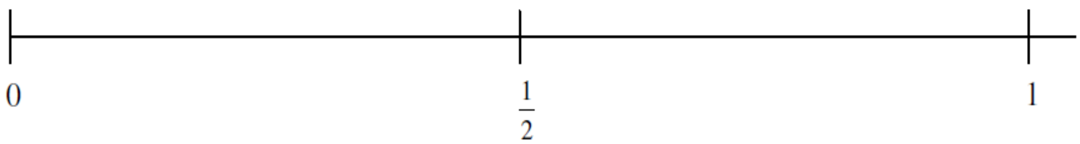


3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

Wong, M. & Evans, D. (2008)

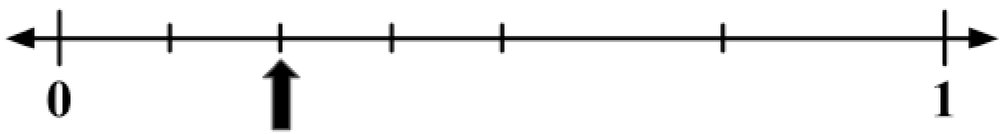
4. a). Name a fraction that is less than ½. ________

b). Place your fraction as accurately as possible on the number line below.



Inspired by Pearn & Stephens (2007)

5. Figure out what this point is called on the number line.



Circle the correct answer:

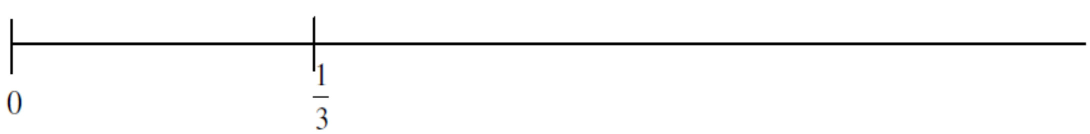
$\frac{2}{6}$ $\frac{2}{7}$ $\frac{1}{4}$ 2 $\frac{2}{4}$

____

How did you figure out the answer?

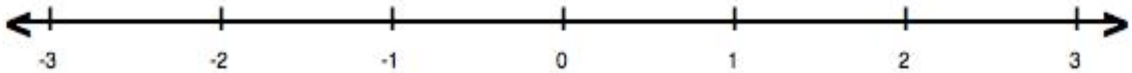
Saxe et al. (2007)

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



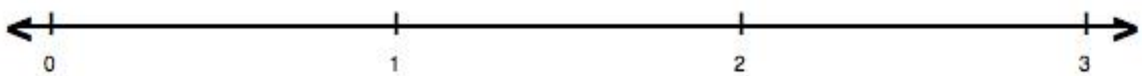
7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.

$$\frac{8}{12} \qquad \frac{8}{3} \qquad \frac{2}{3}$$



Vermont Mathematics Partnership Ongoing Assessment Materials and Resources (2005–2012).
OGAP questions and student work samples. Unpublished professional development materials.

8. Use the number line below to solve the following problem.
Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?



APPENDIX F

IRB APPROVAL FORM

Appendix F: IRB Approval Form

THE UNIVERSITY OF ALABAMA AT BIRMINGHAM

Institutional Review Board for Human Use

Form 4: IRB Approval Form
Identification and Certification of Research
Projects Involving Human Subjects

UAB's Institutional Review Boards for Human Use (IRBs) have an approved Federalwide Assurance with the Office for Human Research Protections (OHRP). The Assurance number is FWA00005960 and it expires on January 24, 2017. The UAB IRBs are also in compliance with 21 CFR Parts 50 and 56.

Principal Investigator:	WITHERSPOON, TAAJAH F
Co-Investigator(s):	
Protocol Number:	**X120807006**
Protocol Title:	*Examining Fifth Graders' Understanding of Fractions Using Number Lines*

The IRB reviewed and approved the above named project on 8-31-12. The review was conducted in accordance with UAB's Assurance of Compliance approved by the Department of Health and Human Services. This Project will be subject to Annual continuing review as provided in that Assurance.

This project received EXPEDITED review.

IRB Approval Date: 8-31-12

Date IRB Approval Issued: 8-31-12

Marilyn Doss

Marilyn Doss, M.A.
Vice Chair of the Institutional Review
Board for Human Use (IRB)

Investigators please note:

The IRB approved consent form used in the study must contain the IRB approval date and expiration date.

IRB approval is given for one year unless otherwise noted. For projects subject to annual review research activities may not continue past the one year anniversary of the IRB approval date.

Any modifications in the study methodology, protocol and/or consent form must be submitted for review and approval to the IRB prior to implementation.

Adverse Events and/or unanticipated risks to subjects or others at UAB or other participating institutions must be reported promptly to the IRB.

470 Administration Building
701 20th Street South
205.934.3789
Fax 205.934.1301
irb@uab.edu

The University of
Alabama at Birmingham
Mailing Address:
AB 470
1530 3RD AVE S
BIRMINGHAM AL 35294-0104

APPENDIX G

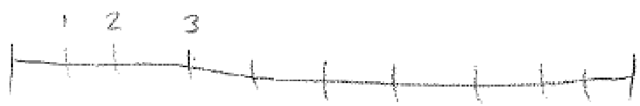
Appendix G: Transcribed Interview Protocols

Student 1

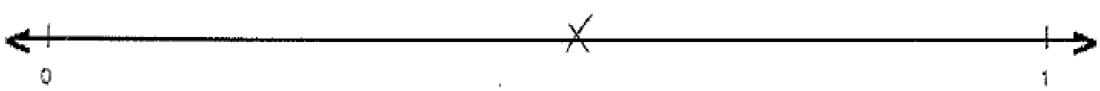
Opening Statement:
Researcher: I would like to start with the number line. a. "How do you 'see' or think of a number line?"

Student: I kind of think of it as a timeline.

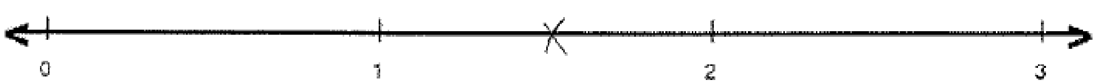
Researcher: Can you draw one? b. Put the numbers 1, 2, and 3 on your number line.



2. This number line shows 0 to 1. Put an (X) where the ½ would be on the number line below.

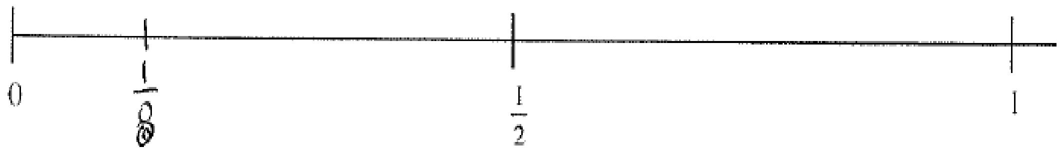


3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

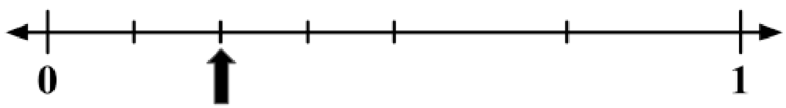


4. a). Name a fraction that is less than ½. ________ S: 1/8

b). Place your fraction as accurately as possible on the number line below.



5. Figure out what this point is called on the number line.



Circle the correct answer:

$\frac{2}{6}$ $\frac{2}{7}$ $\frac{1}{4}$ 2 $\frac{2}{4}$

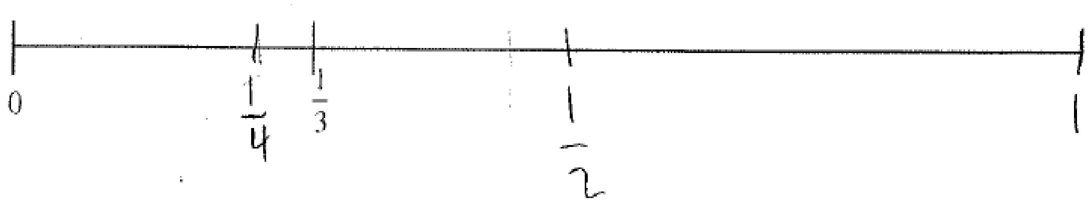
How did you figure out the answer?

S: Do you mean like a fraction?

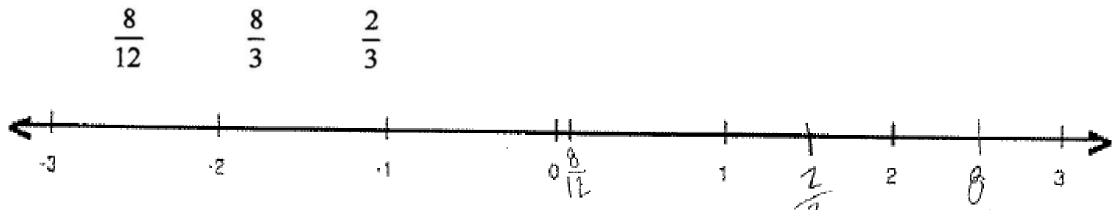
R: (Reread the question.)

S: I knew that was one half and I know that half of half is one-fourth.

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.



Vermont Mathematics Partnership Ongoing Assessment Materials and Resources (2005–2012). *OGAP questions and student work samples.* Unpublished professional development materials.

The student drew an imaginary line between 2 and 3. Then drew a partition between -1 and 0 and labeled 8/3. The student then put 2/3 between 1 and 2. Next the student counted back from 3 to -3 and drew imaginary lines back toward 0. After contemplation, the student placed 8/12 just after 0.

R: How did you know where to place that fraction? {Pointed to 8/3},

S: There's only……. {student paused}

R: What are you thinking?

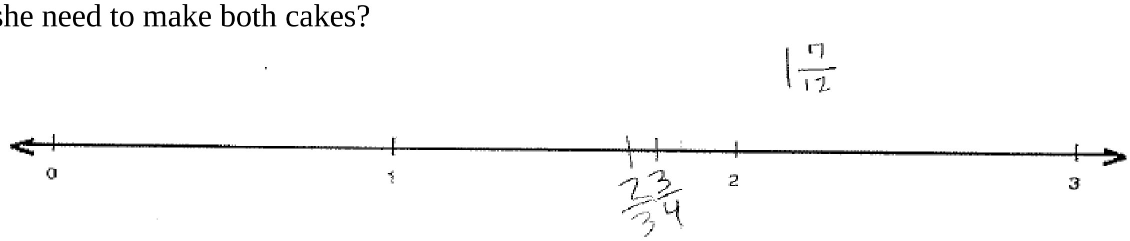
S: Well it would probably be in this area (pointed between 0 to 1) ……. No, I think it's this one (pointed between 2 and 3). May I erase it?

R: Sure. Why do you think it goes there?

S: Because there is three-thirds in this one, three thirds in this one, and three thirds in this one (pointed from 0 to 1, 1 to 2, and 2 to 3). There's 3 wholes, so I was thinking that 8/3 would go into this one.

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?



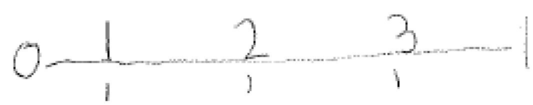
The student drew three imaginary lines between 1 and 2 and then backed up and created a partition for 2/3. Next she drew four imaginary lines and placed ¾ immediately to the right of 2/3. The student wrote 1 then 7 over fraction bar with an extended pause then wrote 12 which resulted in 1 7/12.

Student 2

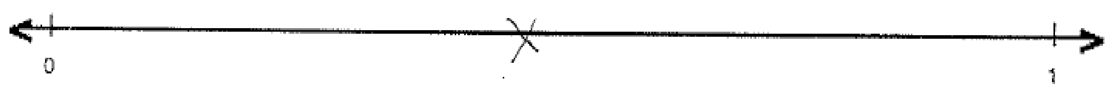
Opening Statement:
Researcher: I would like to start with the number line. a. “How do you ‘see’ or think of a number line?”

Student: uhh like if you start at 0, it’s like a line that you can count up to 1 that helps you figure out some ……

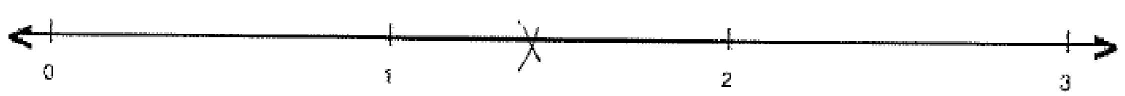
R: Can you draw one? b. Put the numbers 1, 2, and 3 on your number line.



2. This number line shows 0 to 1. Put an (X) where the ½ would be on the number line below.

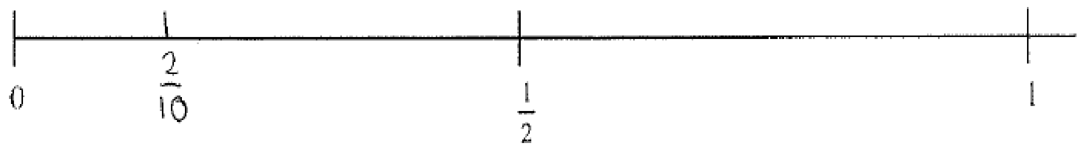


3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.



4. a). Name a fraction that is less than ½. ________ S: 2/10

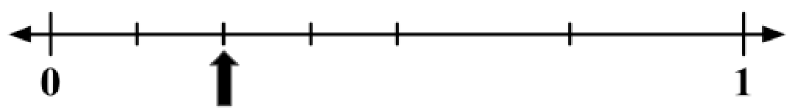
b). Place your fraction as accurately as possible on the number line below.



R: How do you know to put the fraction there?

S: Because umm you could do….. this is 20%, so you can do 10, 20 30, 40, 50 (drew imaginary lines from 0 to ½)

5. Figure out what this point is called on the number line.



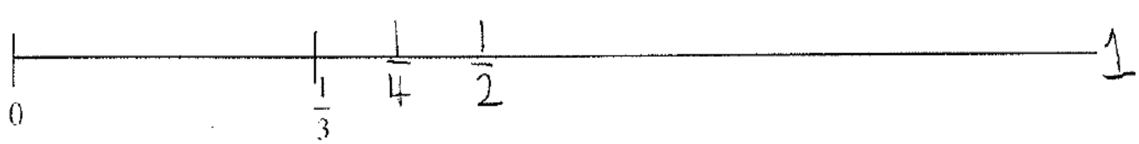
Circle the correct answer:

$\frac{2}{6}$ $\quad$ $\frac{2}{7}$ $\quad$ $\frac{1}{4}$ $\quad$ 2 $\quad$ $\frac{2}{4}$

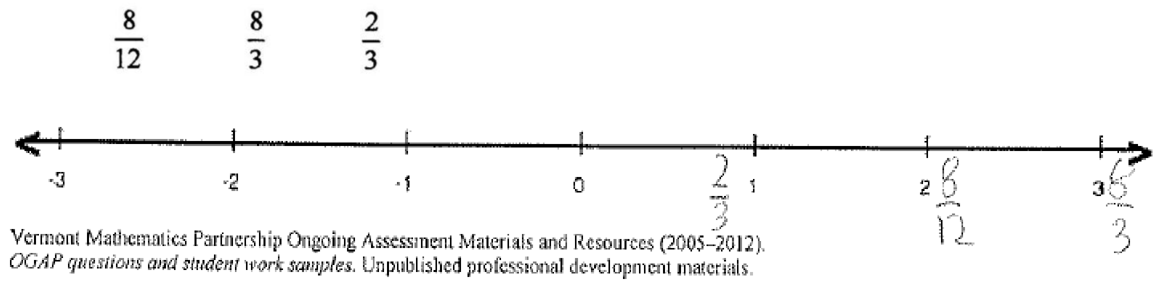
How did you figure out the answer?

S: Uh I thought it wouldn't be ½, because uh ½ is in the middle and I thought 2/6 would be here like 1/6, 2/6 (pointed to first two hash marks).

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.



R: {Pointed to 8/3} How did you know where to place that fraction?

S: Umm 'cause it's more than 100%. Cause you could do 1 and. Oh wait it would be 2 and 1/3…..

R: Ok

8. Use the number line below to solve the following problem.

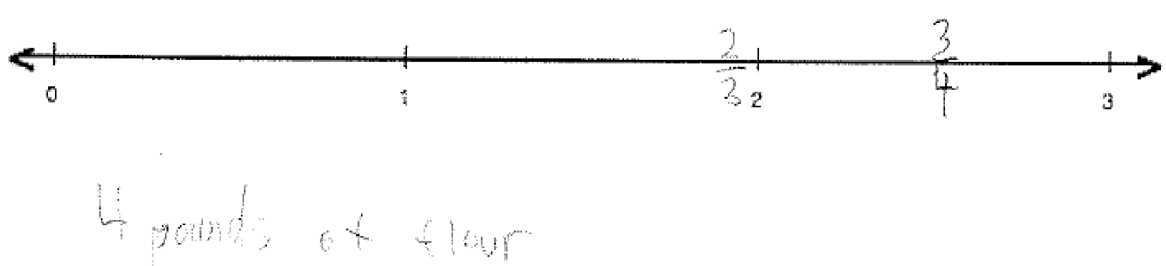
Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?

S: Wait do I write just like in words or do I write?

R: However you want to do it. Do what's best for you?

S: Do I write ¾ (pointed between 0 and 1) 2/3 (pointed between 2 and 3)?

R: {Read the problem again for the student}



S: Wait does this count as a grade?

R: No, it's for my research.

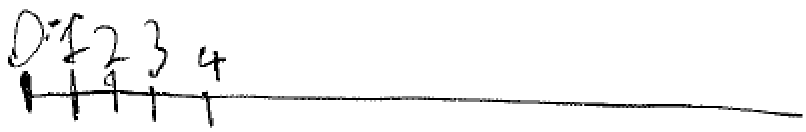
Student 3

Opening Statement:
Researcher: I would like to start with the number line. a. “How do you ‘see’ or think of a number line?”

Student: Kinda like a line that’s counting to a hundred. May I show you it…… with numbers on it?

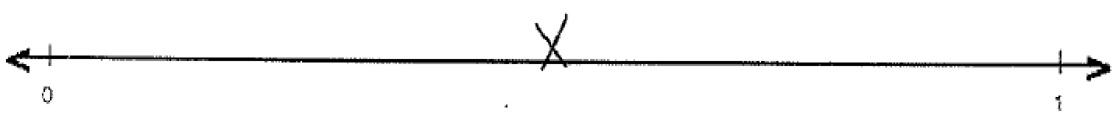
R: Yes, please add numbers 1, 2, and 3 on your number line.

S: Ok, I’m going to put 0 there. Do I do it all the way to a hundred or 50?

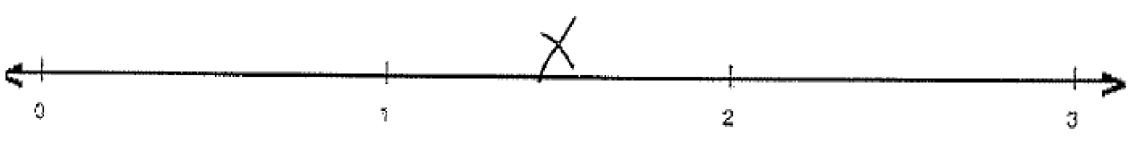


R: No, that’s actually good.

2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.

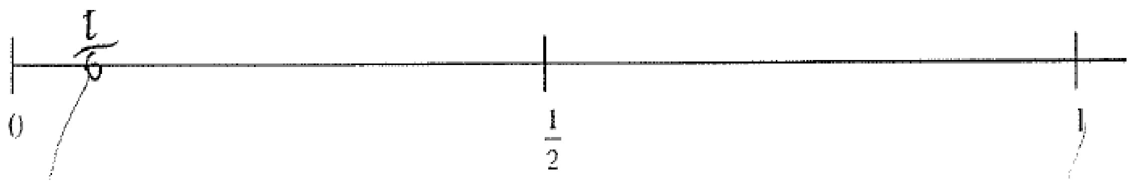


3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.



4. a). Name a fraction that is less than ½. ________ S: uhh 1/6

b). Place your fraction as accurately as possible on the number line below.



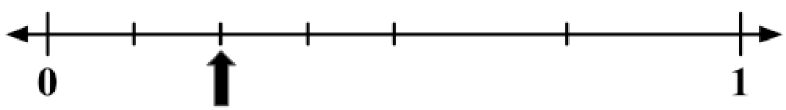
R: Why do you think 1/6 would go there?

S: Because it’s the first one out of 6.

R: So where would the last one go? I’m just curious.

S: {pointed to ½} Umm about right there.

5. Figure out what this point is called on the number line.



Circle the correct answer:

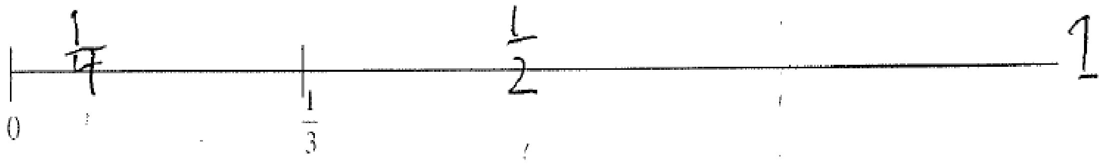
$\frac{2}{6}$ $\frac{2}{7}$ $\frac{1}{4}$ 2 $\frac{2}{4}$

How did you figure out the answer?

S: “Because up here {referred back to image number 4} I put right where that would be and the next would probably be right {pointed to the second partition of number 5}.

R: That’s interesting. Ok.

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



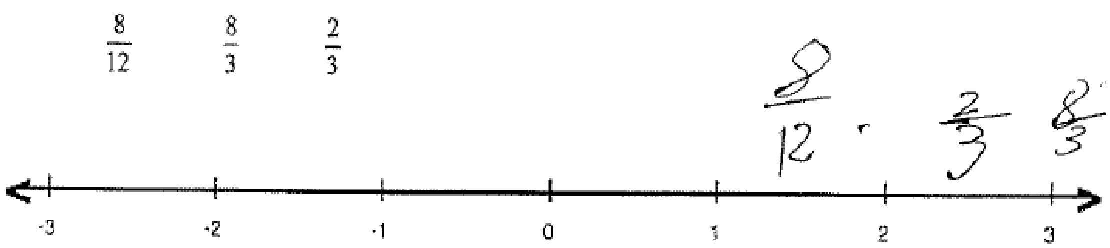
S: ½, ¼, what else?

R: ½, ¼, and 1

S: {wrote 1 at the end of the number line then placed ¼ }
¼ like, ¼ {pointed to ¼} 2 {pointed to ½} and ¾ {pointed in between1/2 and 1} then 4/4 {pointed to 1}.

R: Oh {pointed to ¼, ½, imaginary ¾, and 1} ½, 2/4, ¾, and 4/4

7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.



R: {Pointed to 8/3} How did you know where to place that fraction?

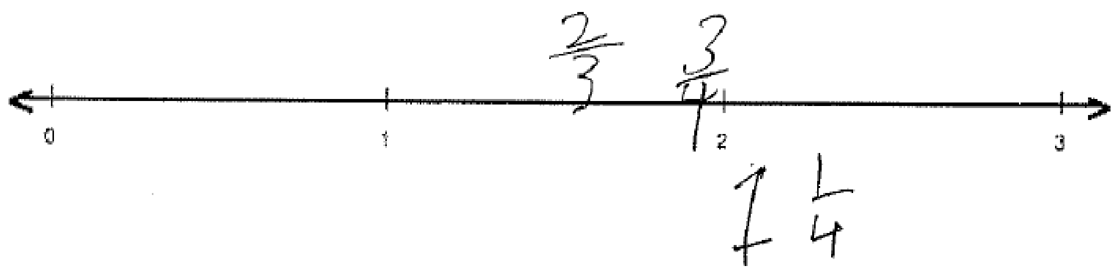
S: Because it's over all of them, so it's like a fraction more than 1.

R: Oh, so you say it's over all the other fractions: When you say over all the other fractions are you saying it's greater than?

S: Yes.

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?



S: It would be 1 and ¼.

T: You can write that if that's what you think.

S: {wrote 1 ¼}

R: How did you get that?

S: I was rounding up and estimating like what 2/4 would be and 1/3. That's probably not right, but I'm just guessing that. So I put 2/3 uh 2/4 right there {pointed to 2/3} that makes 1 whole. And then you have ¼ left right there. So I used that ¼ {pointed to the answer}.

R: Oh O.K. Thanks so much for your time.

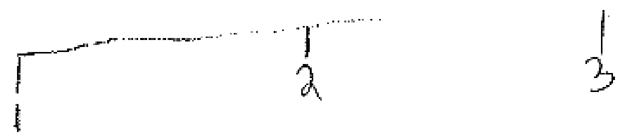
Student 4

Opening Statement:
Researcher: I would like to start with the number line. a. "How do you 'see' or think of a number line?"

Student: Umm, maybe like….like a 0 at the first part and a 1 at the other part. In the middle could be like half with numbers on it.

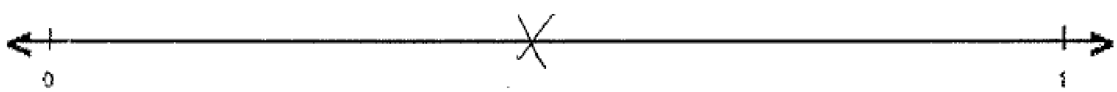
R: Could you draw me a basic number line with numbers 1, 2, and 3 on it?

S: {drew number line with 1,2,3}



R: OK.

2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.

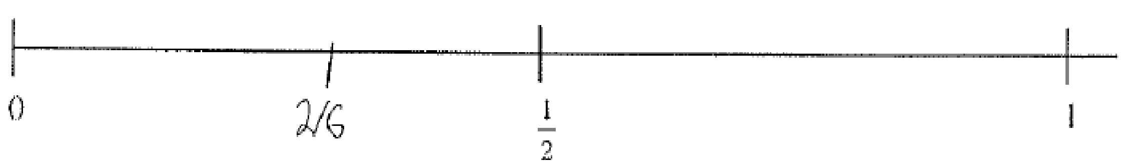


S: {Drew x approximately in the center of the 0 and 1 on the number line.}

3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

4. a). Name a fraction that is less than ½. ________ S: 3/5, I mean 3/6, less than half {chuckle} umm 2/6.

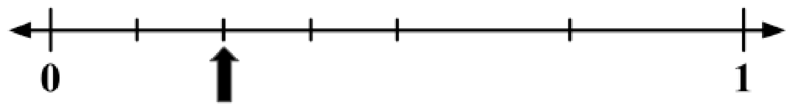
b). Place your fraction as accurately as possible on the number line below.



R: Why did you choose to put it there?

S: Because 1/6 would go there, then 2/6, then 3/6 {pointed at half) would go there.

5. Figure out what this point is called on the number line.



Circle the correct answer:

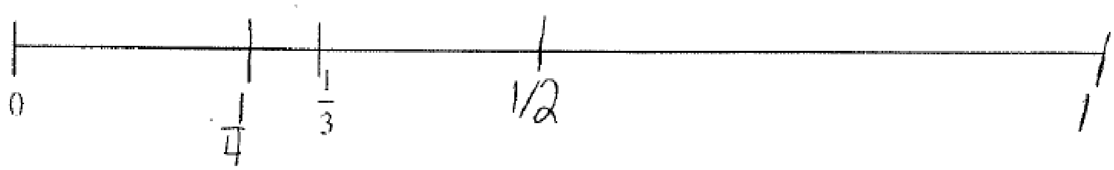
$\frac{2}{6}$ $\frac{2}{7}$ $\frac{1}{4}$ 2 $\frac{2}{4}$

How did you figure out the answer?

R: How did you figure out the answer?

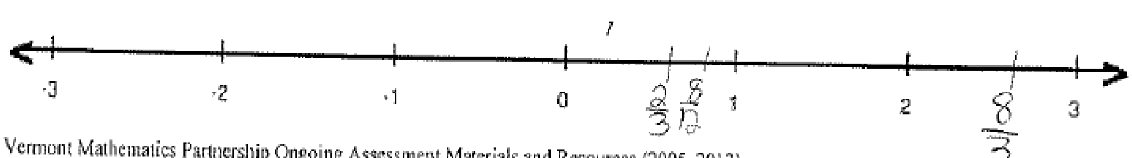
S: Umm, because umm like, umm I was counting like these {pointing to partition} to see what fraction it was. I did 1, 2, 3, 4, and I could tell that skipped, so that would be 5, 6, 7 {stopped at the section right before 1} and that's the second one {pointed at the partition with the arrow}.

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.

$\frac{8}{12}$ $\frac{8}{3}$ $\frac{2}{3}$



Vermont Mathematics Partnership Ongoing Assessment Materials and Resources (2005–2012). *OGAP questions and student work samples.* Unpublished professional development materials.

S: {Drew 3 imaginary vertical lines between 2 and 3, then labeled 8/3} {After 3 attempts to make 12 imaginary vertical lines, he wrote 8/12}.

R: {Pointed to 8/3}, “How did you know where to place that fraction?”

S: Umm because 8/3 is more than 1. Because if it was 3/3 it would be 1. If it was 6/3 it would be 2. So it’s over 2 and then it’s 2/3 left so I thought 2/3 would be there.

R: (Elicitation)– How did you know where to put 2/3 and 8/12?

S: 2/3 is greater than 1, like over half. For 8/12, I just kind of ticked off to 8/12.

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school’s bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?

S: Do you have to use the number line?

R: Solve it the way you want to solve it.

S: {Solved mentally} I got 1 and 5/12.

R: Ok write that down and tell me your thinking.

S: Cause ¾… the num…the denominators 4 and 3 both go into 12. And uh 4 goes into 3, no 4 goes into 12 three times and so I did 3x3 and got 9/12. And then 3 goes into 12 four times, so 4x3 is 12. Wait. Oh wait 3 goes into 12 four times so 2 x4 equals 8. Then I did 8/12 plus 9/12 = 17/12. And then I got, I did 17 is more. If the numerator is greater than the denominator would be over 1 and 5/12.

Student 5

Opening Statement:

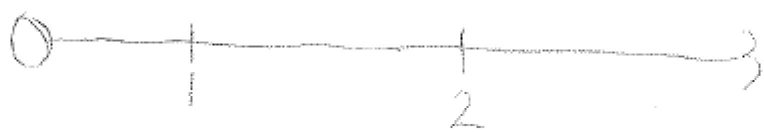
Researcher: I would like to start with the number line. a. “How do you ‘see’ or think of a number line?”

Student: Can I just write it down?

Researcher: I am going to ask you to draw one. Will you add numbers 1, 2, and 3 to your number line?

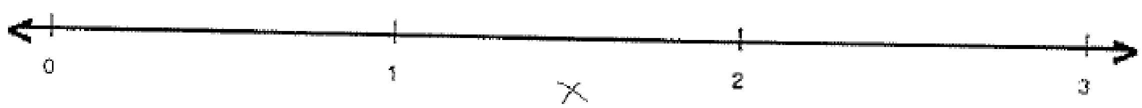
S: So I just draw a number line how I think of it?

R: Sure. Could you include numbers 1,2, 3 to your number line?



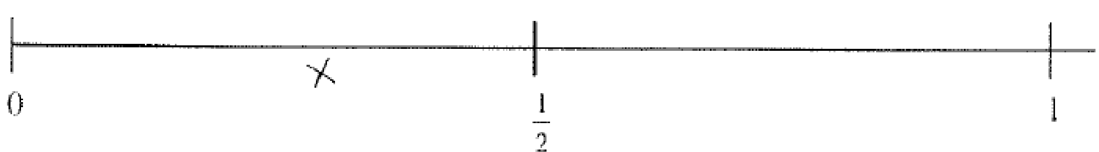
2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.

3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.



4. a). Name a fraction that is less than ½. ________ S: 1/3.

b). Place your fraction as accurately as possible on the number line below.

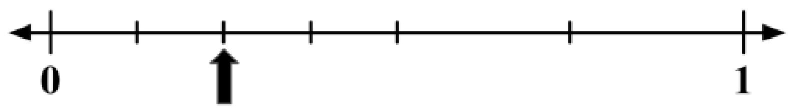


R: How did you know to put it there?

S: Because a half of a half is a fourth and it’s eight and a third more than a fourth.

R: Wow. So it’s 8 and a third more than a fourth. I’ll jot that thinking down.

5. Figure out what this point is called on the number line.



Circle the correct answer:

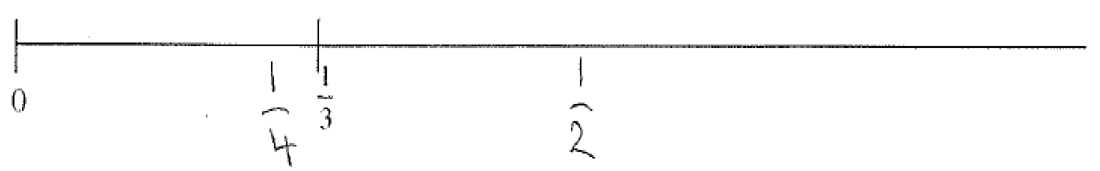
$\frac{2}{6}$ $\frac{2}{7}$ $\frac{1}{4}$ 2 $\frac{2}{4}$

How did you figure out the answer?

R: How did you know that’s the answer?

S: That’s a half {pointed at the half mark} and that’s a half of a half.

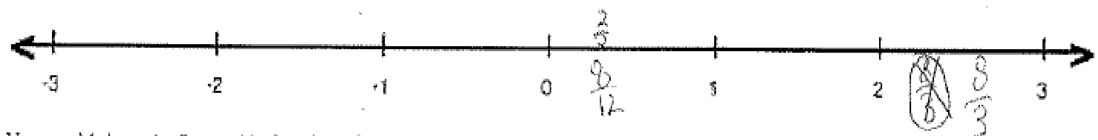
6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



*Video stopped

7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.

$\frac{8}{12}$ $\frac{8}{3}$ $\frac{2}{3}$



8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?

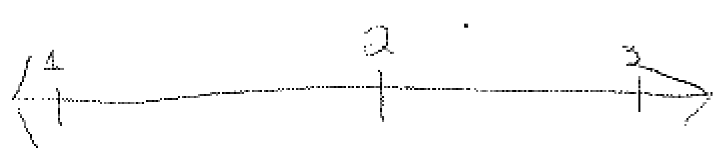
I think it will be more than one

Student 6

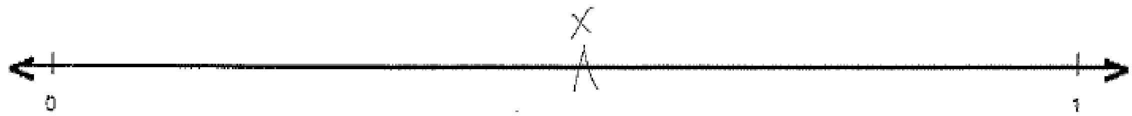
Opening Statement:
Researcher: I would like to start with the number line. a. "How do you 'see' or think of a number line?"

Student: A line with arrows pointing at each end with numbers on it with numbers on it…. with numbers in order.

R: Ok. Could you please draw a number line and put numbers 1, 2, and 3 on it?



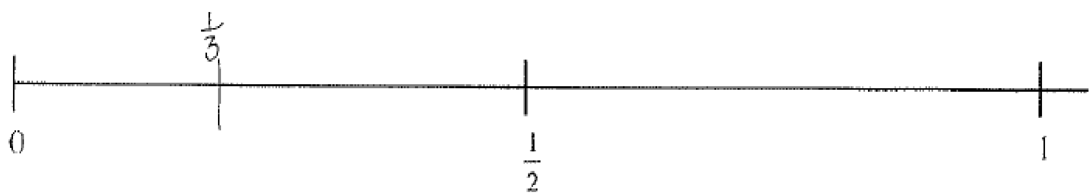
2. This number line shows 0 to 1. Put an (X) where the ½ would be on the number line below.



3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

4. a). Name a fraction that is less than ½. ________ S: umm a third.

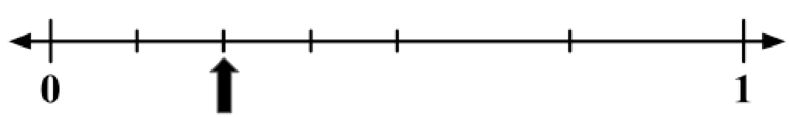
b). Place your fraction as accurately as possible on the number line below.

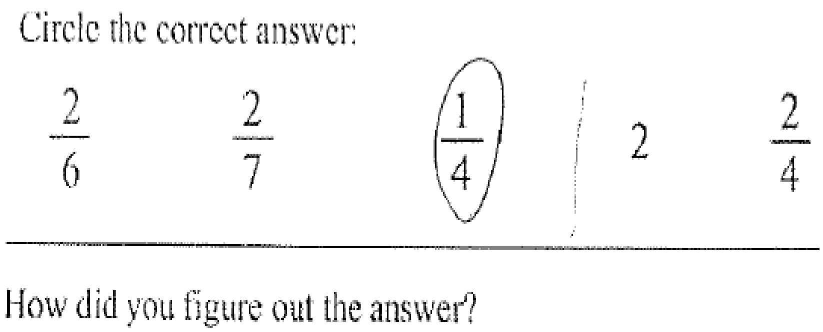


R: How did you know to put it there?

S: Well because I can make uh three spaces {pointed to and created three imaginary spaces between 0 and ½}.

5. Figure out what this point is called on the number line.





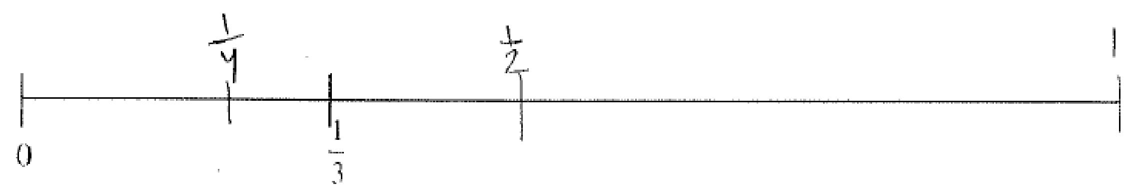
R: How did you know that's the answer?

S: Because I counted 1, 2, 3, 4 {pointed to the first four segments} and that's about half {pointed to the half mark}. So there would be four {pointed to the other half of the number line (actual and imaginary marks)}.

R: So you knew that would be ¼ by doing that.

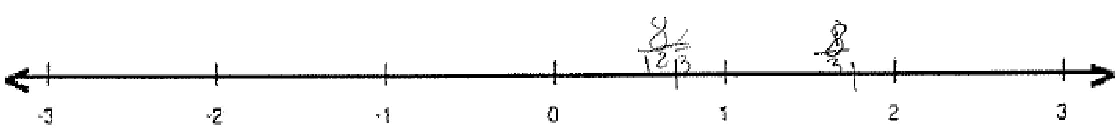
S: Yes.

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.

$\frac{8}{12}$ $\frac{8}{3}$ $\frac{2}{3}$



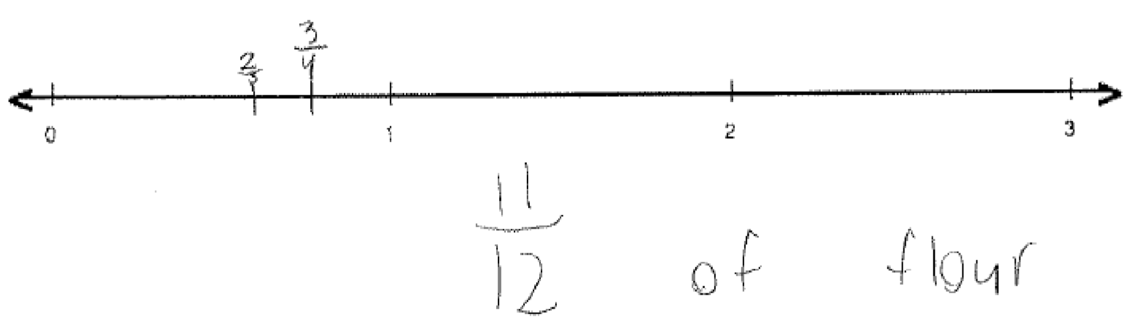
Student thought for a little while and placed 8/12. Next he placed 8/3. The student then drew two imaginary tick marks prior to writing 2/3 in its location.

R: How did you know where to place this fraction {pointed to 8/3}?

S: Umm well because 8 the numerator is uh more than uh 3 the denominator, so 3+3 is 6 and uh 1 more 3 would over the numerator so I could fit 3 into 8 two times. So I knew it was between 1 and 2 and then I knew it was 2 and something. So left over was 2 more. So, 2/3. So I put 2/3 in between 1 and 2.

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?



R: Are you done?

S: Yes, I did it mentally.

R: How did you do it?

S: Well I switched the denominators to uh so to twelfths because they both go into twelfths. Uh 2/3 is 8/12, and oh well I 'plussed' I added 3/4. So 11/12 plus 2/4 is your answer.

Student 7

Opening Statement:
Researcher: I would like to start with the number line. a. "How do you 'see' or think of a number line?"

S: I think of a line. Oh do you want me to write it?
R: Ok you can because I'm going to ask you to draw one and include numbers 1, 2, and 3 on it?

R: Ok. Do you have numbers 1, 2, and 3 on it?

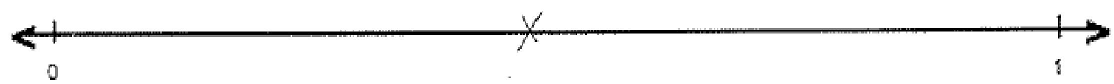
S: Umm it has 1 and 2 on it.

R: Ok. What would it look like if you had 1, 2, and 3 on it?

S: Well the 3 would be here {pointed to the 2 at the end of the number line} and the 2 would be in the middle.

R: Oh. Ok

2. This number line shows 0 to 1. Put an (X) where the ½ would be on the number line below.



S: {Drew x approximately in the center of the 0 and 1 on the number line.}

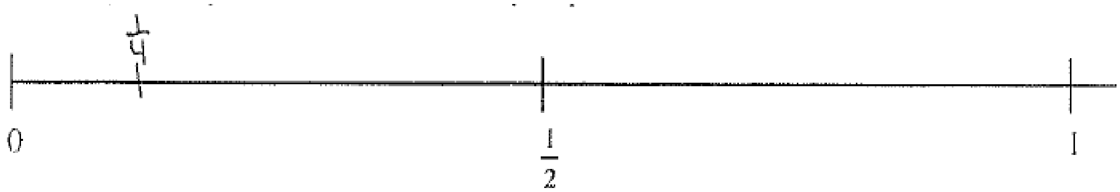
3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

R: How did you know it went there?

S: Umm because instead of half of 1 and 2 it would be half of 0 and 3. So then I know where the half is usually.

4. a). Name a fraction that is less than ½. ________ S: 1/4.

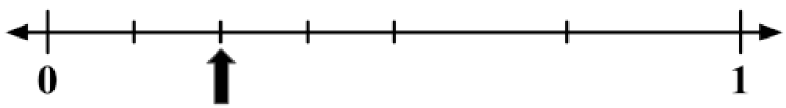
b). Place your fraction as accurately as possible on the number line below.



R: How did you know to put it there?

S: Because I could do ¼, then 2/4 {jumped all the way to the ½ location}, then ¾, and 4/4 {ended the leaps by pointing at the 1}.

5. Figure out what this point is called on the number line.



Circle the correct answer:

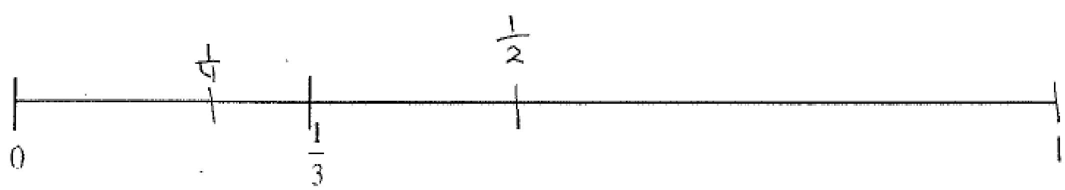
$\frac{2}{6}$ $\quad$ $\frac{2}{7}$ $\quad$ $\frac{1}{4}$ $\quad$ 2 $\quad$ $\frac{2}{4}$

How did you figure out the answer?

R: How did you know that's the correct answer?

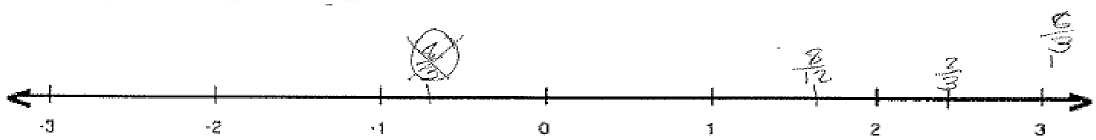
S: Umm because there are six things and that's the second.

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.

$\frac{8}{12}$ $\frac{8}{3}$ $\frac{2}{3}$



R: Ok how did you know where to place this fraction {pointed at 8/3}
S: Umm because I just imagined 8, yea 8, oh now I realized where it would probably go.

R: Where do you think it might go?

S: Like behind that {pointed to 3} because I know that 8/3 is much bigger.

R: Would you like to change your thinking?

S: Yes

R: Would you just circle it and put an X through it? So that we will know that you are changing your thinking.

S: So how did you know it goes there because 3, 6, 9, so it would go right there {pointed to the 3}.

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?

$1\frac{5}{12}$

S: It would be 1 whole and 5/12

R: Would you write that down? How did you get that?

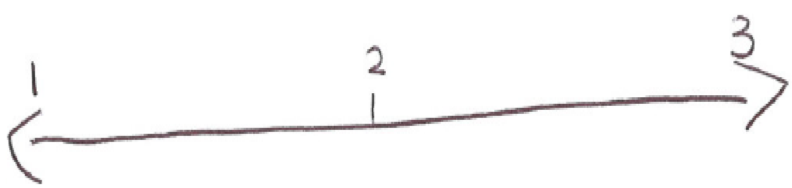
S: Umm because like I did common denominator and so 2x3 would be 8 or 2x4 would be 8. And umm 3x3 would be 9. And So I added those up and then in my head that was one whole. And so then I added those up I think it would be seventeen…twelfths. And I thought in my head how much does it take to get 17/12.

Student 8

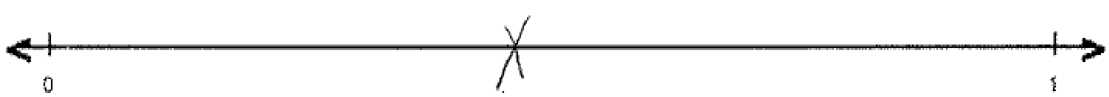
Opening Statement:
Researcher: I would like to start with the number line. a. "How do you 'see' or think of a number line?"

S: Well I just think of a line that you put things in order from smallest to greatest.

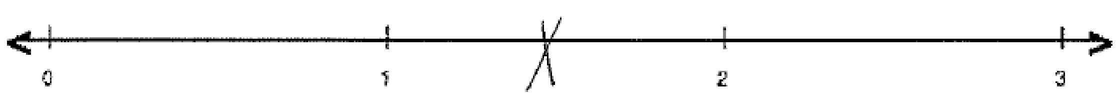
R: Could you draw one and put numbers 1, 2, and 3 on your number line?



2. This number line shows 0 to 1. Put an (X) where the ½ would be on the number line below.

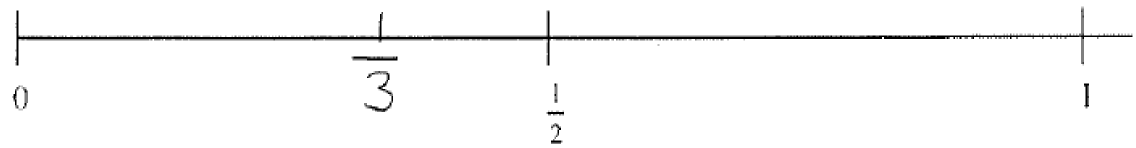


3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

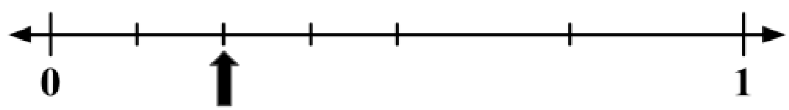


4. a). Name a fraction that is less than ½. ________ S: 1/3.

b). Place your fraction as accurately as possible on the number line below.



5. Figure out what this point is called on the number line.



Circle the correct answer:

$\frac{2}{6}$ $\frac{2}{7}$ $\frac{1}{4}$ 2 $\frac{2}{4}$

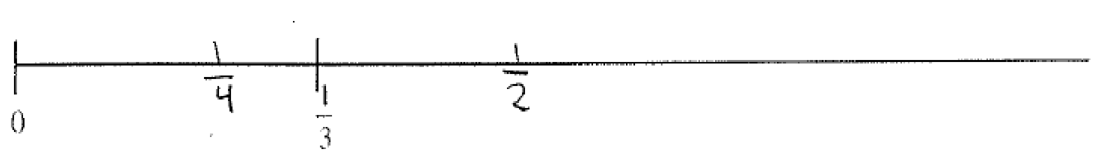
____

How did you figure out the answer?

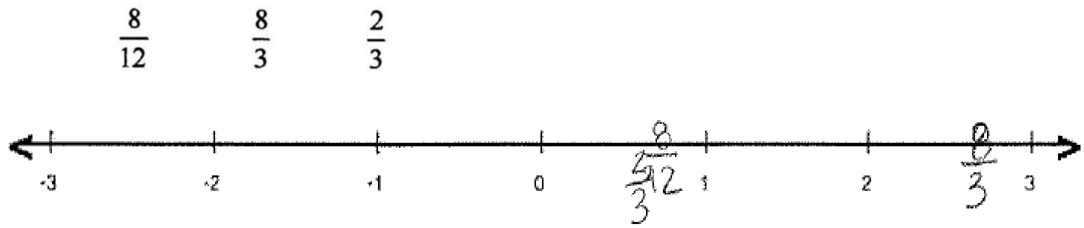
R: How did you know that's the correct answer?

S: Well I did 1, 2, {pointed to the first two partitions} But you could do 1, 2, 3, 4, 5, 6 {pointed to each partition including 1}.

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.



S: So is that like 0 to 1…. 1/12, 2/12, 3/12 {drew imaginary vertical lines}?

R: What ever your're thinking. These are the fractions here {pointed to the fractions}

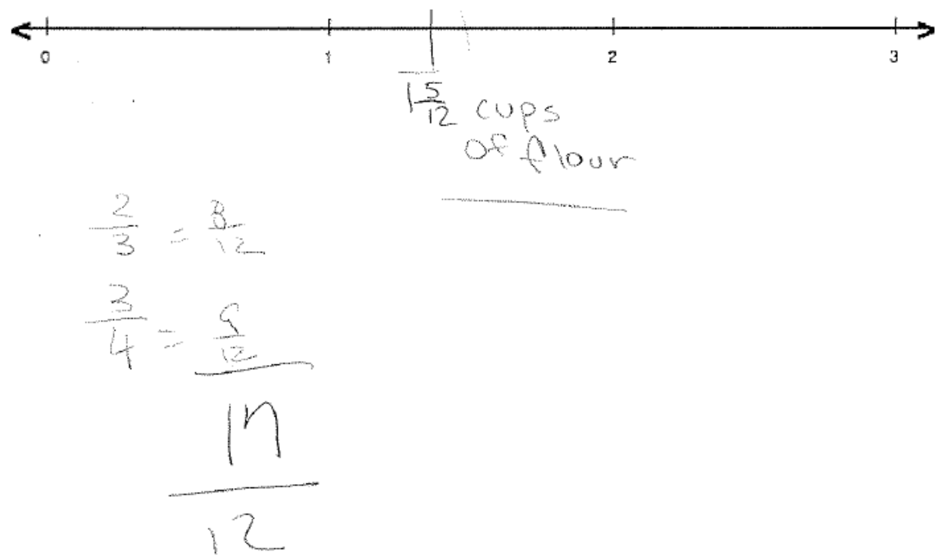
S: I know, I'm just saying like.
S: {Wrote 8/12 with little thought. Placed 2/3 by partitioning into imaginary thirds.

R: How did you know where to place that one? {pointed at 8/3}

S: Well if you did 3/3 that would be 1; 6/3 would be 2; since it's over 6/3 it would be 8/3 so there would be 2 more thirds {pointed at the 8/3}.

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?



S: Can I like add it up?

R: Sure

S: Would I write the….Do I just write a line?

R: Whatever you want to do.

S: Solved “2/3=8/12   3/4 =9/12”….. It’s right there {pointed to answer on number line}

R: What’s your thinking?

S: Well, if you add up the flour you would have 17/12 because you find the common denominator.

R: Did you write 17/12 somewhere?

S: No I added it up in my head

R: Will you write that so I will know?

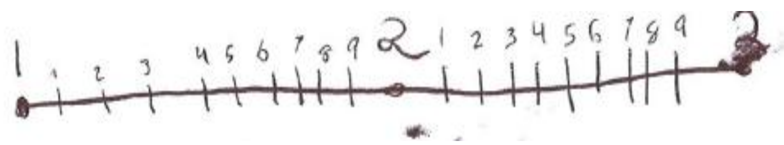
S: And then so there’s 5/12 in between the two. So I know it’s one because 12/12 is 1and 5/12 is one less than half {pointed to the slightly shaded landmark of 1 ½}

Student 9

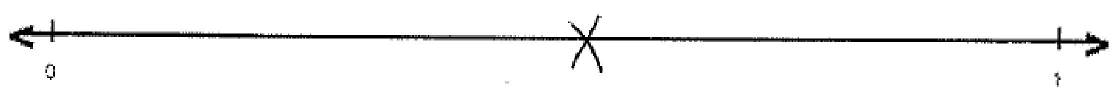
Opening Statement:
Researcher: I would like to start with the number line. a. "How do you 'see' or think of a number line?"

Student: {immediately drew a line}

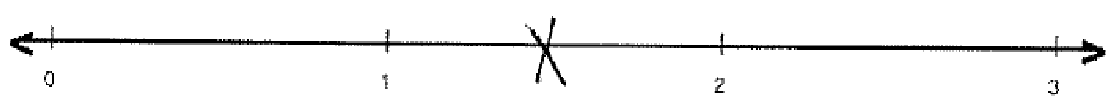
Researcher: Since you are drawing it, could you add numbers 1, 2, and 3 to your number line?



2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.



3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

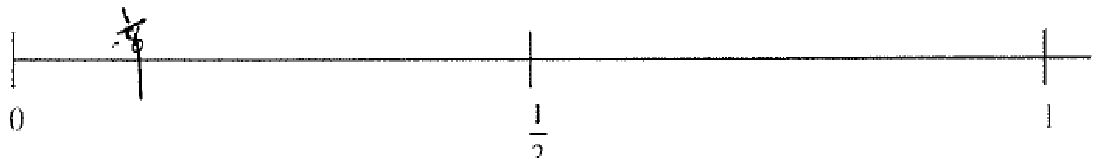


4. a). Name a fraction that is less than ½. ________

R: Can you name one?

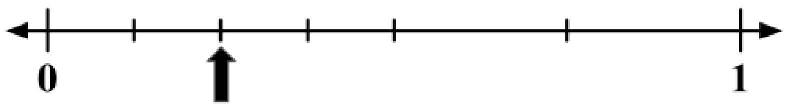
S: Oh. 1/8.

b). Place your fraction as accurately as possible on the number line below.



S: {Placed 1/8 in what appears to be the correct location}

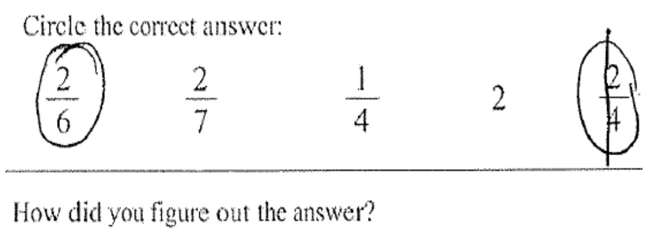
5. Figure out what this point is called on the number line.



S: Does the 1 mean a hundred?

R: What do you think it means?

S: {circled 2/4} Well 2/4 is umm 50 so…. and this has 1, 2, 3, 4, 5, {pointed to each vertical line} oh



Saxe et al. (2007)

R: So are you answering for this point right here? (pointed to the vertical line above the arrow}

S: Uh I think I counted it wrong.

R: Oh Ok. So what do you think it is?

S: Umm maybe ¼.

R: Why do you think that?

S: Because there uh…. because ¼ is 25 and this would be like 10 {pointed to the first vertical line}, be like 10 {pointed to the first vertical line again} then maybe 5 {pointed to the same vertical line again}, 10 and maybe 5 {pointed to the vertical line with the arrow} ‘cause it can be 10 {pointed to the first vertical line labeled with 0}.

R: So 10 would be here {pointed to the first vertical line labeled with 0}. So what would be here {pointed to the first vertical line}?

S: 15

R: {pointed to the vertical line with the arrow}

S: 20

R: Ok, so this will be 5, 10 ({pointed at the first two vertical lines starting with 0}
R: {pointed to the vertical line with the arrow}

S: Whoops…… umm I …..hmm (PAUSE) maybe it would be 2/6

R: Why do you think that?

S: Because there are 1, 2, 3, 4, 5, and this can be 6, {pointed to each vertical line past 0} and this can be 100 {pointed to the 1} So this can be 1, 2 (pointed the first two vertical lines}

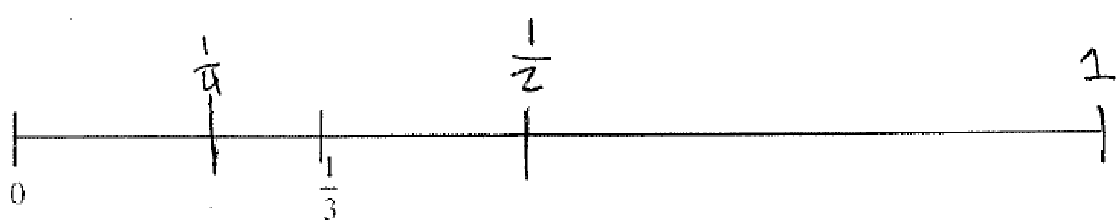
R: Are you changing your thinking?

S: Yes

R: Ok will you draw a line through that {pointed to the 2/4} and put your revised thinking.

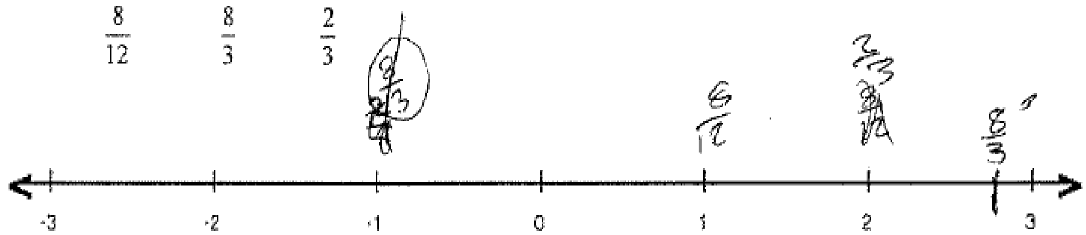
S: {circled 2/6}

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



S: {Wrote 1 at the end, then ½ approximately in the center, placed ¼}

7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.



R: How did you know where to place this fraction {pointed at 8/3}?

S: Because….. Oh, I got it wrong. Because I thought it was 3/8. But really I think it goes here {pointed to 2} or here {pointed to number 3).

R: Ok. So when you thought it was 3/8, you put it there {pointed to above -1). Really you think it should go where?

S: I think it would be right here {pointed to what appeared to about 6/4 between 2 and 3} 3Well if you did 3/3 that would be 1; 6/3 would be 2; since it's over 6/3 it would be 8/3 so there would be 2 more thirds {pointed at the 8/3}.

R: So how did you know it would go there?

S: Because 3/3 would be…because 8 is higher, it would be like 3/3 but 8 goes over the thirds. So I thought it would be 8/3.

R: Ok

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?

S: Do you want me to add it or use the number line?

R: I just want you to solve it.

$$\frac{2}{3} = \frac{4}{12}$$
$$+\frac{3}{4} = +\frac{3}{12}$$
$$\frac{7}{12}$$

R:

R: How did you come to that conclusion?

S: Because something that goes into 3 and 4 is 12 and then 4 goes into 12 three times and 3 goes into the 12 four times. And so I added that together {pointed to problem solved}.

R: And so your answer is ………

S: 7/12.

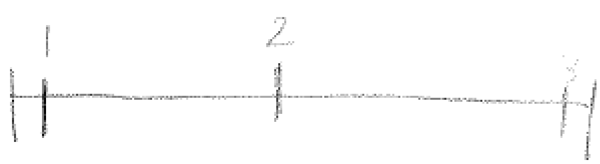
R: 7/12 ok.

Student 10

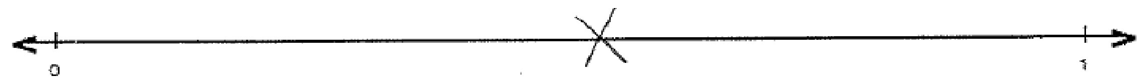
Opening Statement:
Researcher: I would like to start with the number line. a. "How do you 'see' or think of a number line?"

Student: A line with numbers on it.

Researcher: OK. Could you add numbers 1, 2, and 3 on it?



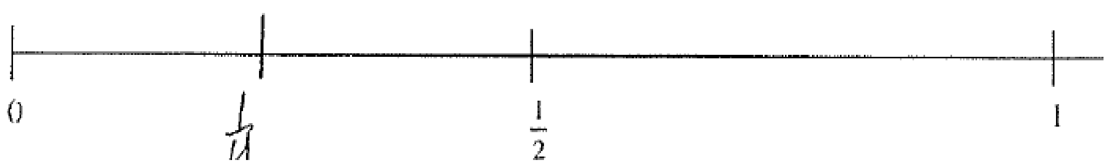
2. This number line shows 0 to 1. Put an (X) where the ½ would be on the number line below.



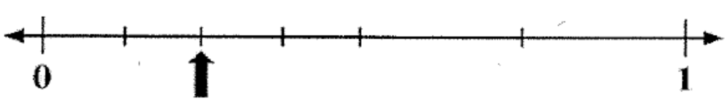
3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

4. a). Name a fraction that is less than ½. _________ 1/4

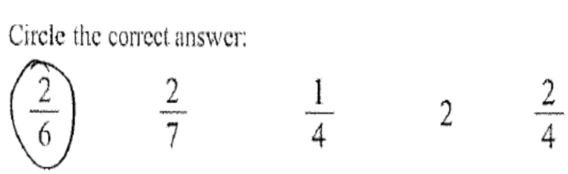
b). Place your fraction as accurately as possible on the number line below.



5. Figure out what this point is called on the number line.



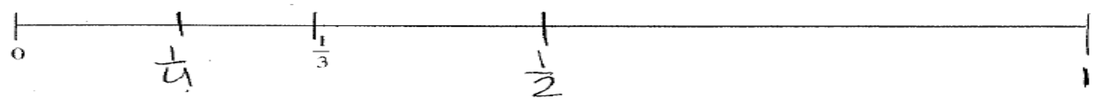
S: Do I have to write it?



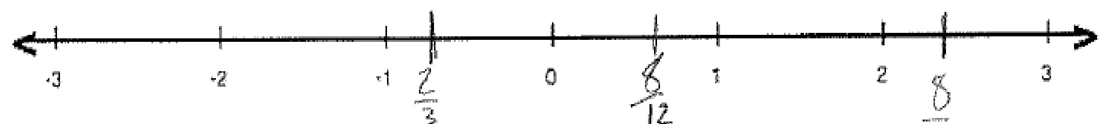
R: How did you figure this out?

S: I counted how many little slashes there are.

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.



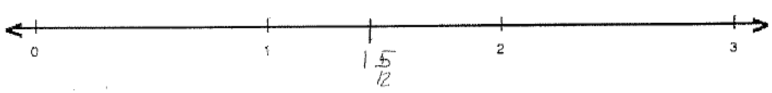
Vermont Mathematics Partnership Ongoing Assessment Materials and Resources (2005–2012). *OGAP questions and student work samples.* Unpublished professional development materials.

R: How did you know where to place this fraction {pointed at 8/3}?

S: Oh I knew it was more than 2 and I knew it would be 2 and 2/3.

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?



$\frac{3}{4\times3}\ \frac{9}{12}$ $\quad \frac{2}{3\times4}\ \frac{8}{12}$

$\frac{9}{12}+\frac{8}{12}=\frac{17}{12}=1\frac{5}{12}$

R: How did you get that?

S: ¾ is equal to umm 9/12 and 2/3 equals to 8/12 and 9/12 plus 8/12 equal 17/12. Which is equal to 1 and 5/12.

R: Ok.

Student 11

Opening Statement:
Researcher: a. “How do you ‘see’ or think of a number line?”

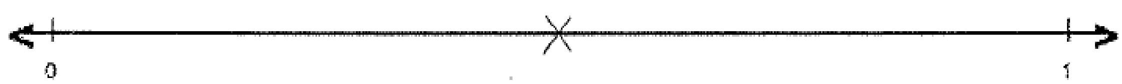
Student: I think of zero to one.

R: Zero to one? Could you add numbers 1, 2, and 3 on it?

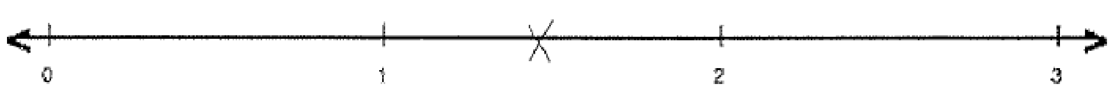
S: I’m done.

R: OK.

2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.

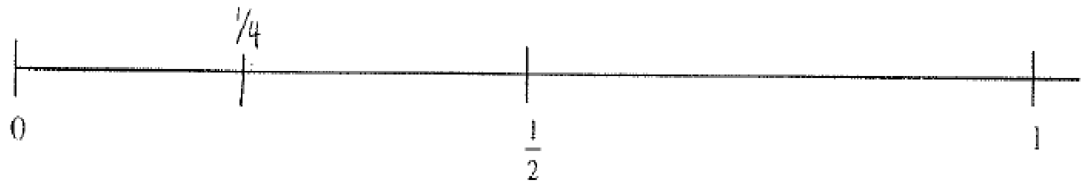


3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

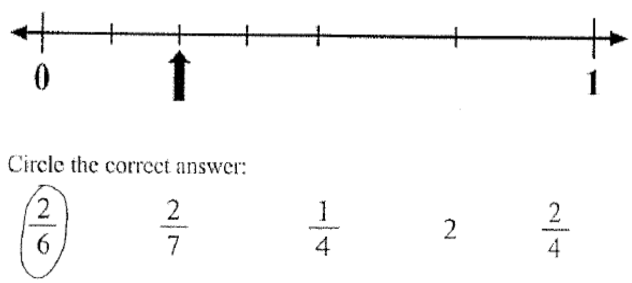


4. a). Name a fraction that is less than ½. ________ 1/4

b). Place your fraction as accurately as possible on the number line below.



5. Figure out what this point is called on the number line.



Saxe et al. (2007)

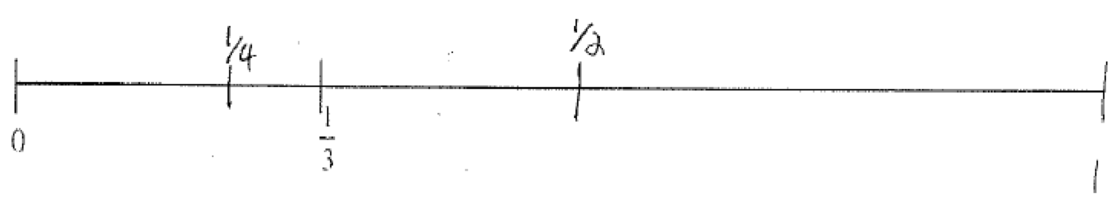
R: How did you figure out the answer?

S: Well I knew this one was two. And I did one, two, three, four (paused pointed at each segment) five and I sort of added lines in between.

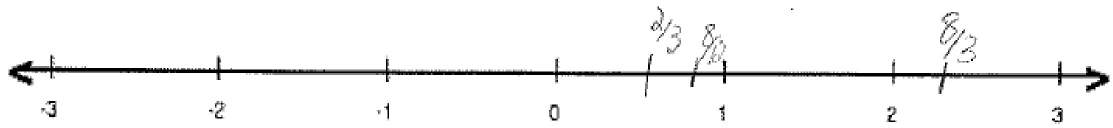
R: Ok. So what's your answer?

S: 2/6.

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.



R: How did you know determine that one {pointed to 8/3}

S: Well, because I knew it was gonna be out of zero to one. So it was 8/3. So I knew 3 (pointed at 0-1 segment) and 6 (pointed at 1 to 2 segment) and then I just did two more.

R: Uh OK. How did you know where to place 2/3?

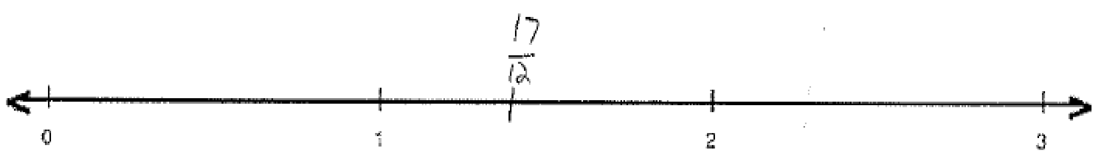
S: Well I knew 2/3 was a little more than 1/2, so I did a little more than that.

R: How did you know where to place 8/12?

S: I know 8/12 is equal to 2/3, but I couldn't fit it in the same spot.

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?



S: Wrote 17/12 about one third of the way past 2.

R: How did you determine that?

S: Well I knew that 2/3 and ¾ both can go into 12. So 2/3 is equal to 8/12 and ¾ is equal to 9/12. And I knew it goes…. Well (erased answer and wrote it in the seemingly correct location) since it's going into one.

R: hmmmm

Student 12

Opening Statement:
Researcher: a. “How do you ‘see’ or think of a number line?”

Student: Just draw it?

R: Sure. And when you draw it, include 1, 2, and 3 on it.

S: Can I hear your question again? I’m sorry. (drew a straight line)

R: A number line. How do you see or think of a number line?

S: A fraction number line?

R: Hmm. Just a number line.

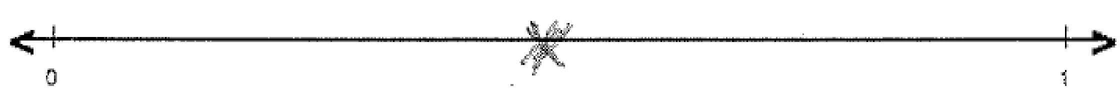
S: OK……. I’m just guessing with the number order.

R: Ok

S: I’m just drawing the line…….. (inaudible).

2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.

S: Umm if I just guess the number line (low voice)



3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

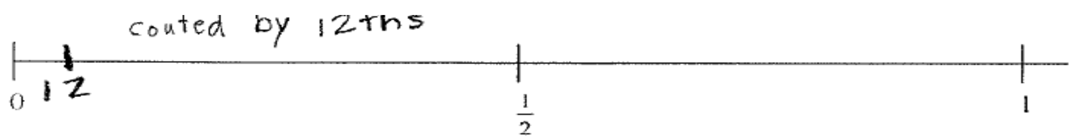
4. a). Name a fraction that is less than ½. ________

S: Yes

R: What is it?

S: 1/12, the first one that popped into my head.

b). Place your fraction as accurately as possible on the number line below.



S: There's like 1/24, 1/100 and it would have to be like tiny little lines. Could I just like put it?

R: Place as accurately as possible

S: Yea I know. I just want to know if it's the twelfths. Like 5/12?

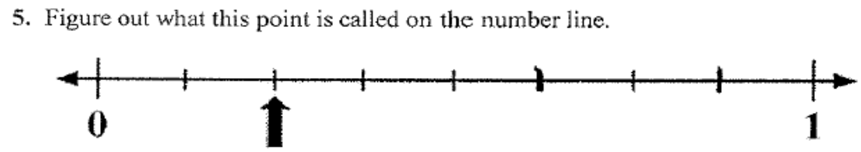
R: oh ok.

S: Will that be OK

R: Whatever you want to put on there is fine.

S: I counted by twelfths. (turned the paper towards the researcher)

5. Figure out what this point is called on the number line.



S: Do I do this based on the twelfths number line I did? (pointed to question #4).

R: This is a totally differently question.

S: Ok. (mumbled) I don't get that. Well I don't know what kind of number line it is. Like is it a sixth or a seventh? I don't know. What kind of Number line it is so I don't really…..

R: Ok do your best.

Circle the correct answer:

$\frac{2}{6}$ $\frac{2}{7}$ $\frac{1}{4}$ 2 $\frac{2}{4}$

R: How did you figure out the answer?

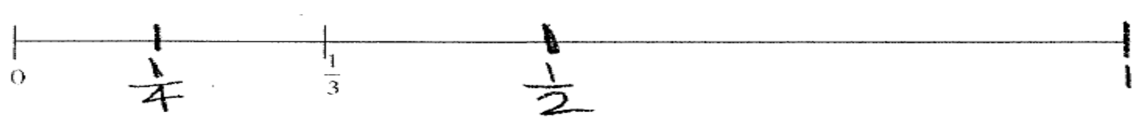
S: On this line right here (pointed to line under circled fraction)

R: You can just tell me.

S: OK. What I did was… Well I counted right here and I kinda made up my own little half lines so they can all equal out. I counted 1, 2, 3, 4, 5, 6, 7, (pointed to the numbers in descending order) and I pointed to the second one. So I got 2/7.

R: Oh, I see your thinking.

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



S: mumbled inaudible…. Where would one-half be? Am I supposed to explain, how I thought of this one?

R: You can just show me.

S: I want to explain…..

R: Ok

S: Ok so I did ½ and since 1/4 is less than 1/3 so I put it right here.(pointed to fraction) Since it's closest to 0.

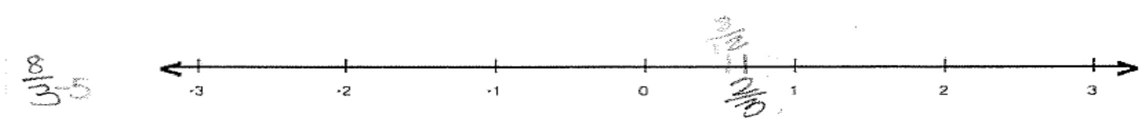
R: OK

7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.

$\frac{8}{12}$ $\frac{8}{3}$ $\frac{2}{3}$

S: Do I have to draw it?...... Wait a minute, I really don't get this, because all of these don't come….. Am I supposed to draw between here, because that's the only place they can fit (pointed between 0 and 1)? Oh ……. (started writing)………. Can I go past negative 3?

R: You can do it however you want to do it.

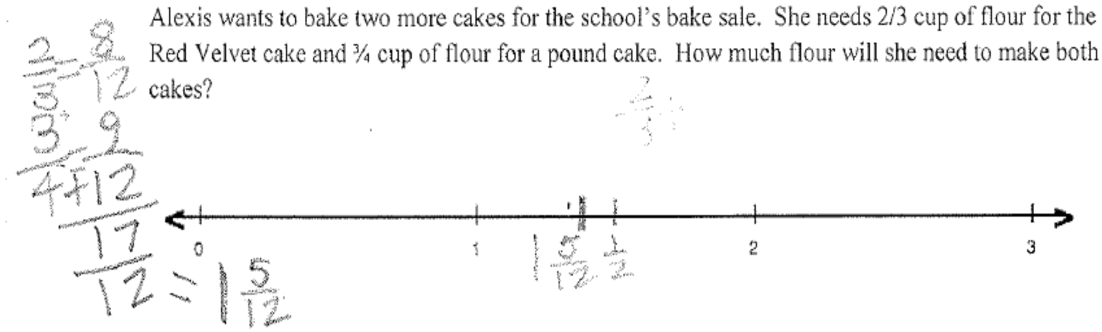


R: OK. Could you explain your thinking as to how you got those {pointed to the answers}?

S: OK 2/3 is 1/3 away from 1. So therefore, it's very close to it. Umm, 8/12 is close to 1 but not as close but it's not as close. So I put it right here not quite in the half.
And over here I noticed (pointed to where 8/3 was located? 9 times.. I mean 3 times 3 is 9. So what I was thinking if it was 9 it would be negative 6. Since it's 8 it's negative 5.

R: OK. Gotcha.

8. Use the number line below to solve the following problem.



S: Am I supposed to add fractions on this one?

R: However you think you should solve it.

S: OK

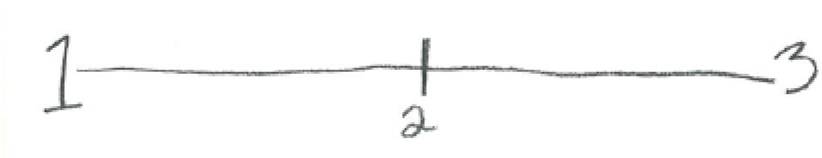
S: Wait a minute, I gotta round them. (inaudible) Does four go into six? Twelve. Wait that doesn't work, it doesn't go into six.

Student 13

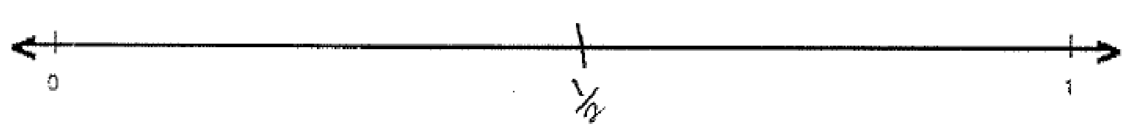
Opening Statement:
Researcher: a. "How do you 'see' or think of a number line?"

Student: Umm I think of like a line that says 0 and 1 at the end. And just like numbers on it.

Researcher: Will you please draw one and include numbers 1, 2, and 3 on it.



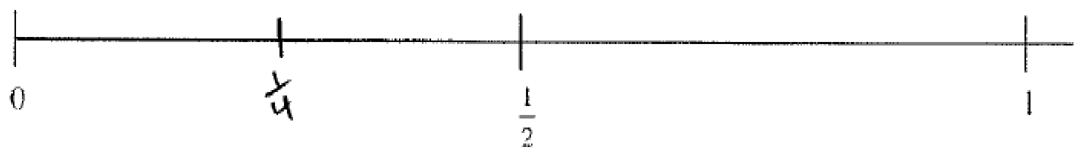
2. This number line shows 0 to 1. Put an (X) where the ½ would be on the number line below.



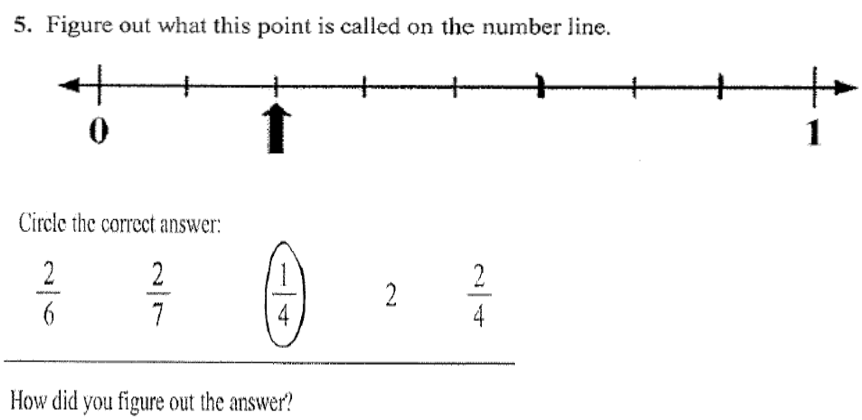
3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

4. a). Name a fraction that is less than ½. ________ umm 1/4

b). Place your fraction as accurately as possible on the number line below.



5. Figure out what this point is called on the number line.


5. Figure out what this point is called on the number line.

0 1

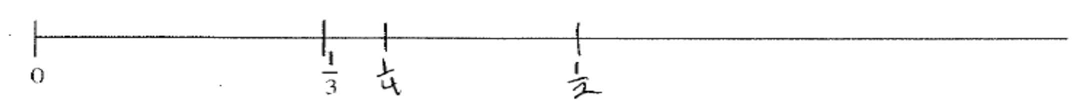
Circle the correct answer:

$\frac{2}{6}$ $\frac{2}{7}$ $\frac{1}{4}$ 2 $\frac{2}{4}$

How did you figure out the answer?

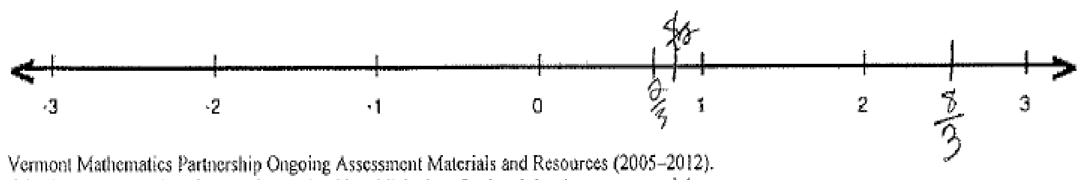
S: I found half of one-half, which is one-fourth.

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.

$\frac{8}{12}$ $\frac{8}{3}$ $\frac{2}{3}$



Vermont Mathematics Partnership Ongoing Assessment Materials and Resources (2005–2012). *OGAP questions and student work samples.* Unpublished professional development materials.

R: OK. Could you explain your thinking as to how you got those? {pointed to the answers}

S: Umm, well I know that 8/3 isn't 2 and half. I know that 2/3 is a little over 1/2 and I knew that 8/12 is a sixth over uh ½. So it's like 2/3.

R: So you are saying this is like 2/3 (pointing at the 8/12)

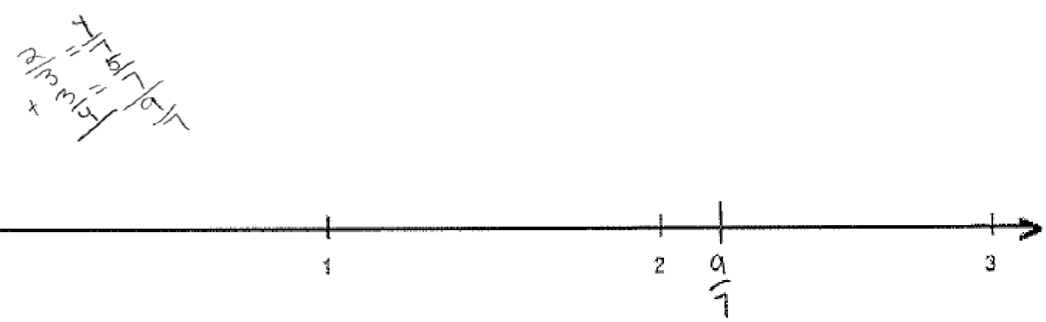
S: yes.

R: OK

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?

S:



R: Ok. Thank you.

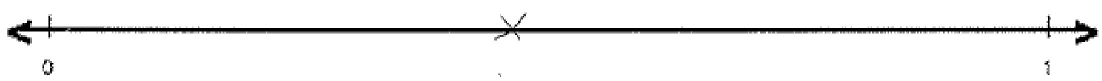
Student 14

Opening Statement:
Researcher: a. “How do you ‘see’ or think of a number line?”

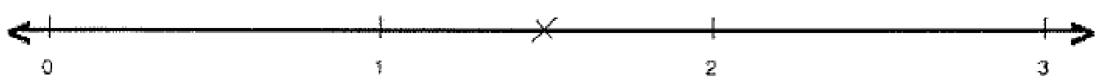
Student: Umm I would think of it as a line and you would have certain little arrows. And then it could have a big one that could describe 2 or maybe 1, 2, 3, 4 and then there’s a bigger one for 5. And the biggest one would be for 1.

Researcher: Will you please draw one and include numbers 1, 2, and 3 on it.

2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.

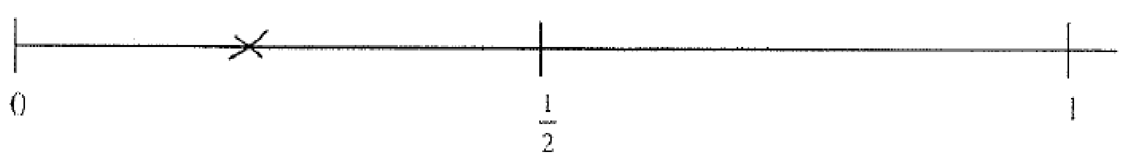


3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.



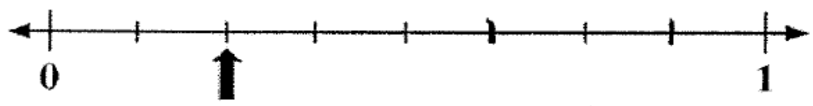
4. a). Name a fraction that is less than ½. ________ umm 1/4

b). Place your fraction as accurately as possible on the number line below.



5. Figure out what this point is called on the number line.

5. Figure out what this point is called on the number line.



Circle the correct answer:

$\frac{2}{6}$ $\frac{2}{7}$ $\frac{1}{4}$ 2 $\frac{2}{4}$

How did you figure out the answer?

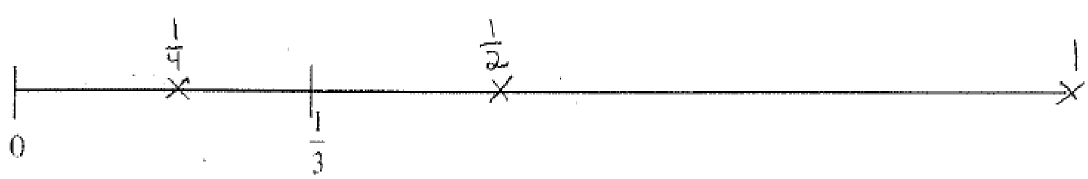
S: Umm here's one and zero and I know that half is going to be in the middle. And 2/6 would be like right before it and ¼ would be right behind it. Because I know that 3/6 is equivalent to 1/2. And 2/6…. 4 and 6 could both go into 12. 2/6 is equal to…. I would have to figure that out.

R: So how did you know where to place ¼?

S: I know that ¼ is half of ½.

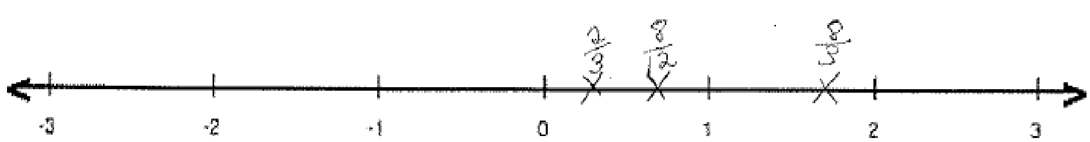
R: I see your thinking.

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.

$\frac{8}{12}$ $\frac{8}{3}$ $\frac{2}{3}$



R: OK. Could you explain your thinking as to how you got those{pointed to the answers}?

S: I know that 8/12 would be equal to 2/3 of 1. Well this 2/3 should probably be right here (pointed to the 8/12) because they are both equivalent

R: Oh, so what would that look like if they are both equivalent?

S: They can both be two-thirds.

R: OK

S. And then…I'm still trying to figure out 8/3

R: What are you thinking about 8/3?

S: Well I know it's going to be more than 1. Umm….

R: How did you decide to put it right there?

S: Umm……. Well maybe… Now I'm thinking like that it would be well I think it would go right there(pointed to 8/3). It would be 8/12 because it can't be like 3 and 6 because 3, 6, then 9. And so it can't be equivalent to a 2/3. Well it could be, but not like…… in a way to where you could say it was 2/3.

R: Ok

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes?

S: Uh… it would be 17/12 cups of flour. So it would be 1 and
R: Could you write that for me?

$$\frac{3}{4}=\frac{9}{12}+\frac{2}{3}=\frac{8}{12}$$

$$\frac{3}{12}+\frac{8}{12}=\frac{17}{12}\quad 1\frac{1}{5}$$

S: And it would be 1 and 1/5.

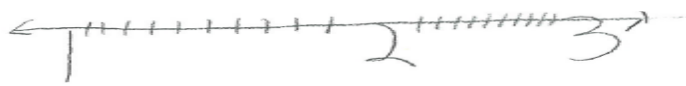
R: Ok.

Student 15

Opening Statement:
Researcher: a. “How do you ‘see’ or think of a number line?”

Student: Umm.. like two numbers, one or two, and lots of smaller numbers in between them.

Researcher: Will you please draw one and include numbers 1, 2, and 3 on it.

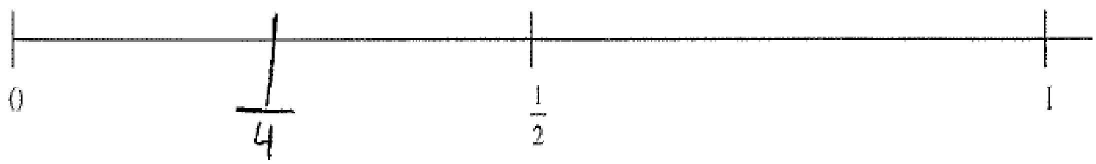


2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.

3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

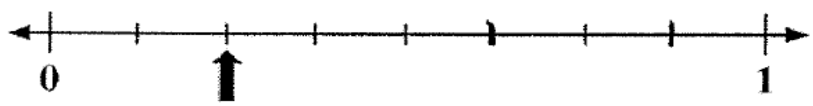
4. a). Name a fraction that is less than ½. ________ 1/4

b). Place your fraction as accurately as possible on the number line below.



5. Figure out what this point is called on the number line.

5. Figure out what this point is called on the number line.



Circle the correct answer:

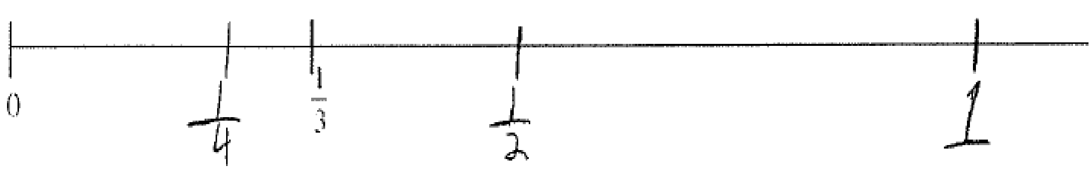
$\frac{2}{6}$ $\frac{2}{7}$ $\frac{1}{4}$ 2 $\frac{2}{4}$

How did you figure out the answer?

S: Because there are 7, well there are 5 but there's one with a couple missing. So there would be one there and one in there (pointed to the blank spaces). So it would be 2/7.

R: Oh. So that's 2/7?

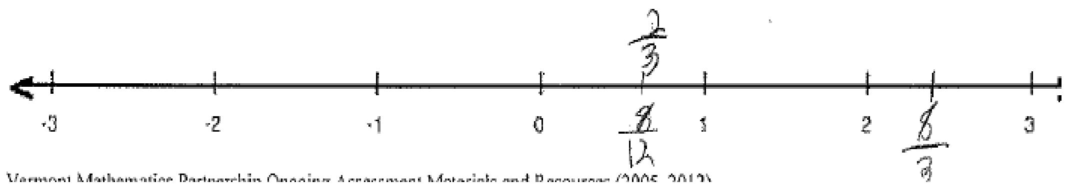
6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.

$\frac{8}{12}$ $\frac{8}{3}$ $\frac{2}{3}$

S: placed 2/3 in the approximate ¼ position and placed 8/12 and 8/3 in their current locations.



R: OK. Could you explain your thinking as to how you got those {pointed to the answers}?

S: For 8/12, I knew that 4, 8, and 12 would be 2/3. And it's not more than one. And so I just found…. .oh wait (erased 2/3 and wrote above 8/12). And so. Ok, I knew 2/3 and 8/12 were both equal, so I put those in the same spot. And so 8/3, I knew that 3, 6 sets would be more than two, but that would be 9. So I knew it would be 2 and 2/8 or 2 and 2/5 or 2 and 2/4. So I put it right there.

R: OK

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for th Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make bot cakes?

$1\frac{5}{12}$ cups of flour

$\frac{2}{3} = \frac{8}{12}$

$\frac{3}{4} = \frac{9}{12}$

$\frac{17}{12}$

S: Can I write on it?

R: Yea, OK.

R: Ok. Thank you.

Student 16

Opening Statement:
Researcher: a. “How do you ‘see’ or think of a number line?”

Student: Umm.. I think of it as… umm there’s a line and there’s …..numbers on it. And it umm… It helps you with your strategies.

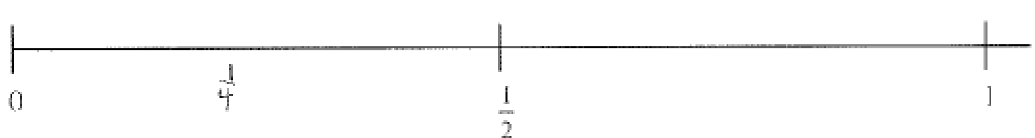
Researcher: Will you please draw one and include numbers 1, 2, and 3 on it?

2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.

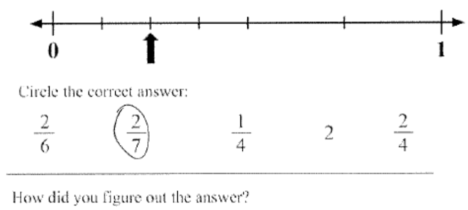
3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

4. a). Name a fraction that is less than ½. ________ ¼

b). Place your fraction as accurately as possible on the number line below.



5. Figure out what this point is called on the number line.

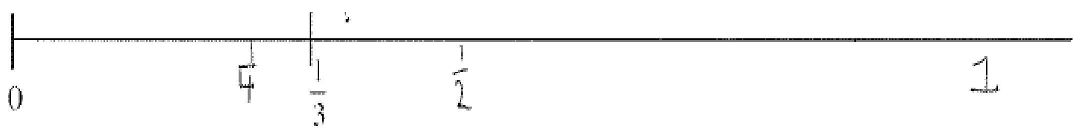


R: You don't have to write it. You can just tell me.

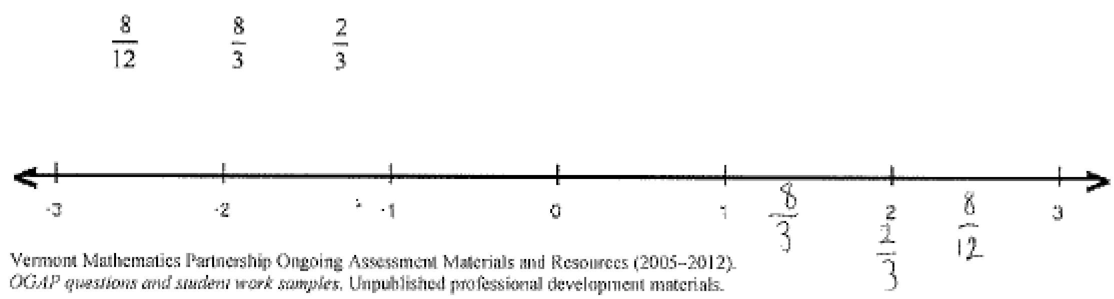
S: Ah, I saw that there were seven lines total. And it was on the second line.

R: Oh. So that's 2/7?

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.



R: OK. Could you explain your? {pointed to the answers}

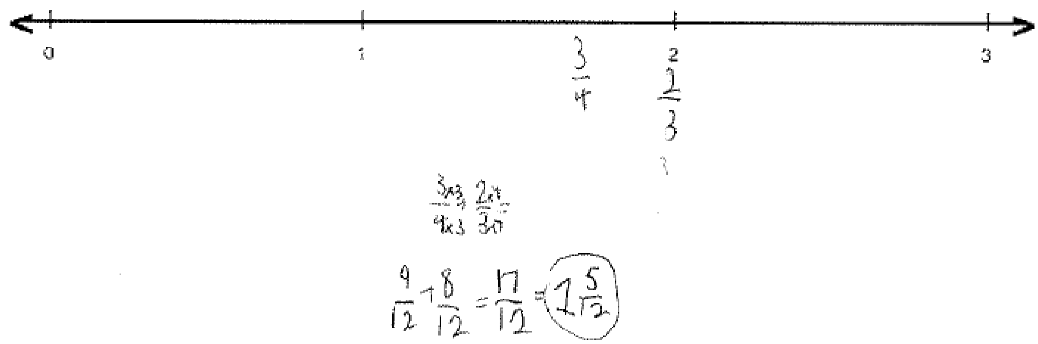
S: Uh, I knew that 8/12 could be converted to 4/6, and so I knew that could be converted to uh 2/3. And so I put 2/3 right there (pointed to it). And then I uh did 8/3 and I got a little confused on that one.

R: Oh 8/3 was a little confusing.

S: yes

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet Cake and 3/4 cup of flour for a pound cake. How much flour will she need to make both cakes?



R: Are you done?

S: Yes

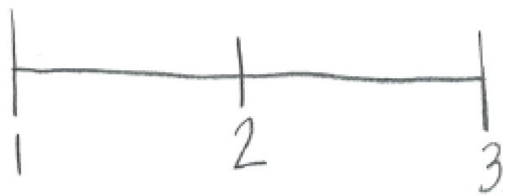
R: Ok. Thank you.

Student 17

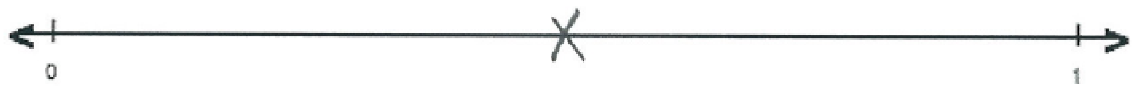
Opening Statement:
Researcher: a. "How do you 'see' or think of a number line?"

Student: Well I think of it as a helping strategy when you have like a really big…. Like with fractions, it could be a really big number line (inaudible).

Researcher: Will you please draw one and include numbers 1, 2, and 3 on it.



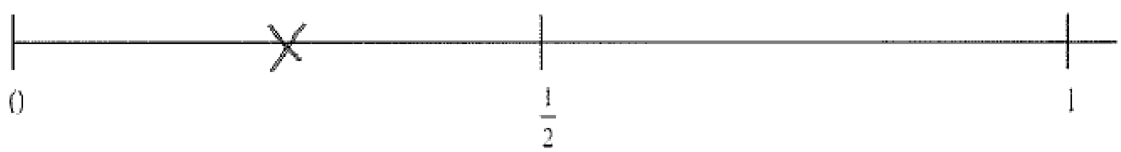
2. This number line shows 0 to 1. Put an (X) where the ½ would be on the number line below.



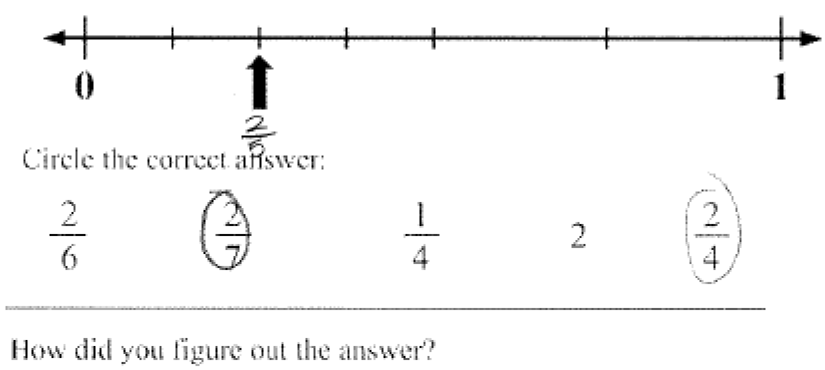
3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

4. a). Name a fraction that is less than ½. ________ ¼

b). Place your fraction as accurately as possible on the number line below.



5. Figure out what this point is called on the number line.



S: (Student wrote 2/5 underneath mark.)

R: (Reminded student to circle the correct answer).

S: Oops… I meant 2/4 (circled 2/4)

R: How did you figure out the answer?

S: Oh actually…. (erased 2/4 and circled 2/7)

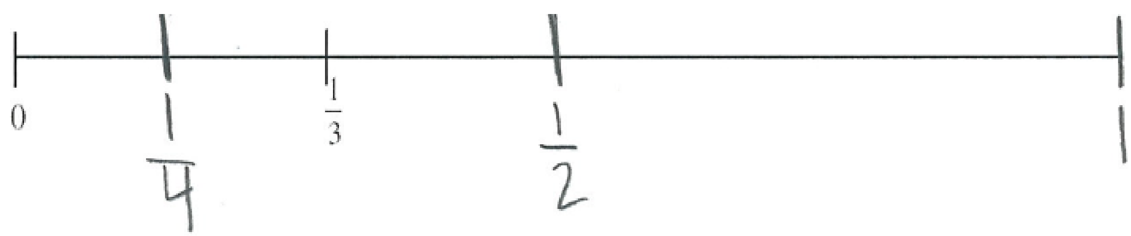
R: So how did you figure out the answer?

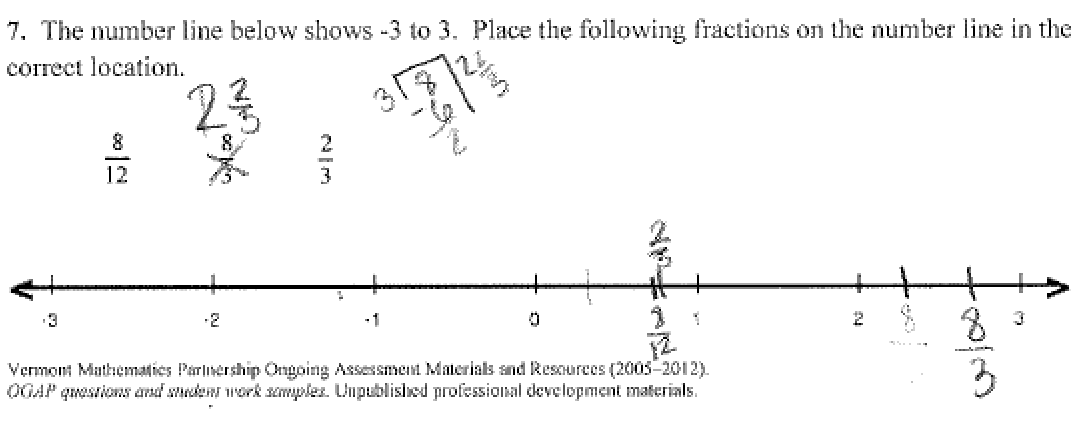
S: Do I need to write it down?

R: No you can just tell me.

S: Well I decided since these are this far apart (pointed to the first four hash marks one at a time) it wouldn't just stop there. So I decided it would be (pointed to where other hash marks would be) so I came up with 2/7.

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.




7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.

$\frac{8}{12}$ $\frac{8}{3}$ $\frac{2}{3}$

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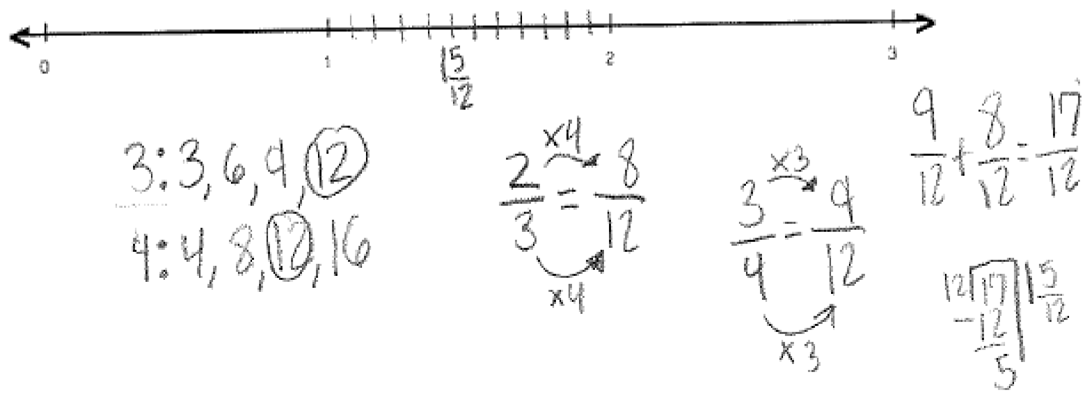
The student counted twelve imaginary lines and wrote 8/12. Next she solved 8 ÷3 with long division and found the quotient of 2 2/3. She then counted two imaginary lines and wrote 2/3 adjacent to 8/12. She corrected 8/3 to the current location.

R: OK. I can see your thinking.

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet Cake and 3/4 cup of flour for a pound cake. How much flour will she need to make both cakes?

Student drew twelve hash mark , counted over to 5, and wrote 1 5/12.



R: Are you done?

S: Yes

R: Ok. Thank you.

Student 18

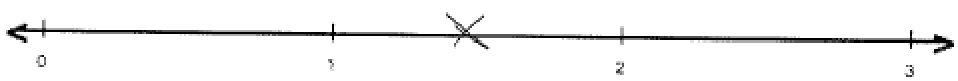
Opening Statement:
Researcher: a. "How do you 'see' or think of a number line?"

Student: Umm.. as…like a line with numbers ranging from a certain distance.

Researcher: Will you please draw one and include numbers 1, 2, and 3 on it.

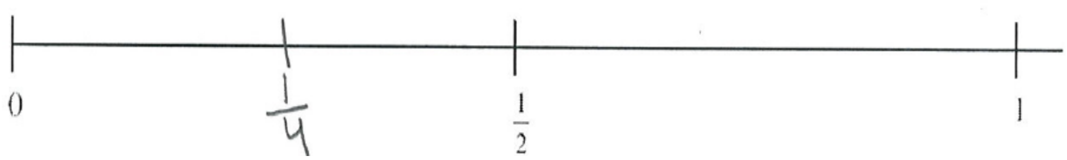
2. This number line shows 0 to 1. Put an (X) where the ½ would be on the number line below.

3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.



4. a). Name a fraction that is less than ½. ________ ¼

b). Place your fraction as accurately as possible on the number line below.



5. Figure out what this point is called on the number line.

Circle the correct answer:

$\frac{2}{6}$ $\frac{2}{7}$ $\frac{1}{4}$ 2 $\frac{2}{4}$

____

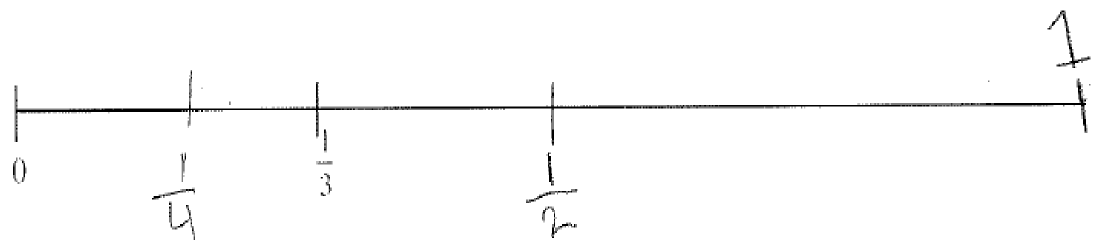
How did you figure out the answer?

S: Write it?

R: No, just tell me

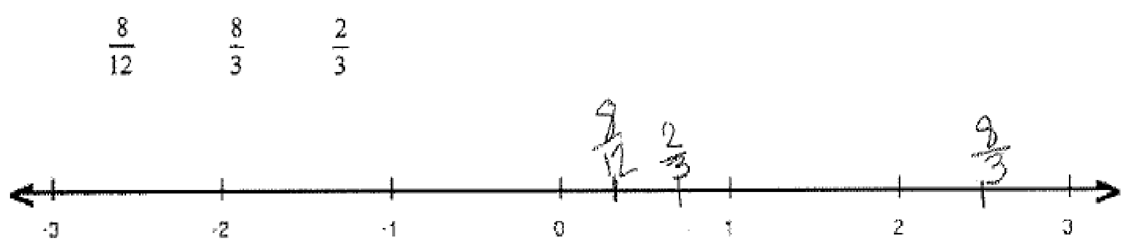
S: I counted the lines and the line that's pointed is 2 out of 6.

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



7. The number line below shows -3 to 3. Place the following fractions on the number line in the

correct location.



R: OK. Could you explain your thinking? How did you know where to place the fractions? {pointed to the answers}

S: Cause 8/3 is a mixed number so I did umm make it into a … into an improper fraction …So I got 2 and 2/3.

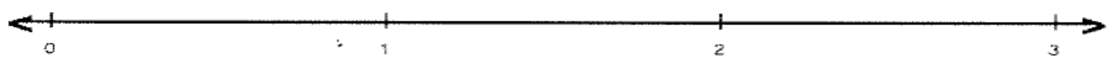
R: So how did you get those? (pointed to 8/12 and 2/3)

S: 2/3 is close to one and it only takes a third to get there. So I put it kind of close to the 1.
And 8/12 is uh..close to one also. It only needs four to go to twelve. So its one, two, three, four….(pointed from the 0 four spaces to 8/12)

R: Ok

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet Cake and 3/4 cup of flour for a pound cake. How much flour will she need to make both cakes?



$$\frac{2}{3} \overset{+}{\times} \frac{3}{4} = \frac{5}{12}$$

S: Do I use the number line? I don't know how to add on the number line.

R: Ok. Do it the best you know how.

Student 19

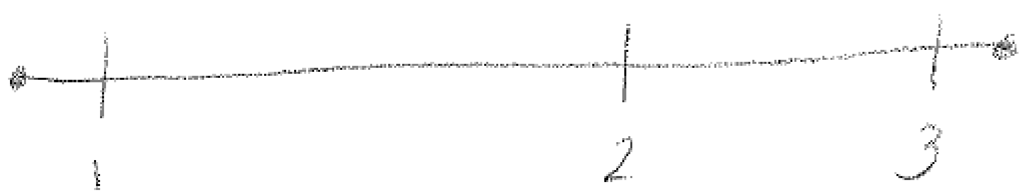
Opening Statement:
Researcher: a. "How do you 'see' or think of a number line?"

Student: What?

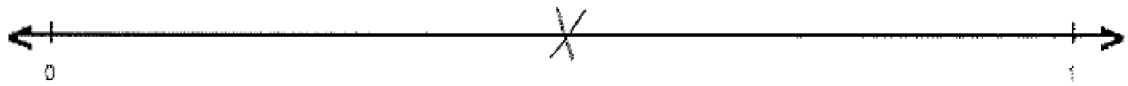
R: A number line.

S: Oh, I see it as a way to get back and forth to numbers…. out of numbers

Researcher: Will you please draw one and include numbers 1, 2, and 3 on it.



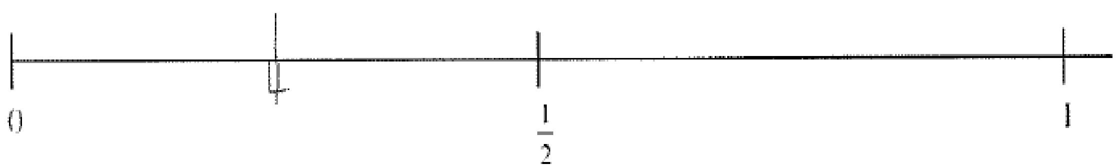
2. This number line shows 0 to 1. Put an (X) where the ½ would be on the number line below.



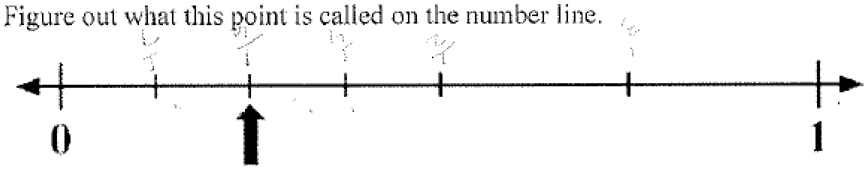
3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

4. a). Name a fraction that is less than ½. ________ ¼

b). Place your fraction as accurately as possible on the number line below.



5. Figure out what this point is called on the number line.



Circle the correct answer:

$\frac{2}{6}$ $\frac{2}{7}$ $\frac{1}{4}$ 2 $\frac{2}{4}$

How did you figure out the answer?

S: I figured out that this is one-third (pointed to the three-fourths marker), one-half (pointed to one-half marker), one…. (pointed to the three-eighths marker)…… I messed up.

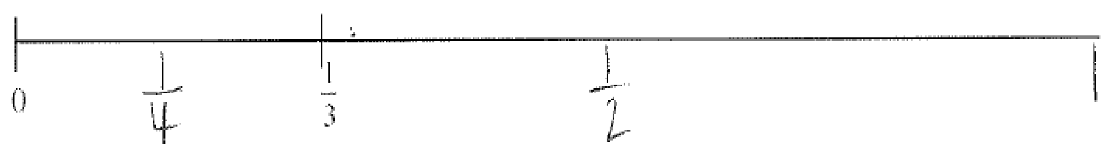
R: So you think this is one-third, one half (pointed to markers as he named)? (Continued to point at three-eighths marker.)

S: One-fourth (pointed to three –eighths location), one-sixth (pointed to one-fourth location), one seventh (pointed to one-eighth location).

R: So what do you think?

S: Its…. Or…. I think it is 2/4

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.

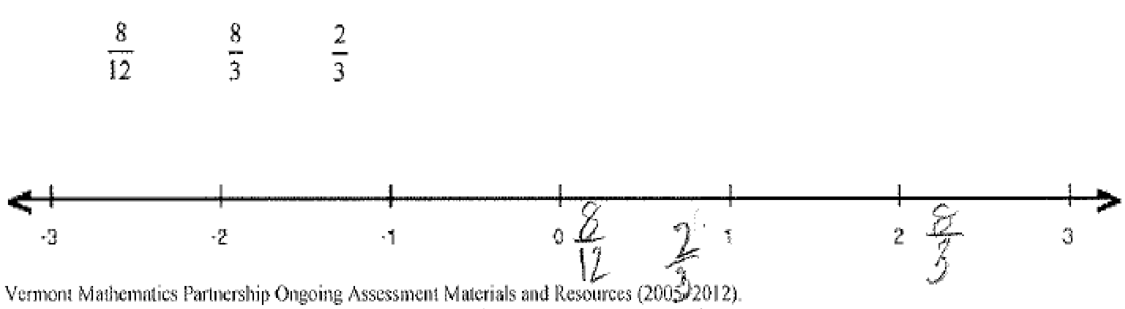


Student completed ¼ then, ½.

R: Where would the 1 be? (pointed to the 1)

S: Oh (added 1 to the number line).

7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.



Vermont Mathematics Partnership Ongoing Assessment Materials and Resources (2003-2012). *OGAP questions and student work samples*. Unpublished professional development materials.

The student immediately placed 8/12, 2/3, and 8/3 next to each other between 0 and 1.

R: OK. Could you explain….

S: Oh (erased 2/3 and 8/3).

R: Oh I'm sorry .

S: (Rewrote as seen above)

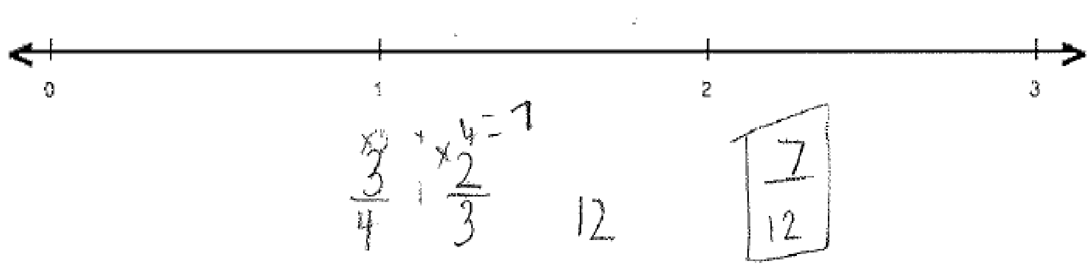
R: Could you explain your thinking as to how you knew where to place your fractions?

S: I knew that 8/12 is not a whole yet, so… and it's the lowest. And 2/3 is one away from one whole. So I put it right next to the 1. 8/3 is over a whole, it's over two wholes. So it's one away from three wholes.

R: OK

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet Cake and 3/4 cup of flour for a pound cake. How much flour will she need to make both cakes?



R: Are you done?

S: Yes

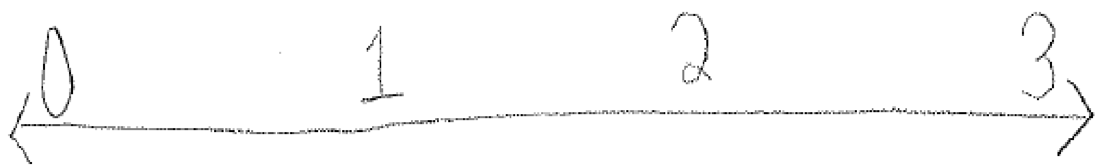
R: Ok. Thank you.

Student 20

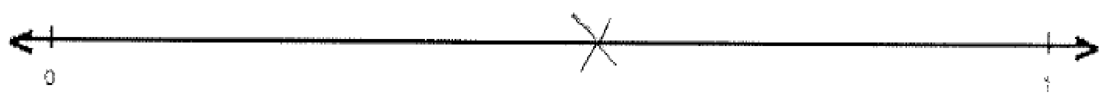
Opening Statement:
Researcher: a. “How do you ‘see’ or think of a number line?”

Student: Uh.. I think of two whole numbers and in between it there will be decimals and fractions.

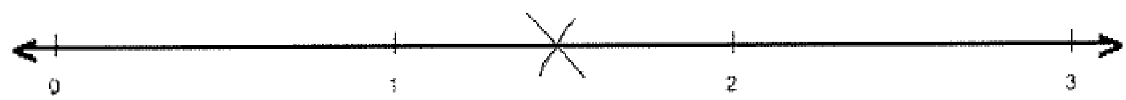
Researcher: Will you please draw one and include numbers 1, 2, and 3 on it.



2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.

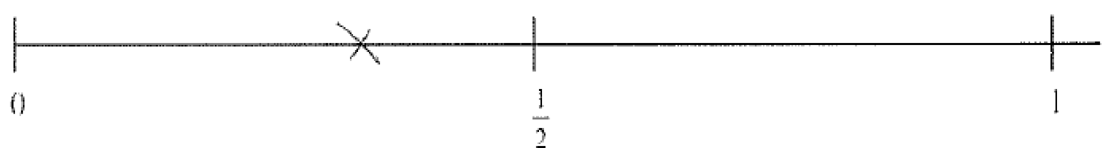


3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

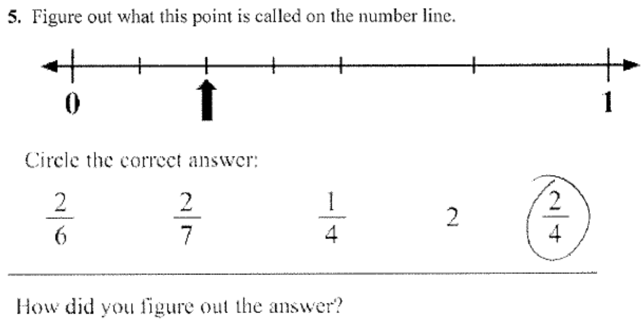


4. a). Name a fraction that is less than ½. _________ ¼

b). Place your fraction as accurately as possible on the number line below.



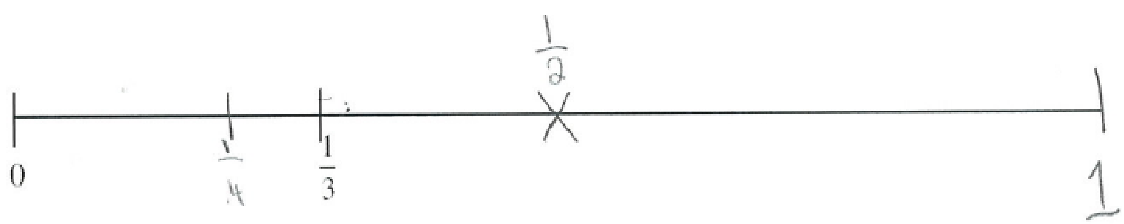
5. Figure out what this point is called on the number line.



S: (immediately circled 2/4) Well I saw that there were one sections, two sections, three sections, four sections, that would be half. So it was less than half. Then I just counted up how many sections. It gave me two.

R: Ok

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.

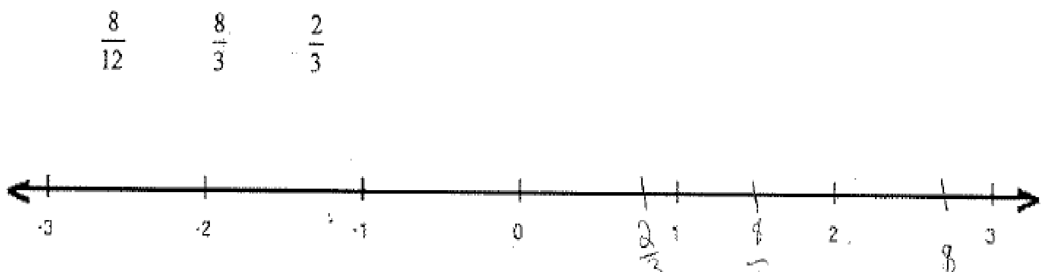


Student completed ½ then ¼.

R: What about this one (pointed to the 1)? Where would it be?

S: (inserted the 1)

7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.



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R: OK. Could you explain your thinking as to how you got those? {pointed to the answers}

S: Is this including the whole number line or negative to three or is it just including the positives?

R: Uh… you tell me

R: Could you explain your thinking how did you know where to place them?

S: Well I knew that it was an improper fraction since 8 is bigger than 3. So it's going to be 2 and two-thirds. So I started splitting them into thirds (pointed to the section between 2 and 3).

R: Oh. How did you know where to place those?

S: Well then I thought about…. I started thinking about if I should include from the point here (-3) to the point here (3). So I measured up how much would be a third (scanned from -3 to 1 for a third). I got a little more than this. I didn't know exactly where to place it. And then I have 8/12. I tried to split into twelfths.

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet Cake and 3/4 cup of flour for a pound cake. How much flour will she need to make both cakes?

$\frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$ cups of flour

R: Are you done?

S: Yes

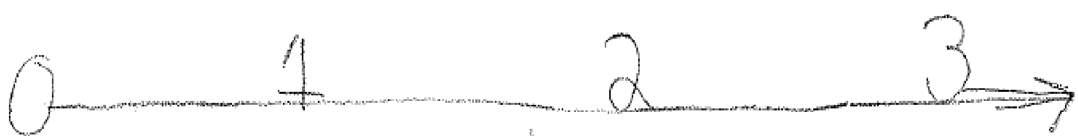
R: Ok. Thank you.

Student 21

Opening Statement:
Researcher: a. “How do you ‘see’ or think of a number line?”

Student: Umm.. I think of a scale between 0 and 1.

Researcher: Will you please draw one and include numbers 1, 2, and 3 on it.



2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.

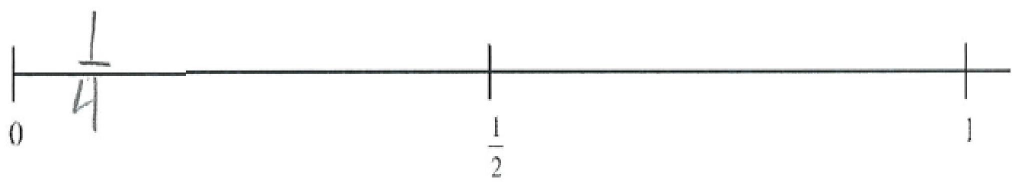
3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

4. a). Name a fraction that is less than ½. ________ ¼

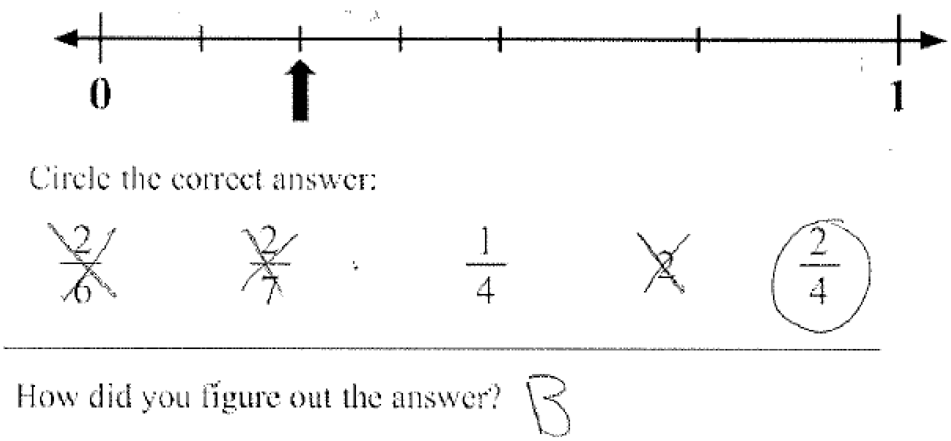
b). Place your fraction as accurately as possible on the number line below.

S: Does it half to be lowest?

R: It has to be less than ½.



5. Figure out what this point is called on the number line.



S: Can I 'X' out some of the answers.

R: It's up to you.
R: Ok. How did you figure that out?

S: (Started to write)

R: You can just tell me.

S: I had thought of it because I knew that 2 could be eliminated because two is not in this (pointed to the number line by scanning) And that 2 is a whole number.

And 2/7… 2/7 couldn't be… it's probably right here (pointed to a space right before number 1). I thought ¼ could be close to right here (pointed to the ¼ location). Because I thought this could have been 20 percent (pointed to the 1/8 location) and this would have been 25 (pointed to the ¼ location) But I had just thought 2/4 because 2/4 could be in this spot right here (pointed to the 3/8 location). And 2/4 is bigger than ¼.

R: Ok

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.

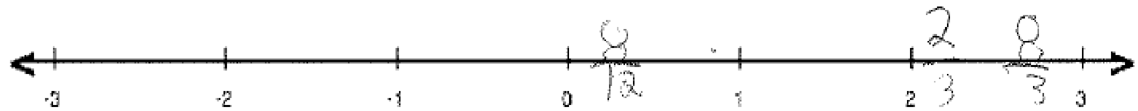
S: Does it have to be in between this (pointed to the space between 0 and 1/3)?

R: It can be wherever you think it should be.

7. The number line below shows -3 to 3. Place the following fractions on the number line in the

correct location.

$\frac{8}{12}$ $\frac{8}{3}$ $\frac{2}{3}$



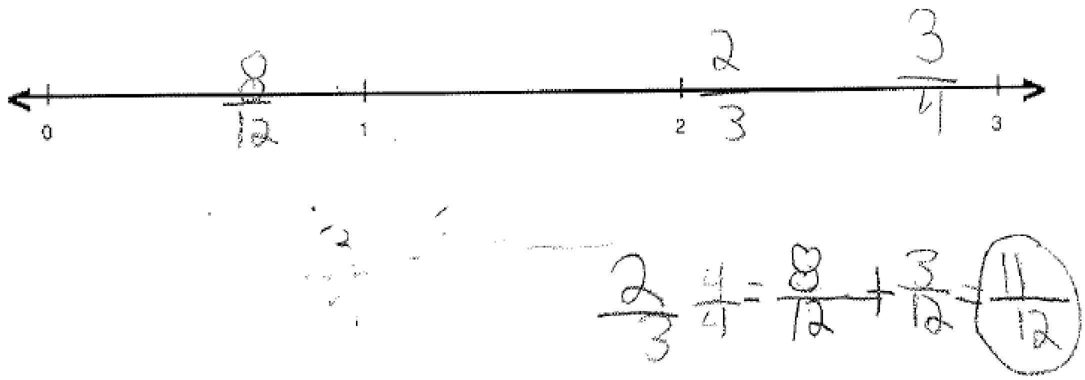
R: How did you know where to place your fractions?

S: Because I knew that all three of these fractions can't be where the negatives are because a negative plus a negative equals a positive and….. 2/3 is bigger than 8/3. But uh 8/3 can't be simplified. But 8/12 can. So I had to put that near 1… 1 and 0.   And 2/3 is close to two wholes.

R: OK

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet Cake and 3/4 cup of flour for a pound cake. How much flour will she need to make both cakes?



S: (stopped writing)

R: Are you done?

S: Oh no this problem is hard. (Passed completed paper to researcher.)

Student 22

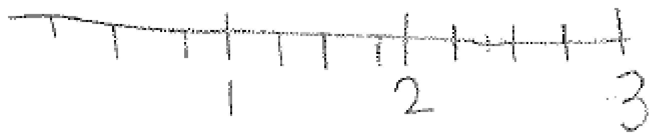
Opening Statement:
Researcher: a. "How do you 'see' or think of a number line?"

Student: What?

R: a. "How do you 'see' or think of a number line?"

S: I see it has like numbers, kind of like a ruler. Like numbers dash……. and some in the middle like one and a half in the middle.

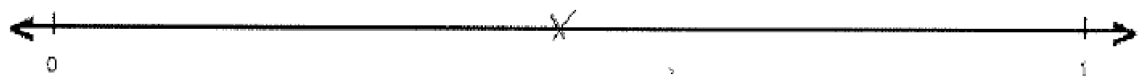
Researcher: Will you please draw one and include numbers 1, 2, and 3 on it.



S: It’s not perfect.

R: OK

2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.



S: like right here (placed x on the number line)

3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

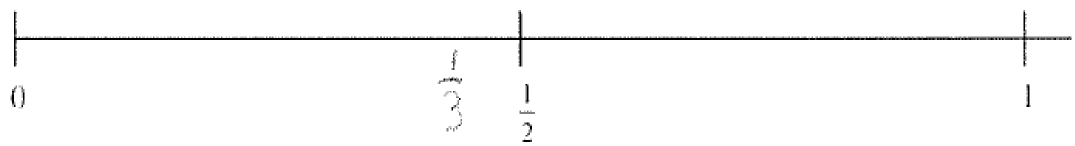
4. a). Name a fraction that is less than ½. ________ 1/3

b). Place your fraction as accurately as possible on the number line below.

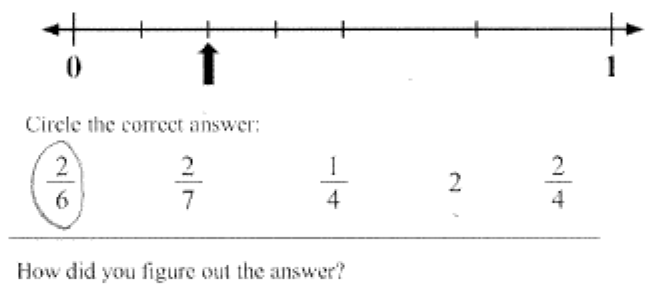
S: Like you put a dash and then you put a 1/3, kind of like this? (pointed to 1/3 as written)

R: Just put it as accurately as possible on the number line. Where do you think it would go?

S: Like right here (she inquired as she wrote)?



5. Figure out what this point is called on the number line.



S: Figure out…ok
This would be….. I know this would be like ½ of something (pointed to the half mark)
First figure out…. Tenths
I know it's not like the 2 wholes because it's over one
I don't think it's 2/7
2/4 would probably be closer to a half
I think it would be 2/6 (circled)

R: OK. How did you figure it out?

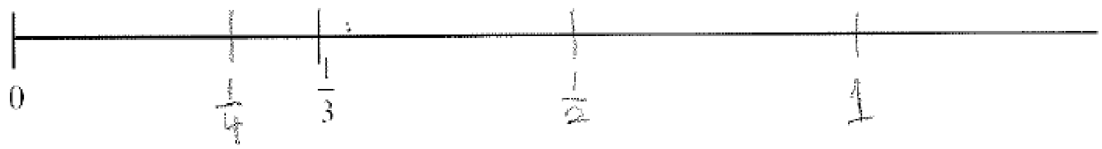
S: Well first I crossed out the wholes because it's over 1
And then 2/4 might be closer to a half umm
Same with ¼
And 2/7 maybe like….. 2/7 is smaller than a half by a lot…. 'cause it's so close to 1/10

R: OK

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.

S: 0 and 1/3 …. Accurately as possible
I think 1 is somewhere over here (pointed to the end of the line)

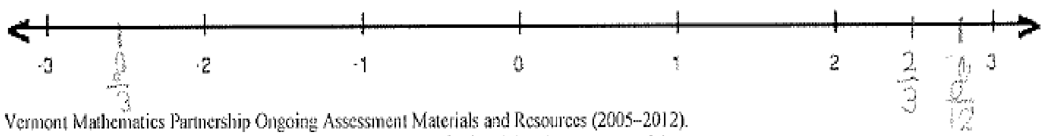
I'm not sure….. (inaudible)….half
A third is bigger than a fourth so….. (drew ¼ …… 1)



7. The number line below shows -3 to 3. Place the following fractions on the number line in the

correct location.

$\frac{8}{12}$ $\frac{8}{3}$ $\frac{2}{3}$



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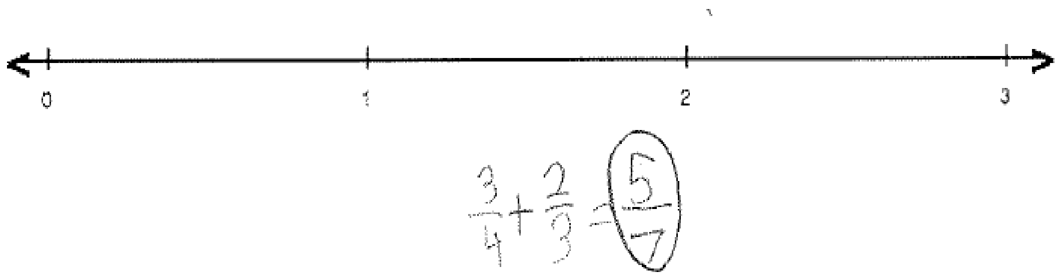
S: (stated fractions). 8/3 is an improper fraction so it may go like.. somewhere bigger………(pointed between 2 and 3) 8/12 and 2/3… 2/3 might be going somewhere in here …. Might (pointed between 2 and 3). (wrote 2/3)…..
Let's see I'm going to do 8/12 now…I know 8/12 simplified ….. is like umm….. 2/3
I'm thinking twelfths might be somewhere like (wrote 8/12)
8/3 is a little bit harder… I think I can get it….. (wrote 8/3)
Ok

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet Cake and 3/4 cup of flour for a pound cake. How much flour will she need to make both cakes?



S: This is a hard problem. I think I might know this…. Might…Do I like do it at the top or the bottom or use the number line?

R: Do it however you want to do it. Are you done?

S: Is it whole numbers or fractions

R: You have to tell me

S: man (wrote answer)

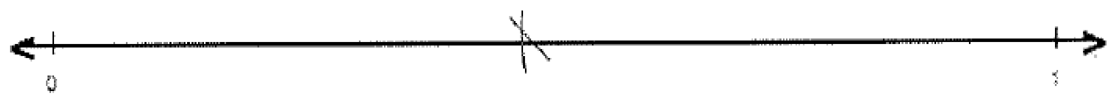
Student 23

Opening Statement:
Researcher: a. "How do you 'see' or think of a number line?"

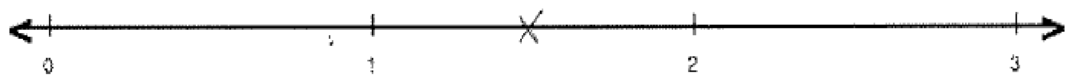
S: Umm the smaller number is on the left hand side and the bigger numbers are on the right hand side.

Researcher: Will you please draw one and include numbers 1, 2, and 3 on it.

2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.

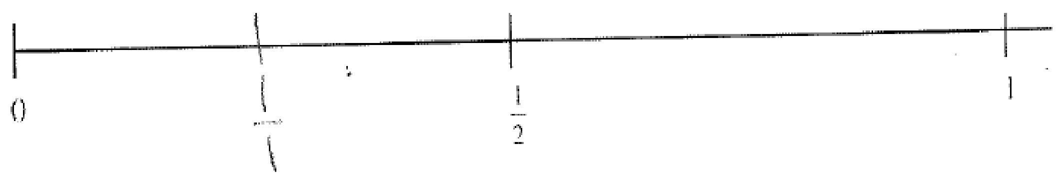


3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

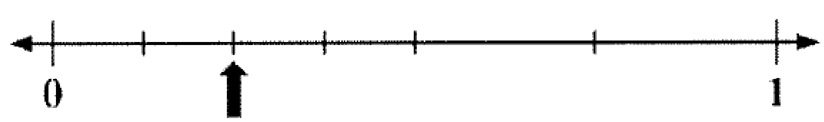


4. a). Name a fraction that is less than ½. ________ 1/1 (one-first)

b). Place your fraction as accurately as possible on the number line below.



5. Figure out what this point is called on the number line.



Circle the correct answer:

$\frac{2}{6}$ $\frac{2}{7}$ $\frac{1}{4}$ 2 $\frac{2}{4}$

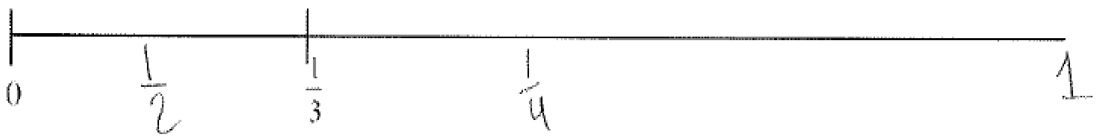
How did you figure out the answer?

S: Wait, where do you want me to write it?

R: You just circle the correct answer.

S: Umm there are seven uh little umm lines (traced first two hash marks) and it places where one of the numbers is …..and there it goes (pointed to first two hash marks) 2 and here is 7…. So 2/7

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



S: Can I erase? (erased ½ from behind 1/3)

R: Sure

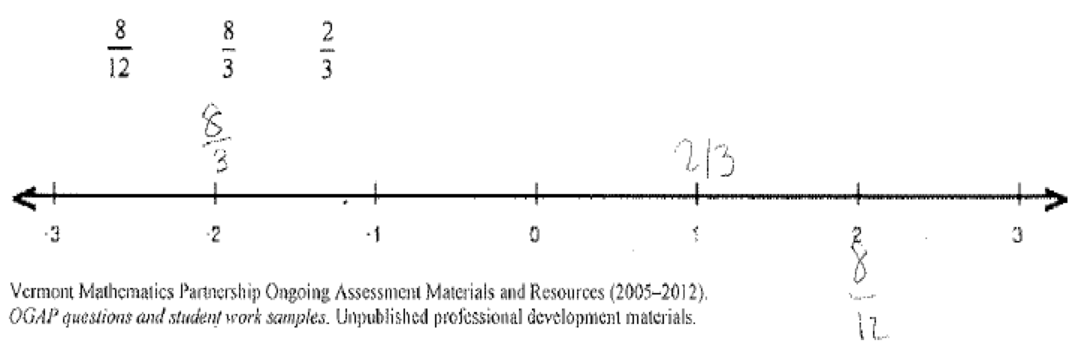
R: What about that one? (pointed to the 1) Where would you put that number?

S: OK

Student wrote number 1.

7. The number line below shows -3 to 3. Place the following fractions on the number line in the

correct location.



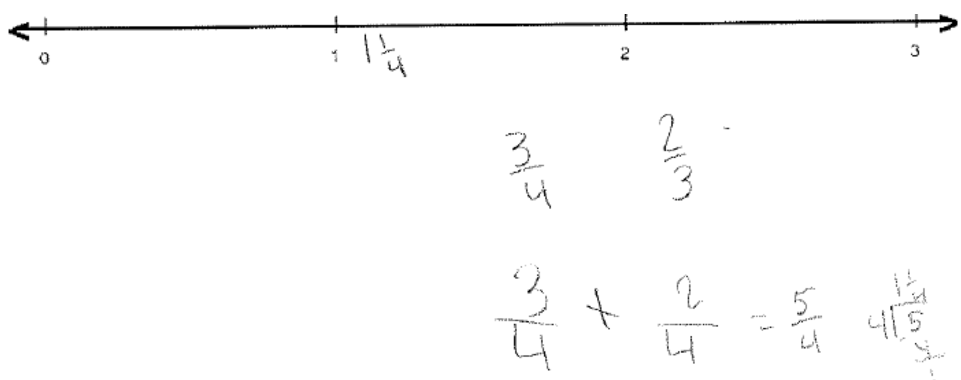
R: OK. Could you explain your thinking as to how you got those {pointed to the answers}?

S: Umm, I…. did …. 8/12 is probably equal to or if you're… if 8/12 is equal to 4/6 and … oh I did it wrong it should be right there (pointed to 2/3 that he had written). And umm

umm 1, 2, 3, 4, 5, 6, 7, (pointed to each hash mark) there's seven. And I did … I put 8/3 right there because…. umm three will go into 8 two times with a remainder of 2. And I thought that two would be the right answer because there's two and …. I don't know.

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet Cake and 3/4 cup of flour for a pound cake. How much flour will she need to make both cakes?



S: Umm. Can I like solve it?

R: Yes

R: Are you done?

S: Yes

R: Ok. Thank you.

Student 24

Opening Statement:
Researcher: a. "How do you 'see' or think of a number line?"

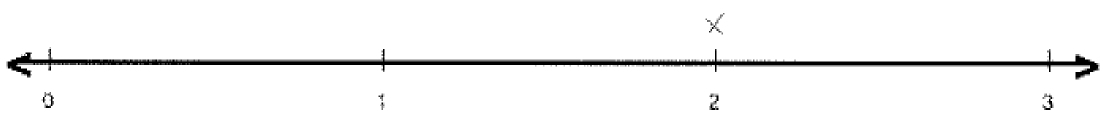
S: It places the numbers in order

Researcher: Will you please draw one and include numbers 1, 2, and 3 on it.

2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.

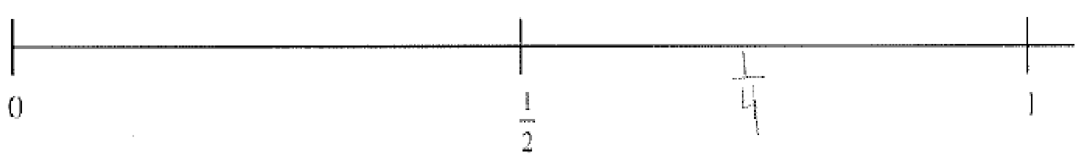
S: Like right here (placed x on the number line)

3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

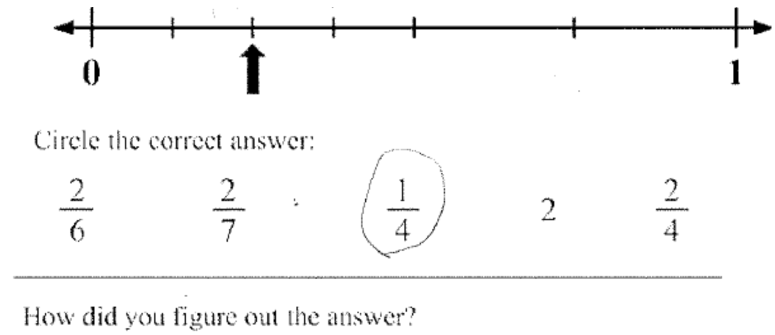


4. a). Name a fraction that is less than ½. _________ ¼

b). Place your fraction as accurately as possible on the number line below.



5. Figure out what this point is called on the number line.



S: Because that's one-half so underneath… so it's less than one - half.

R: OK. Where is one-half again?

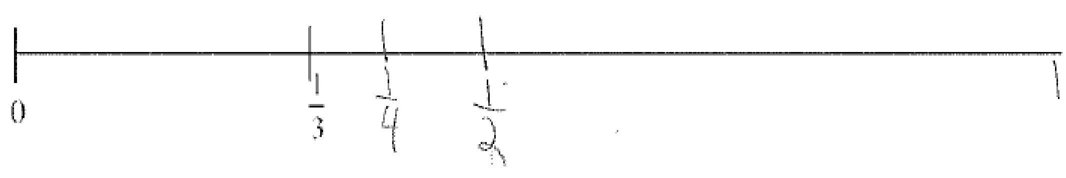
S: There (pointed to 3/8 location).

R: Ok so this is one- half. How did you get that?

S: Because it's less than one-half.

R: Ok

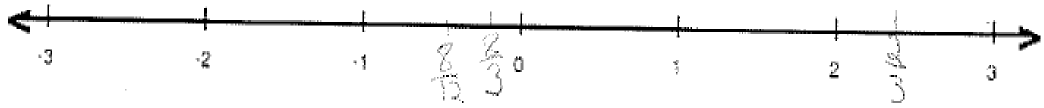
6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



Student erased the original ¼ and ½ to move closer to 1/3.

7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.

$\frac{8}{12}$ $\frac{8}{3}$ $\frac{2}{3}$



R: OK. Could you explain your thinking as to how you got those? {pointed to the answers}

S: 8/12 would be less than 1… and 8/3 would be less than 1… (pointed to 1)

And 2/3 would be more than 2 (pointed to 2).

R: OK.

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet Cake and 3/4 cup of flour for a pound cake. How much flour will she need to make both cakes?

$$\frac{2}{3}+\frac{3}{4}=\frac{18}{12}=1\frac{6}{12}$$

R: Are you done?

S: Yes

R: Ok. Thank you.

Student 25

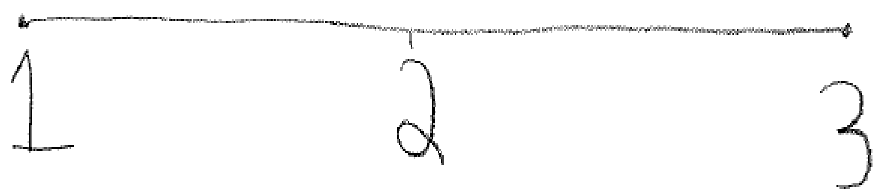
Opening Statement:
Researcher: a. “How do you ‘see’ or think of a number line?”

Student: What?

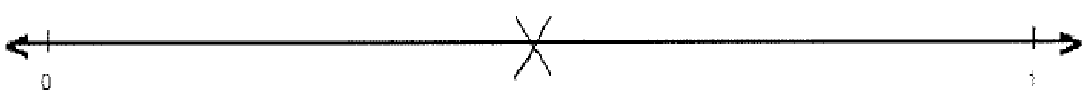
R: “How do you ‘see’ or think of a number line?”

S: I think of a number line as like.. a line with numbers between 0 to 1 and like 1 and 2.

Researcher: Will you please draw one and include numbers 1, 2, and 3 on it.



2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.



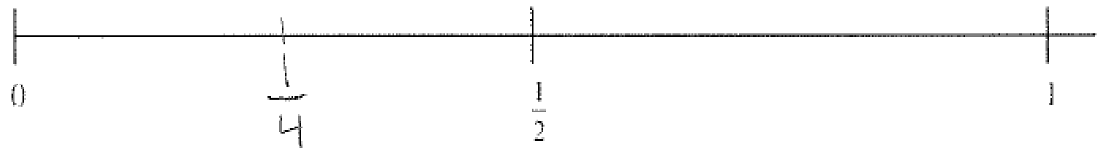
S: Just like right there?

R: Yes, put an X.

3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

4. a). Name a fraction that is less than ½. ________ one-fourths

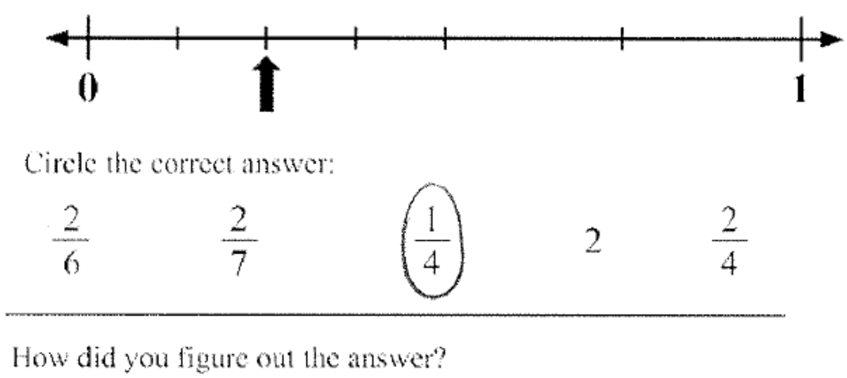
b). Place your fraction as accurately as possible on the number line below.



S: One-fourths?

R: Yes.

5. Figure out what this point is called on the number line.



S: (Circled 2/6)

R: How did you figure out your answer?

S: Oh no, I want to change my answer (erased the circle from 2/6…… circled ¼).

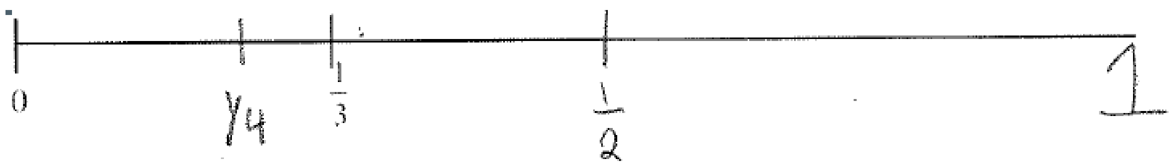
R: OK. How did you get that?

S: (started to write)

R: You can just tell me.

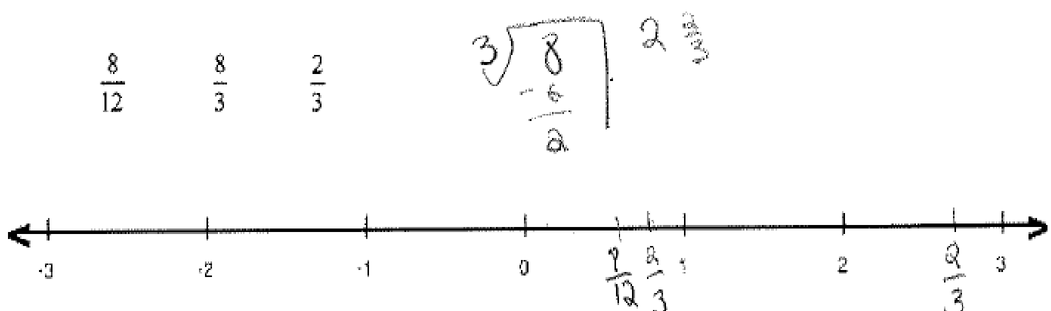
S: Well I divided ……(inaudible) I divided like a number line into four groups (pointed and counted four equal sections) and so the point right there is ¼.

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.



7. The number line below shows -3 to 3. Place the following fractions on the number line in the

correct location.



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R: OK. Could you explain your thinking as to how you got those {pointed to the answers}?

S: For 8/3?

R: Well 8/12…. I just divided 1, 2, …..0 to 1 into like 12 small groups twelfths (partitioned imaginary lines in between 0 and 1). So then I put the eight 1/12's (inaudible). And then for 2/3, I divided it into like three groups…. so. And the 8/3, I noticed it was an improper fraction. So I changed that into a mixed number. And let's see… can I write ….making sure I got it right… I'm not that good at mental stuff…. (Student solved the division problem 8/3…… erased 8/3 and moved it from in front approximately 1 ½ location and changed it to 2/3). Because it's 2 and 2/3.

R: Ok. So I noticed that you changed this to read 2/3, why did you do that?

S: What?

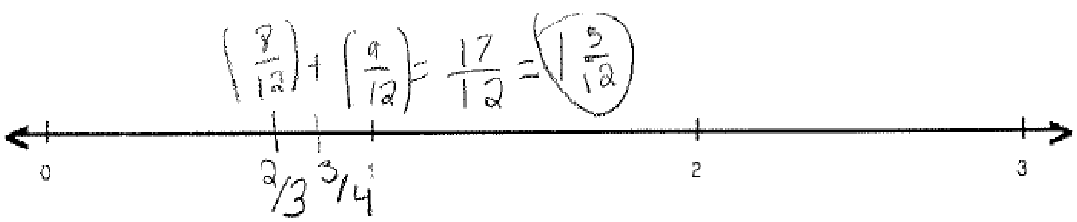
R: Why did you name that 2/3?

S: Because the answer was 2 and two-thirds. So there's the 2 (pointed to the 2) and there's the 2/3 (pointed to her written response).

R: Ok

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet Cake and 3/4 cup of flour for a pound cake. How much flour will she need to make both cakes?



S: I'm done.

R: Ok. Thank you.

Student 26

Opening Statement:
Researcher: a. “How do you ‘see’ or think of a number line?”

S: Like a line with numbers on it and you can use it to subtract back or up to find an answer.

Researcher: Will you please draw one and include numbers 1, 2, and 3 on it.

2. This number line shows 0 to 1.   Put an (X) where the ½ would be on the number line below.

S: Like right here? (Placed x on the number line.)

3. This number line shows 0 to 3. Put an (X) where the ½ would be on the number line below.

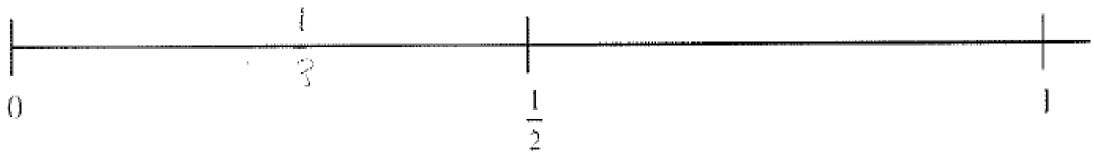
R: Tell me how did you know to put it there.

S: Because one-half is after 1.

R: OK.

4. a). Name a fraction that is less than ½. ________ 1/3

b). Place your fraction as accurately as possible on the number line below.

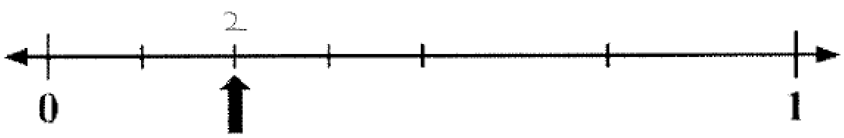


R: How did you know to put 1/3 there?

S: Well it would be 0, 1, 2 and then 1/3. (pointed to spaces on the number line)

R: Oh. OK.

5. Figure out what this point is called on the number line.



Circle the correct answer:

$\frac{2}{6}$ $\frac{2}{7}$ $\frac{1}{4}$ 2 $\frac{2}{4}$

How did you figure out the answer?

S: 2

R: OK. Would you circle the correct answer?

S: (circled 2)

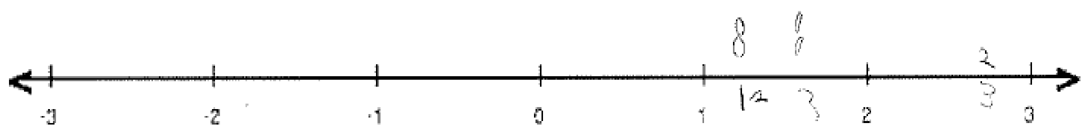
R: How did you figure that out?

S: 0, 1, 2 (pointed to hash marks on the number line).

6. The number line shows 0 and 1/3. Put ½, ¼, and 1 as accurately as possible on the number line below.

7. The number line below shows -3 to 3. Place the following fractions on the number line in the correct location.

$\frac{8}{12}$ $\frac{8}{3}$ $\frac{2}{3}$



R: How did you know to put those there?

S: 8/12 is the smallest and then 8/3 is kinder larger and 2/3 is the largest.

R: OK

8. Use the number line below to solve the following problem.

Alexis wants to bake two more cakes for the school's bake sale. She needs 2/3 cup of flour for the Red Velvet cake and ¾ cup of flour for a pound cake. How much flour will she need to make both cakes? $\frac{2}{3} + \frac{3}{4}$ $\frac{5}{7}$

R: Are you done?

S: Yes.

R: Ok. Thank you.